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Nurse Rostering via Mixed-Integer Programming

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Abstract. Nurse scheduling is a topic widely studied due to its overall effect on patient care and hospital performance. This research focuses on formulating a Mixed Integer Programming (MIP) workforce scheduling model as a nurse rostering problem. The model incorporates multiple objectives of individual nurse preference and qualification. This is approached by categorizing nurses into different hierarchical levels based on their qualifications and positions; Head Nurse, Senior Nurse, Nurse, and Assistant Nurse. Moreover, each nurse's holiday preferences are accounted for in the model. The proposed MIP model is solved to generate a schedule to meet hospital demand and individual nurse preferences. A MIP solver using Python 3 was used to find the optimal solution by cutting planes. The sensitivity analysis and computational results reflect different scenarios and scheduling to fit all hospital environments and demands.

Keywords. MIP, Scheduling, Nurse Roster, Healthcare

1. Introduction

The challenges of providing high-quality healthcare arise due to the increasing healthcare costs of material and staff salaries. Therefore, advanced decision-making techniques are studied to minimize expenses. One of the crucial factors that needs special attention is the scheduling of nurses. Nurses in hospitals are essential medical professionals that significantly affect patient care and overall hospital performance. Therefore, optimizing nurse-related issues is widely studied in the literature. They consist of scheduling individuals over a given time horizon to fit hospital nurse demand. Throughout the week, nurses can experience physical and psychological fatigue. Several issues must be addressed when setting up a weekly schedule, such as individual nurse preference, minimum working hours, and maximum working hours. In the next section, a literature review on nurse scheduling is done to study these different factors and form a Mixed Integer Programming (MIP) model.

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The main objectives of this research are as follows. First, we focus on scheduling nurses and assigning nurses according to their qualifications and roles. To our knowledge, previous research generally focuses on a homogeneous group of nurses rather than creating a detailed schedule for each nurse based on their qualifications, planned vacations, and working hours preferences. The proposed MIP model explores the trade-off between nurse demand, preferences, qualifications, and working hours. Using the MIP proposed in Section 4 on the generated data, nurse efficiency can be improved and better service levels obtained. Based on the constraints, the model should positively impact nurses' schedules to meet their requests and the hospital demand. We also examine the performance of the model for different parameters settings. This research concludes that minimizing overtime and meeting nurse preferences positively impact hospital performance.

The remainder of the paper is organized as follows. Section 2 gives an overview of how nursing problems have been approached in the literature and defines the contribution. Then Section 3 introduces the problem statement, the scope of the paper, and the model assumptions. Section 4 describes the MIP formulation. Section 5 presents the data used in this investigation and the solution approach. The computational results and outcomes are also discussed. Finally, Section 6 summarizes the findings, limitations, and future work.

2. Literature Review

The review examines MIP models that address nurse scheduling problems, which are typically formulated with an objective function aimed at either maximizing or minimizing certain factors. Maximization is typically used to prioritize nurse performance preferences, while minimization typically targets various system costs.

Effective management of the healthcare system is highly dependent on proper scheduling of nurses. Once the scheduling model is formulated, various methods are available to obtain the optimal schedule. Guessoum et al. [1] propose a two-phase model with predefined constraints for nurse rostering. In the first phase, a generic variable fixing heuristic is utilized to form a second sub-problem, which, in its turn, can be solved by a general-purpose MIP solver. This leads to a swift and efficient optimization approach for nurse rostering. However, it is crucial to consider the individual preferences of nurses as a significant factor [2]. This can only be accomplished by planning a sufficient number of nurses fairly for each duty. Additionally, the qualifications of the employees must be considered when creating MIP models and generating schedules. A separate study explores the integration of various nurse qualifications and competencies in a MIP-based nurse scheduling problem [3]. The model was validated and solved using actual data from a hospital, where it was successful in meeting the hospital's demand.

Overall, effectively managing nurses and their schedules has a significant impact on nurse productivity. Another MIP model was developed and solved to enhance nurse productivity, using extensive datasets from a Chinese hospital [4]. The study demonstrated that there is a trade-off between performance criteria. Moreover, different constraints must be defined as either hard or soft for a well-defined nurse roster. The models presented in this study are discussed in research that addresses the International Nurse Rostering Competition [5], where some of the essential scheduling constraints for nurses and healthcare providers are utilized in our research. Additionally, another recent work [6] has focused on modifying typical constraints found in the literature to promote the concept of balanced work routines for all nurses. The authors present a sophisticated and intricate schedule arrangement that aligns with the actual requirements of hospital units.

On the application side, Belin et al. [7] integrate nurse scheduling and operating room scheduling using an MIP model. The proposed branch-and-price approach uses column generation to find the exact solution. Therefore, they were able to achieve promising cost savings results with a large number of computational experiments. Similarly, Maenhout2010 et al. [8] used the branch-and-price approach for a different MIP model. Another interesting set of studies integrates nurse scheduling with patient availability. Schoenfelder et al. [9] consider scheduling changes and unit assignments of cross-trained float nurses, where patient availability is an uncertain parameter. Adyas et al. [10](2021), in their turn, use historical data to estimate non-stationary patient arrivals. The resulting dynamic nurse staffing policy had 3% cost savings compared to fixed staffing policies.

Nurse scheduling problems have also been upgraded to be solved by metaheuristic optimization algorithms. A genetic algorithm method was used to obtain the optimal nurse schedule [11]. Two-point crossover and random mutation were used. Eventually, the schedule was compared to an existing hospital schedule, resulting in savings of 12% and 13% in staffing expenses per month and the number of nurses, respectively. A recent review, [12], provides a comprehensive exploration of potential research prospects on nurse scheduling. Mathematical models are solved using math-heuristics and other advanced methods. The authors also emphasize the importance of operational research for healthcare communities. Therefore, this also highlights the importance of nurse scheduling problems and their evolution for future research to aid in managing nurses efficiently.

The presented research focuses on formulating an MIP model for a nurse scheduling problem. The initial objective is to minimize the costs associated with nurse working hours. This objective includes quantitative measures and results in lower costs. In addition, qualitative measures are included in this research to increase healthcare quality. According to Maenhout et al. [8], during nursing personnel scheduling, the head nurses often face problems when there are multiple goals and priorities. Therefore, this research focuses on scheduling nurses considering their qualifications and their vacation preferences. The problem formulated is solved to make it easy and efficient to find the optimal schedule.

3. Problem Definition

In this article, we consider a hospital with a set of nurses. Each nurse has a different position depending on her or his qualifications. According to Bester et al. [13] research, there are six different nurse positions with decreasing qualifications. These are: Head Nurse, Senior Nurse, Nurse, Staff Nurse, Senior Nurse Assistant, and Nurse Assistant. In the presented research, these categories are abbreviated into four: Head Nurse, Senior Nurse, Nurse, and Nurse Assistant. The hospital has a specific demand for different nurses on every shift. However, to maintain the best care provided by the hospital, the preference for the working hours of the nurses should also be considered. Therefore, a schedule must be generated that considers the hospital's demand, nurses' qualifications, and nurses' preferences.

This is done by formulating and solving an MIP model. The main objective of the model is to minimize the total allocations of nurse slots. Several assumptions are considered to form the objective function and the set of constraints illustrated in Section 4. The following is a summary of some of the assumptions.

1. Nurses are classified into different positions according to their qualifications.
2. There are four types of nurse positions in a hospital (Head Nurse, Senior Nurse, Nurse, and Nurse Assistant).
3. There is a demand for nurses who have different qualifications.
4. Nurses enter their working hour availability and holiday preference.
5. A nurse cannot exceed the maximum daily working hours.
6. A nurse must meet the minimum daily working hours requirement.
7. Nurses have maximum overtime hours they can not exceed.
8. The created schedule is based on the results of the MIP model.

4. Mathematical Formulation

The defined problem is formulated as an MIP mathematical model. The model consists of indices, parameters, decision variables, an objective function, and constraints. The optimal solution that meets hospital demand is obtained, and a nurse schedule is developed depending on the model's time span. The model is then solved to form an optimum schedule for nurses to meet the hospital's demand.

First, the model indices are the set of nurses, the shift times, and the days. Moreover, a set of pre-defined parameters is listed below.

Indices:

n – nurse index ($n = 1, \ldots, N$),
q – qualification index ($q = 1, \ldots, 4$),
t – time index ($t = 0, \ldots, T$),
d – day index ($d = 0, \ldots, D$).

Parameters:

$De_{t,d}$ – demand for nurses at time t on day d,
α_n – minimum working hours for nurse $n \in N$ per week,
β_n – maximum working hours for nurse $n \in N$ per week,
δ – minimum working hours for a nurse per day,
π – maximum working hours for nurse per day,
τ – maximum overtime hours,
σ – maximum shortage hours,
$\gamma_{n,d}$ – preferred number of hours for nurse $n \in N$ to work on day $d \in D$,
λ_q – demand for nurses with qualification $q \in Q$,
$P_{q,n}$ – binary indicator that nurse $n \in N$ has the qualification $q \in Q$.

For this model, we expect an outcome telling us the shifts each nurse is assigned to. Hence, the two main binary decision variables are defined. The first, $X_{n,t,d}$, indicates whether nurse n works at time t on day d. The second, $Y_{n,t,d}$, indicates that defines the time each nurse starts. The decision variables, model objective function, and constraints are illustrated below.

Decision variables:

$$X_{n,t,d} = \begin{cases} 1 & \text{Nurse } n \text{ works at time } t \text{ on day } d \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{n,t,d} = \begin{cases} 1 & \text{Nurse } n \text{ begins work at time } t \text{ on day } d \\ 0 & \text{otherwise} \end{cases}$$

Mixed Integer Programming Model formulation:

$$\min \sum_{n\in N}\sum_{t\in T}\sum_{d\in D} X_{n,t,d} \tag{1}$$

$$\text{s.t.} \sum_{n\in N} X_{n,t,d} \geq De_{t,d} \qquad \forall t \in T, d \in D \tag{2}$$

$$\sum_{t\in T} Y_{n,t,d} \leq 1 \qquad \forall n \in N, d \in D \tag{3}$$

$$Y_{n,t,d} \geq X_{n,t,d} - X_{n,t-1,d} \qquad \forall n \in N, t \in T, d \in D \tag{4}$$

$$\sum_{n\in N} P_{q,n} X_{n,t,d} \geq \lambda_q \qquad \forall q \in Q, t \in T, d \in D \tag{5}$$

$$\sum_{t\in T} X_{n,t,d} \leq 2\pi \qquad \forall n \in N, d \in D \tag{6}$$

$$\sum_{t\in T} X_{n,t,d} \geq 2\delta \sum_{t\in T} Y_{n,t,d} \qquad \forall n \in N, d \in D \tag{7}$$

$$\sum_{t\in T}\sum_{d\in D} X_{n,t,d} \leq 2(\beta_n + \tau) \qquad \forall n \in N \tag{8}$$

$$\sum_{t\in T}\sum_{d\in D} X_{n,t,d} \geq 2(\alpha_n - \sigma) \qquad \forall n \in N \tag{9}$$

$$\sum_{t\in T} X_{n,t,d} \leq 2\gamma_{n,d} \qquad \forall n \in N, d \in D \tag{10}$$

The objective function (1) is defined to minimize the total number of slots to which nurses are assigned. A set of MIP constraints composed to enforce the demand and fairness requirements. Constraints (2) guarantee that general hospital demand is met and that there are enough nurses available. Constraints (3) ensure that nurses' shifts start at most once a day. Constraints (4) connect the beginning of nurses' shifts to the consequent time slots. Constraints (5) require that there be enough nurses for each qualification q. Constraints (6) ensure that nurses work a maximum of π hours per day. Since the slots in the model are 30 minutes each, the right-hand side of some of the constraints is doubled. Constraints (7) take care of the minimum working hours per day, assuming that nurse n starts working that day. Constraints (8) and (9) consider the entire week and the minimum and maximum overtime and shortage hours that each nurse is allowed to take. Finally, constraints (10) take into account the preferences of each nurse in working hours for each day.

5. Discussion of results

5.1. Input Data

The data used in this research are based on available in literature and on logical assumptions. Initially, it is assumed that there are 18 nurses available in a specific department. Two are qualified as nurse assistants, and ten are nurses. Only 4 and 2 are qualified to be senior and head nurses, respectively. In addition, Excel generates random numbers that reflect the preferences of each nurse for the working days and the number of hours. However, hospital demand may differ from one department to another [13]. The four categories of nurses have a demand that fluctuates between 1-4 nurses per shift. Hence, using this data, a demanding schedule may be generated. Initially, Six days of the week are managed; however, this value may change based on the scheduling requirements. Moreover, each nurse's maximum and minimum working hours per week are set up like a typical 48 hours per week working schedule, with a maximum of 60 hours and a minimum of 48.

5.2. Solving Approach and Results

As discussed earlier, MIP models can be solved in multiple ways, such as branch-and-price, column generation, different (meta)heuristics, etc. In this study, Gurobi Optimizer 9.1.2 was used to solve the proposed MIP model. Although it is an automatic solver that uses libraries to solve the model, it is fast and efficient. The solver uses cutting planes within minutes to find an optimal solution for the generated data. The first set of data resulted in a minimum solution of 1728. This optimal solution implies the costs incurred by nurses. The slots covered are the nurse timings, and their sum is the slots covered by the nurses all week. After the optimal solution is found, the decision variables, X and Y, are used to generate a plan for all nurses. Figure 2 illustrates a schedule generated after solving the MIP model. The schedule includes all four categories and the timing of each nurse and shift.

Different scenarios are applied to change the roster schedule for further analysis and the optimal solution and schedule are compared. The first case study examines how the number of nurses hired affects the optimal solution and the increase in costs. There is a continuous increase in total cost as the number of nurses is hired. This is illustrated in the plot in Figure 1.

The following case study studies the relationship between demand per shift and total cost. Two types of shift demands are considered. During the day, the nurse demand is assumed to be higher. Therefore, 7 AM to 7 PM has a different nurse demand than 7 PM to 7 AM. The initial demand per shift is 6 for the first 12 hours of the day and 5 for the next. However, this has changed and increased by around 15%. When the demand per

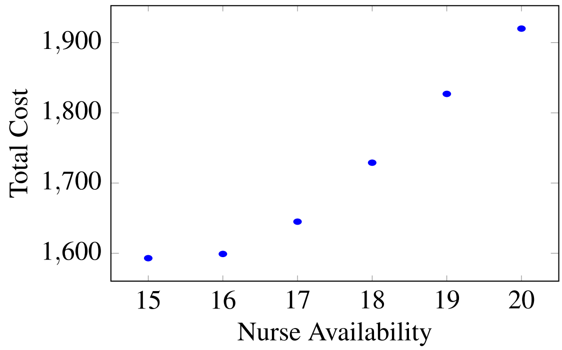

Figure 1. Total cost versus the total number of nurses available in the department

shift increases, the total cost does not increase drastically. The difference in the optimal solution is presented in Table 1. Therefore, adding more demand per shift is more managerial than hiring new nurses in this case. A similar analysis can be applied to other case studies, hospital demands, and departments to make the right decision.

Table 1. Total demand per shift.

Total Demand	Total Cost
7AM to 7 PM: 5 nurses 7PM to 7 AM: 4 nurses	1728
7AM to 7 PM: 6 nurses 7PM to 7 AM: 5 nurses	1729

However, not all nurses prefer to work the same number of hours. Some nurses do not mind working 12 hours, while others may lose function after 10 hours. This has been applied after generating random preferences in Excel. The number of hours per shift is based on employee preference. Table 2 summarizes the relationship between the minimum solution and the difference in nurse preferences. It can be concluded that there is no difference in the optimal solution when nurses have different choices in the number of working hours per shift.

Table 2. Variability in the number of hours per shift per nurse.

Maximum hours/shift	Total Cost
12	1729
10/12	1728

Finally, the demand per nurse qualification is defined in the problem. Initially, there was a demand of 2,2,1,1 for Assistant Nurse, Nurse, Senior Nurse, and Head Nurse, respectively. However, the demand for the primary nurse type, Nurse, is analyzed. Figure 3 shows the change in the objective function plot. The optimal solution increases slightly as the demand increases. However, the total cost increases significantly once it reaches a specific number, 6. This is due to the limited availability of nurses hired.

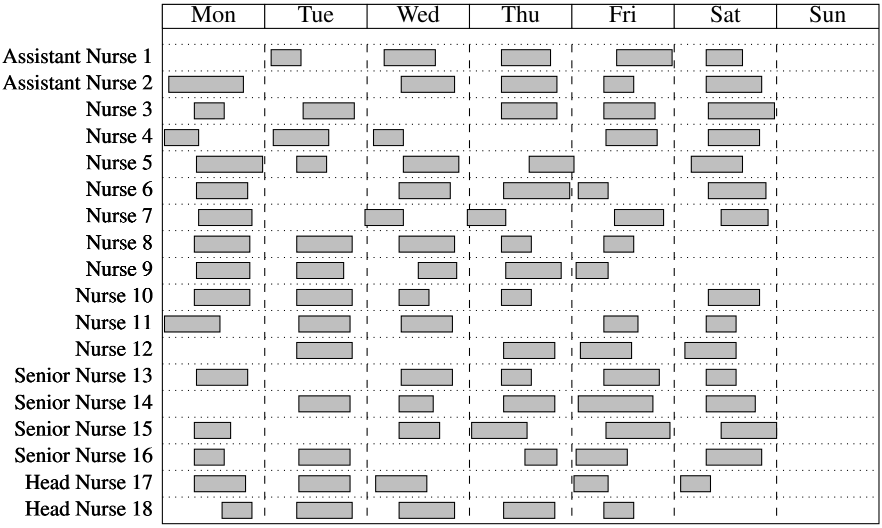

Figure 2. One week employee plan (Nurse Roster)

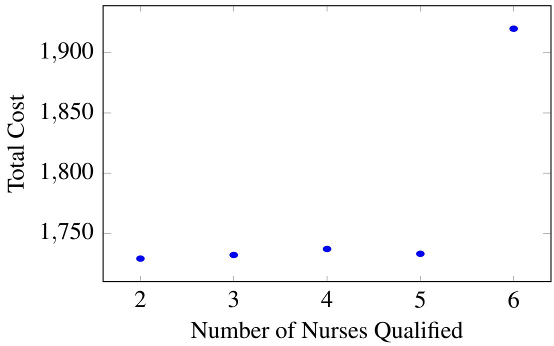

Figure 3. Total cost versus the demand for qualified nurses

6. Conclusion

In conclusion, this study develops a framework to reduce the number of nursing slots scheduled for all nurses. Data were compiled on the basis of assumptions from the literature. Additionally, a schedule was made to meet the hospital's demand for different nurse qualifications and preferences after formulating and solving the MIP quickly and efficiently.

This study faces two main limitations. First, the data used are not real-life hospital data due to the limitations of accessing hospital information and confidentiality. Another limitation was the costs of having nurses work and their wages, which were not considered when building the MIP model.

In terms of quality of healthcare care, future work includes integrating nurse scheduling with other facilities such as operation room scheduling. Moreover, other column generation methods may be used to have optimal solutions when solving the MIP model. Finally, the study can be extended to use efficient heuristics or metaheuristics.

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