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The EE-COV-SSI Method for the Exact Identification of Model Order in Modal Analysis

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Abstract. The covariance-driven stochastic subspace modal parameter identification method has been widely used in the field of engineering structures. Effective determination of the model order of the structural system is the key to applying this method to identify the modal parameters. It is particularly difficult to determine the model order for unstable systems affected by noise disturbances and computational errors. In order to effectively determine the model order, an exponential eigenvalue entropy incremental covariance-driven stochastic subspace identification (EE-COV-SSI) algorithm is proposed. The condition number of the state matrix is used to determine the degree of perturbation of the response signal to the system stability. Meanwhile, the identification accuracy of the modal parameters is reflected by calculating the modal frequency coefficient of variation. Finally, the method is applied to the modal analysis of a four-story frame structure. The results show that the method can accurately identify the model order and improve the identification accuracy of the modal parameters.

Keywords. EE-COV-SSI, Model order, Condition number, Entropy increment

1. Introduction

In recent years, the problem of structural health monitoring has attracted a lot of attention from scholars. The modal parameters based on vibration data analysis are an important basis for assessing the health condition of structures. Covariance-driven Stochastic Subspace Identification (COV-SSI) method has the advantages of less input parameters and fast computation, and is widely used in structural modal parameter identification. COV-SSI, as a time-domain modal analysis method based on state-space model, requires prior determination of the system model order. Since the structural vibration data inevitably contains a large amount of noise, it poses a great difficulty to determine the model order.

Usually, the number of non-zero singular values from Singular Value Decomposition (SVD) is used to determine the model order [1]. In practice, the number of non-zero singular values increases due to noise interference, which makes it impossible to determine the model order accurately. To solve this problem, scholars have proposed SVD-based model order determination methods, the singular value averaging method [2], the singular value rate of change method [3], and the singular

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value percentage adjacent increment ratio method [4]. However, these methods still cannot accurately determine the model order in the low signal-to-noise ratio case. Since entropy can be used to analyze non-stationary linear signals and has the characteristic of being resistant to noise interference [5-7]. However, for some vibrating systems, there is non-convergence of the singular entropy, resulting in the inability to fix the order.

In order to solve the problem of model order determination, the EE-COV-SSI method is proposed. The model order is determined from the eigenvalue entropy increment at convergence, which verifies the effectiveness of the EE-COV-SSI method based on the modal parameter identification results of the four-story frame structure.

2. Covariance-driven stochastic subspace modal parameter identification

The covariance-driven stochastic subspace algorithm is based on the discrete-time state space model, and the state matrix and output matrix are obtained by the calculation. The eigenvalue decomposition of the state matrix is performed to obtain the modal parameters of the system.

2.1. Discrete State-Space equations

The inputs can be considered as random white noise for structural vibration tests under ambient excitation. The discrete state space model for an n-degree-of-freedom system excited by white noise can be expressed as

$$\begin{cases} x_{k+1} = Ax_k + w_k \\ y_k = Cx_k + v_k \end{cases} \tag{1}$$

where $x_k \in R^{2n\times 1}$ is the discrete-time state vector at time instant k, $A \in R^{2n\times 2n}$ is the state transition matrix, $w_k \in R^{2n\times 1}$ is the process noise vector, $y_k \in R^{r\times 1}$ is the measurement vector obtained by r sensors, $C \in R^{r\times 2n}$ is the observation matrix, $v_k \in R^{r\times 1}$ is the measurement noise vector.

2.2. Modal Parameter Identification

In modal analysis, the state matrix and output matrix can be obtained by analyzing and calculating the vibration data. The frequencies and damping ratios are obtained from the state matrix decomposition, and the modal vibration patterns are obtained from the output matrix and the corresponding eigenvectors.

The eigenvalue decomposition of the state matrix yields

$$A = \Psi\Lambda\Psi^{-1} \tag{2}$$

where Λ is the diagonal matrix, Ψ is the matrix composed of eigenvectors.

The jth eigenvalue of the state matrix takes the form

$$\lambda_j = exp\left[\left(-\xi_j w_j \pm \sqrt{1-\xi_j^2}\right)\Delta t\right] \tag{3}$$

where w_j intrinsic frequency, ξ_j damping ratio, Δt time step

$$w_j = |\ln(\lambda_j)| / \Delta t \tag{4}$$

$$\xi_j = -real(\ln(\lambda_j)) / w_j \Delta t \tag{5}$$

$$\Phi_j = C\Psi_j \tag{6}$$

where Ψ_j is the complex eigenvector of A corresponding to the eigenvalues λ_j, and Φ_j is the jth order modal oscillation.

3. Exponential Eigenvalue Entropy Incremental COV-SSI

The covariance-driven stochastic subspace identification method based on discrete state space equations requires the determination of the model order before identifying the modal parameters. The model order is not correct, and the modal parameters cannot be extracted accurately. Therefore, a reasonable model order is the key to accurately identify the modal parameters.

3.1. Exponential Eigenvalue Entropy Increment Method

The entropy can be used to represent the uncertainty of the state of things and is an effective metric for measuring the amount of information contained in a signal.

The system has m states, each with probability P_i of existence (i=1, 2 ,..., m), and the entropy E can be described as

$$E = -\sum_{i=1}^{m} p_i \, ln(p_i) \tag{7}$$

In Shannon's theory, the measure of information gain is taken as entropy [8]. Eq. (7) shows that the system does not converge when the probability is equal to 0. The amount of statistical information can be expressed by instead of which solves the problem of non-convergence of information entropy gain, and the convergence can be accelerated by using exponential entropy gain operation.

The eigenvalue diagonal array obtained from the eigenvalue decomposition of the state matrix, the eigenvalue exponential entropy increment ΔE_i at order i can be expressed as

$$\Delta E_i = -\left(\lambda_i \Big/ \sum_{i=1}^{m} \lambda_i\right) exp\left(\lambda_i \Big/ \sum_{i=1}^{m} \lambda_i\right) \tag{8}$$

When the eigenvalue exponential entropy increment decreases to asymptotically zero, the amount of effective feature information of the signal tends to saturate. The model order is determined based on the incremental entropy of the eigenvalue index at convergence to zero.

3.2. Condition Number and Modal Coefficient of Variation

The condition number is to judge the influence of data perturbation on the system stability, the smaller the condition number, the higher the system stability, while the frequency variation coefficient reflects the accuracy of identification results. The formula is defined as follows.

$$k = |\lambda_1| / |\lambda_i| \tag{9}$$

$$\delta_f = \sum_{i=1}^{n} \sigma_i / \mu_i \tag{10}$$

where n is the model order, σ_i and μ_i correspond to the standard deviation and mean of the ith order of the mode at the model order, respectively, k is the state matrix condition number, λ_1 and λ_i are the first eigenvalue and the ith eigenvalue of the ith order of the mode, respectively.

4. Identification of Modal Parameters of Four-Story Frame Structures

To verify the feasibility and applicability of the EE-COV-SSI method, modal parameters are identified for a four-story frame structure is shown in Fig.1.

Figure 1. Experimental partial picture.

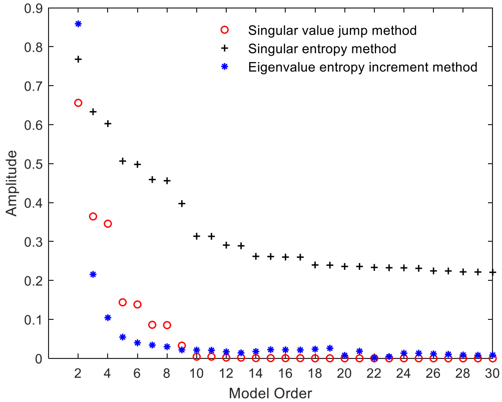

Figure 2. Determining the order.

As shown in Fig.2, the model order determined by the singular value jump method is 6, and the order of the model determined by the singular entropy method is 10, which is 16 for the eigenvalue entropy increment method.

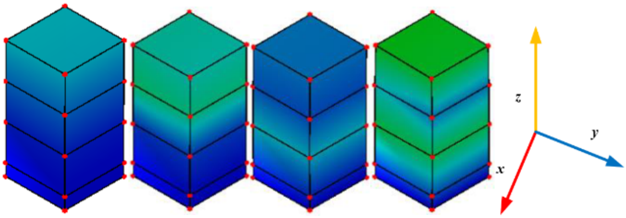

Figure 3. (a) First order, (b) Second order, (c) Third order (d) Fourth order, (e) Orientation diagram

The finite element model of the structure was built using ANSYS commercial software. The first four modal vibrations are shown in Fig. 3.

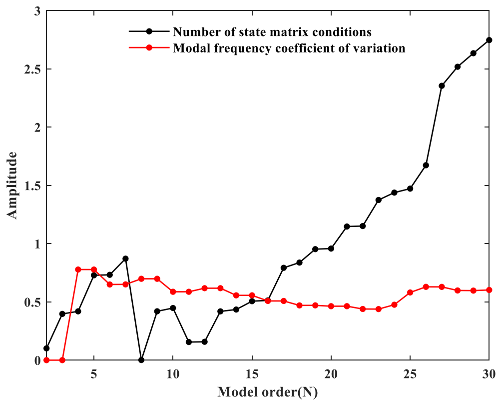

Figure 4. Number of conditions and frequency coefficient of variation

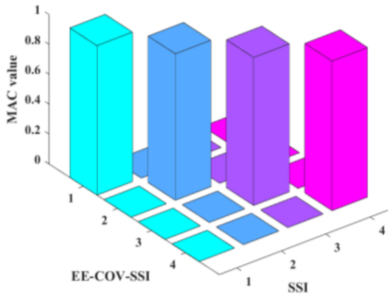

Figure 5. The MAC values between EE-COV-SSI and SSI method for the first four modes.

As shown in Fig. 4, the condition number and frequency coefficient of variation are smaller and the same for order 16, which indicates that the parameter identification accuracy is higher at that order. The values of the vibration MAC obtained by the EE-COV-SSI method and SSI are shown in Fig. 5, and they correlate well.

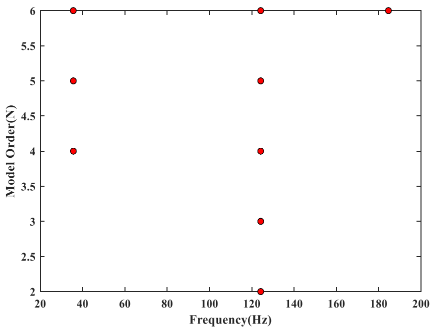

Figure 6. SSI stability diagram.

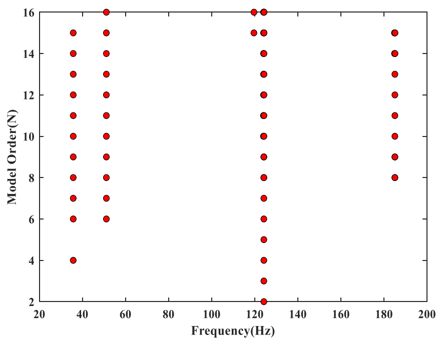

Figure 7. EE-COV-SSI stability diagram.

As shown in Fig. 6, when the model order is 6, the SSI method can only recognize two orders of modes. As shown in Fig. 7, when the model order is 16, the EE-COV-SSI method can completely identify the first four orders of modes, which verifies the effectiveness of the EE-COV-SSI method. (Red '●'indicates stable poles)

Table 1 lists the EE-COV-SSI and SSI methods to identify the values. As can be seen from Table 1, the EE-COV-SSI method can identify the modal parameters completely and with high accuracy, which the SSI method cannot identify the modal parameters accurately and has a large relative error with the theoretical values.

Table 1. Comparison of modal parameters identified via different methods.

	Order	Reference values	SSI	EE-COV-SSI
Frequency(Hz)	1	35.74	35.55	35.75
	2	51.16	----	51.12
	3	124.12	124.35	124.21
	4	185.10	----	185.07
Damping ratio (%)	1	0.24	0.28	0.24
	2	0.29	----	0.29
	3	0.19	0.21	0.21
	4	0.18	----	0.17

The paper proposes an EE-COV-SSI method for overcoming the inaccuracy of model order determination by traditional methods. The model order is determined by entropy increment, and the experimental results verify the effectiveness of the EE-COV-SSI method.

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