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Supply Chain Coordination with Multiple Retailers Under Combined Revenue Sharing and Buy-Back Contracts

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Abstract. This paper mainly proposes a coordination mechanism for a supply chain system containing multiple retailers and one supplier. According to the proposed coordination mechanism, lateral transshipments are performed among retailers while revenue-sharing contract is used to allocate the revenue caused by lateral transshipment. Meanwhile, buy-back contract is used to coordinate the retailer and supplier. We use a supply chain system with a supplier and two retailers as an example to demonstrate the effectiveness of the proposed coordination mechanism. The numerical example shows that the proposed coordination mechanism can effectively coordinate the discussed supply chain.

Keywords. non-cooperative, game;lateral, transshipment, buy-back contract, revenue sharing contract, supply chain coordination

1. Introduction

With the development of economy, market competition has become increasingly fierce. As market customers pay more attention to time, retailers prefer a less time-consuming replenishment strategy when facing stock shortages. In this context, lateral transshipment has received more attention as a replenishment strategy that can increase or maintain the supply capacitywhile reducing the total cost. Generally speaking,lateral transshipment is an stock management mode in which companies at the same level in the supply chain share stock with each other[1].

In order to cope with the severe market competition, retailers not only need faster replenishment speed and higher profits, but also need to have a stronger ability to deal with stock risks. Because the buy-back contract can effectively reduce the retailer's stock risk, it has received more attention from enterprises. Generally speaking, a buy-back contract refers to a mechanism by which a supplier buys back products that the retailer has not sold at the end of the sales season at a price lower than the wholesale price to compensate the retailer's income. It reduces the retailer's stock risk and improves the stability of the supply chain by incentivizing retailers to increase their orderat the beginning ofthe period. TheBuy-back contract has become one of the most convenient contracts for coordinating the supply chain because they are easy to implement between

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suppliers and retailers. Also, buy-back contract used canadd the supplier to the non-cooperative game model and make the management strategy more scientific.

Nowadays, many people have conducted research on lateral transshipment.Peng Wan et al.[2] study the preventive lateral transshipment inventory strategies by considering two retailers who can update the forecast information of demand when the manufacturer provides two ordering opportunities at different prices. Liao Yi et al[3] studys a basic inventory management strategy-lateral transshipment-under decentralized systems, which play an important role in dealing with stockouts during unexpected crises. In order to solve large-scale MDP, Zhen Li et al[4] propose a comprehensive heuristic lateral transshipment policy.Dehghani and Abbasi[5] propose a new lateral-transshipment policy for perishable items based on the age of the oldest item in the system to improve supply chain performance.The buy-back contract has also triggered some discussions in recent days. In a recent paper, Huo Zhiyu [6] discussed whether the profit of the supply chain after the introduction of the buy-back contract has improved compared with the traditional decentralized procurement model. Lei Xie et al[7] studies how the yield uncertainty and the relative bargaining power affect the performance of buy-back contract when the buyer faces uncertain demand and yield.Xinlin Dong and Qi Xin[8] conducted a research on the two-stage ordering strategy under the buy-back contract and discussed the role of the buy-back contract in reducing retailer risk.Chunhai Yun et al[9] studies the coordination decision-making of two-stage closed-loop supply chain based on buy-back contract.

At present, most of the studies on lateral transshipment are based on the optimal ordering strategy that aims at maximizing the expected revenue of each retailer under lateral transshipment. Most studies on buy-back contracts only discuss the impact of buy-back contracts on the profits of non-transshipment supply chain systems. And we want to uses revenue sharing contract to realize the coordination of lateral transshipment among multiple retailers and adds the supplier to the game model through the buy-back contract.

The remainder of this paper is organized as follows. Section Ⅱ introduces the basic model and coordination conditions. In Section Ⅲ, we give the algorithm. Section Ⅳ presents the numerical examples, followed by the results analysis. Section Ⅴ summarizes the findings.

2. Mathematical Model

2.1. Problem Description

There is a supply chain composed of a supplier and two retailers. Assuming that the demand between two retailers is independent of each other, the supplier's supply capacity is unlimited. at the beginning of the period, retailers decide simultaneously on their individual order quantity from the supplier.There is an agreement between retailers that in situations where a reailer has surplus stock and another reailer is stocked out, it is desirable to transfer surplus stock from the former to the latter,and the former needs to pay the latter according to the revenue sharing contract.at the end of the period, if the retailers has excessstock, the supplier will buyback this part of the stock..

2.2. Mathematical Modeling

The related parameter symbols are shown in Table 1:

Table 1. Symbol description table

Symbol	Meaning
r_i	Unit selling price to the customer($r_i > c_i$)
q_i	Retailer's orders
d_i	Retailer's demand
l_i	Unit shortage cost
c_i	Unit purchase cost from the supplier
λ_i	Unit transshipment price
δ_i	Revenue sharing ratio
v_i	Unit marginal value
τ_{ij}	Unit transshipment cost from retailer i to retailer j, incurred by retailer i
m	Unit manufacturing cost
T_{ij}	Quantity of transshipment from retailer i to retailer j
b_i	Unitbuy-back price to the supplier
s_i	Unit salvage value of remaining inventory at the end of the period($s_i < c_i$)

Revenue-sharing contract: Suppose there is transshipment between retailers and retailer i transships excess stock to retailer j. Retailer j's expenditure consists of two parts. First, retailer j needs to pay retailer i the transshipment price$\lambda_i$for each item transshipped from retaileri.Second, retailer j needs to pay the revenue of the transshipped items retailer i in proportion to δ_j.

Buy-back contract: at the end of the period,the supplier needs to buy the remaininginventory of retailer i at the price of b_i. The buy-back price b_i is less than the product wholesale price c_i and greater than the product residual value s_i.

The parameters λ_i, δ_j, and b_i are formulated by a third-party company. The third-party company should observe the retailer's order quantity q_i and demand quantity d_i . Then, they can specify λ_i, δ_j and b_i and based on the observation information and other cost parameters.If the retailer's revenue under thiscase is not as good as the retailer's revenue under the classic newsvendor model, the retailer will choose to refuse touse the contract.

From the above conditions, we can get that the total expected profit of the system is:

$$\pi^t = \sum_{i=1}^{n} E[r_i\, min(\, q_i^t, d_i) - l_i(d_i - q_i^t)^+ + s_i(q_i^t - d_i)^t - \sum_{j=1}^{n} \tau_{ij} T_{ij} - mq_i] \tag{1}$$

q_i^t in the formula (1) represents the net order quantity of retailer i after transshipment between retailers, $q_i^t = q_i - \sum T_{ij} + \sum T_{ji}$. In formula (1), $r_i\, min(\, q_i^t, d_i)$ represents the sales revenue of the system.$l_i(d_i - q_i^t)^+$ represents the loss due to unsatisfied customer demand.$s_i(q_i^t - d_i)^t$ represents the salvage value of remaining inventory at the end of the period.$\sum_{j=1}^{n} \tau_{ij} T_{ij}$ is the transshipment cost between retailers,and mq_i represents the manufacturing cost of the supplier.

The supplier's expected revenue is:

$$\pi^b = \sum_{i=1}^{n}[(s_i - b_i)(q_i^t - d_i)^+ + (c_i - m)q_i] \quad (2)$$

In formula (2), $s_i(q_i^t - d_i)^+$ represents the salvage value of the supplier's remaining inventory at the end of the period. $b_i(q_i^t - d_i)^+$ represents the cost of items buyed back from retailers.

The expectedprofitof retailer i is:

$$\pi_i^d = E[r_i\, min(q_i^t, d_i) - l_i(d_i - q_i^t)^+ + b_i(q_i^t - d_i)^+ - \rho_i^1(Q,\delta)] - c_i q_i + \sum_{j=1}^{n}(\lambda_i - \tau_{ij})T_{ij} - \sum_{j=1}^{n}\lambda_j T_{ji} + \rho_i^2(Q,\delta) \quad (3)$$

Among them, $\sum_{j=1}^{n}\lambda_j T_{ji}$ and $\rho_i(Q,\delta)$ represent the fees that retailer i needs to pay for items transshipped from another retailer. $\sum_{j=1}^{n}\lambda_i T_{ij}$ and $\rho_j(Q,\delta)$ represent the revenue that retailer i get for items transshipped to another retailer.

$$\rho_i^1(Q,\delta) = \sum_{j=1}^{n}\delta_i(v_i - \tau_{ji})T_{ji}$$

according to the two cases of $q_i \geq d_i$ and $q_i \leq d_i$, we can simplifies formula (1), formula (2) and formula (3) , and get the following formulas:

let's set

$$\alpha_i = Pr(d_i \leq q_i)$$
$$\beta_i = Pr(q_i + q_j - d_j \leq d_i \leq q_i)$$
$$\gamma_i = Pr(q_i \leq d_i \leq q_i + q_j - d_j)$$

System benefits:

$$\rho_i^2(Q,\delta) = \sum_{j=1}^{n}\delta_j(v_j - \tau_{ij})T_{ij}$$

$$\pi^t = \sum_{i=1}^{n}E[r_i\, min(q_i, d_i) - l_i(d_i - q_i)^+ + s_i(q_i - d_i)^+ + \phi_i(Q) - mq_i$$

Calculate derivative of the above formula with respect to q_i, the outcome is

$$\frac{\partial \pi^t}{\partial q_i} = (v_i - m) + (s_i - v_i)\alpha_i + (v_j - s_i - \tau_{ij})\beta_i + (s_j + \tau_{ji} - v_i)\gamma_i$$

Retailer i revenue:

$$\pi_i^d = E[r_i\, min(q_i, d_i) - l_i(d_i - q_i)^+ + b_i(q_i - d_i)^+ + \varphi(Q,\lambda) + \psi(Q,\delta)] - c_i q_i$$

$$\varphi(Q,\lambda) = \begin{cases} \sum_{j=1}^{n}(\lambda_i - b_i - \tau_{ij})T_{ij} if\ q_i \geq d_i \\ \sum_{j=1}^{n}(v_i - \lambda_j)T_{ji} if\ q_i \leq d_i \end{cases}$$

$$\psi(Q,\delta)=\begin{cases}\sum_{j=1}^{n}\delta_j\left(v_j-\tau_{ij}\right)T_{ij} & if\ q_i\geq d_i\\ \sum_{j=1}^{n}-\delta_i\left(v_i-\tau_{ji}\right)T_{ji} & if\ q_i\leq d_i\end{cases}$$

$$\phi_i(Q)=\begin{cases}\sum_{j=1}^{n}\left(-s_i-\tau_{ij}\right)T_{ij} & if\ \ q_i\geq d_i\\ \sum_{j=1}^{n}v_iT_{ji} & if\ \ q_i\leq d_i\end{cases}$$

Calculate derivative of the above formula with respect to q_i, the outcome is

$$\frac{\partial\pi_i^d}{\partial q_i}=(v_i-c_i)+(b_i-v_i)\alpha_i+[\lambda_i-b_i-\tau_{ij}+\delta_j(v_j-\tau_{ij})]\beta_i+[\lambda_j-v_i+\delta_i(-\tau_{ji})]\gamma_i$$

Supplier benefits:

$$\pi^b=\sum_{i=1}^{n}[(s_i-b_i)(q_i-\sum T_{ij}+\sum T_{ji}-d_i)^+ +(c_i-m)q_i]$$

Calculate derivative of the above formula with respect to q_i, the outcome is

$$\frac{\partial\pi^b}{\partial q_i}=(s_i-b_i)(1-\beta_i)+(s_j-b_j)\gamma_i+c_i-m$$

Let $\partial\pi^t/\partial q_i=\partial\pi^b/\partial q_i$, we have that

$$(1-\beta_i)b_i+\gamma_ib_j=(c_i+s_i-v_i)+(v_i-s_i)\alpha_i+(\tau_{ij}-v_j)\beta_i+(v_i-\tau_{ij})\gamma_i \tag{4}$$

Similariy, we have that

$$(1-\beta_j)b_j+\gamma_jb_i=(c_j+s_j-v_j)+(v_j-s_j)\alpha_j+(\tau_{ji}-v_i)\beta_j+(v_j-\tau_{ji})\gamma_j \tag{5}$$

By considering the formula (4) and (5), we can get

$$b_i=\frac{[(c_i+s_i-v_i)+(v_i-s_i)\alpha_i+(\tau_{ij}-v_j)\beta_i+(v_i-\tau_{ij})\gamma_i](1-\beta_j)}{(1-\beta_i)(1-\beta_j)-\gamma_i\gamma_j}-\frac{[(c_j+s_j-v_j)+(v_j-s_j)\alpha_j+(\tau_{ji}-v_i)\beta_j+(v_j-\tau_{ji})\gamma_j]\gamma_i}{(1-\beta_i)(1-\beta_j)-\gamma_i\gamma_j}$$

Let $\partial\pi^t/\partial q_i=\partial\pi_i^d/\partial q_i$,we have that

$$\beta_i\lambda_i+\gamma_i\lambda_j=(c_i-m)+(s_i-b_i)\alpha_i+[v_j-s_i+b_i+\delta_j(\tau_{ij}-v_j)]\beta_i+[s_j+\tau_{ij}+\delta_i(\tau_{ji}-v_i)]\gamma_i \tag{6}$$

Similariy, we have that

$$\beta_j\lambda_j + \gamma_j\lambda_i = (c_j - m) + (s_j - b_j)\alpha_i + [v_i - s_j + b_j + \delta_i(\tau_{ji} - v_i)]\beta_j + [s_i + \tau_{ji} + \delta_j(\tau_{ij} - v_j)]\gamma_j \quad (7)$$

By considering the formula (6) and (7), we can get

$$\lambda_i = \frac{(s_i - b_i)\alpha_i\beta_j - (s_j - b_j)\alpha_i\gamma_i + A\beta_i\beta_j + B\gamma_i\gamma_j + C\beta_j\gamma_i + D}{\beta_i\beta_j - \gamma_i\gamma_j}$$

$$A = v_j - s_i + b_i + \delta_j(\tau_{ij} - v_j)$$

$$B = -s_i - \tau_{ji} - \delta_j(\tau_{ij} - v_j)$$

$$C = s_j + \tau_{ij} + \delta_i(\tau_{ji} - v_i) - [v_i - s_j + b_j + \delta_i(\tau_{ji} - v_i)]$$

$$D = (c_i - m)\beta_j - (c_j - m)\gamma_i$$

The expected revenue of the retailer using the revenuesharing contract is higher than their expected revenue in the newsvendor model,or else retailers will choose to refuse to use the contract. π_i^{ld} represents the expected profit of the retailer corresponding to the newsvendor model.

$$\pi_i^{ld} = \sum_{i=1}^{n} E[r_i\,min(q_i, d_i) - l_i(d_i - q_i)^+ + s_i(q_i - d_i)^+ - \sum_{j=1}^{n} \tau_{ij}T_{ij} - c_iq_i]$$

3. Heuristic Algorithm

Through the previous discussion, we find a simple heuristic algorithm to solve the coordination parameters. The algorithm cannot only calculate the optimal order quantity, trasnsshipment price and expected profit, but also change the relevant parameters to show the impact of changes in related parameters on coordination. Specific steps are as follows:

(1) Input the retailer's demand d_i.

(2) Input the necessary parameters c_i, m, τ_{ij}, v_i.

(3)Input the initial Revenue sharing ratioδ_i^0 and the iteration step sizeη_i, and set a=0.

(4)Calculating λ_i, the system optimal order quantity q_i^s and the retailer's optimal order quantity q_i^d.

(5) Determine whether $q_i^s = q_i^d$holds. If yes, go to (6); otherwise go to (8).

(6) Determine whether $\pi_i^d \geq \pi_i^{ld}$ holds. If yes, go to (7); otherwise go to (8).

(7) Output the order quantity, expected profit and other information, let $\delta_i^0 = \delta_i^0 + \eta_i$, a=a+1.

(8) Judge whether $0 \leq \delta_i^0 \leq 1$, $0 \leq \lambda_i \leq 1$ holds. If yes, go to (4); otherwise, stop the program.

4. Numerical Analysis

It is known that the demand of two retailers obeys the uniform distribution of [0,200]. Setting $r_1 = r_2 = 40$, $s_1 = s_2 = 10$, $l_1 = l_2 = 0$, $\tau_{12} = \tau_{21} = 2$, $c_1 = c_2 = 20$, $v_1 = v_2 = 40$, $m = 15$.

Let $q_i^m = q_i/200, q_j^m = q_j/200$,then

$$\alpha_i = \begin{cases} q_i^m & if \ \ 0 \le q_i^m \le 1 \\ 1 & otherwise \end{cases}$$

$$\beta_i = \begin{cases} q_i^m\left(1 - 0.5q_i^m - q_j^m\right) & if \ \ 0 \le q_i^m + \ q_j^m \le 1 \\ 0.5 \ \ (1 - q_j^m) \ \ ^2 & if \ \ 1 - q_i^m \le q_j^m \le 1 \\ 0 & otherwise \end{cases}$$

$$\gamma_i = \begin{cases} 0.5(q_j^m)^2 & if \ \ 0 \le q_i^m + \ q_j^m \le 1 \\ 0.5(1 - q_i^m)\left(q_i^m + 2q_j^m - 1\right) & if \ \ 1 - q_i^m \le q_j^m \le 1 \\ 0.5(1 - q_i^m)\left(q_i^m + 2q_j^m - 1\right) - 0.5(1 - q_i^m)^2 & if \ \ q_i^m \le q_j^m \le 2 - q_i^m \\ 0 & otherwise \end{cases}$$

4.1. Identical parameters

According to the above parameter description and calculation method, we can find that the optimal order quantity is $q_i = q_j = 143.7$, and the corresponding optimal solution of the newsvendor model is $q_i = q_j = 133.3$. In the newsboy model, the retailer's profit and the system profit are respectively π_i^{ld}, π_i^{lt}.

Then, We analyzes the impact of transshippment price and revenue sharing ratio on the retailer's expected profit under the coordination situation, as shown in Table 2.

Table 2. The impact of revenue sharing ratio on expected profit

δ_i	λ_i	π_i^d	π_i^{ld}	π_i^{lt}	π^t
0	27.26	1671.7	1666.7	4381.6	4364.7
0.1	23.46	1671.7	1666.7	4381.6	4364.7
0.2	19.66	1671.7	1666.7	4381.6	4364.7
0.3	15.86	1671.7	1666.7	4381.6	4364.7
0.4	12.06	1671.7	1666.7	4381.6	4364.7
0.5	8.26	1671.7	1666.7	4381.6	4364.7
0.6	4.46	1671.7	1666.7	4381.6	4364.7

It can be seen from Table 2 that under the condition that all the parameters are completely symmetrical, as revenue sharing ratio increases, the coordinated transshipment price decreases, and the expected profits of the retailer, supplier, and system all remain unchanged. The expected profit of the supplier and the retailers is greater than the expected profit of the retailer of the newsvendor model. Therefore, we conclude that when all parameters of the two retailers are equal, it is a better choice for retailers to use the contract.

4.2. Not identical parameters

Actually, identical parameter values are often difficult to achieve. Therefore, we separately explored the changes in transshipment price and expected profits under the asymmetric conditions of transshipment cost and marginal values.We give the impact of transshipment cost and marginal value on the transshippment price, and the results are shown in Fig.1 and Fig. 2.

Table 3. Sensitivity analysis for τ_{ij}

τ_{ij}	λ_i	π_i^d	π^b	π^t	b_i
0	33.90	1711.9	1048.6	4367.5	14.221
0.1	27.26	1671.7	1048.3	4364.7	14.452
0.2	24.43	1644.0	1048.5	4358.5	14.588
0.3	22.89	1622.1	1048.0	4350.6	14.677
0.4	21.93	1603.2	1047.4	4341.7	14.740
0.5	21.21	1586.8	1046.8	4332.1	14.779
0.6	20.80	1570.0	1046.0	4322.3	14.823

Figure 1. Sensitivity analysis for τ_{ij}

As shown in Table 3 and Fig. 1, as the transshipment cost increases, the retailer's profits and transshipment price falls sharply, and the supplier's profits declines slightly. When the transshipment cost is too high, the implementation of the lateral transshipment strategy may not make the retailer profitable and they may refuse to use the contract.Therefore, the third-party companie should carefully consider when formulating cost parameters.

Table 4. Sensitivity analysis for v_i

v_i	λ_i	π_i^d	π^b	π^t	b_i
40	27.26	1671.7	1048.3	4364.7	14.452
42	23.75	1844.4	1049.1	4559.5	14.658
44	21.59	2023.6	1047.2	4755.6	14.831
46	20.29	2208.7	1043.9	4952.5	14.971
48	19.48	2398.5	1040.1	5150.2	15.083
50	18.98	2591.5	1036.3	5348.4	15.171
52	18.64	2786.7	1032.8	5546.9	15.242
54	18.42	2983.3	1029.6	5745.7	15.300
56	18.26	3181.0	1026.8	5944.8	15.348

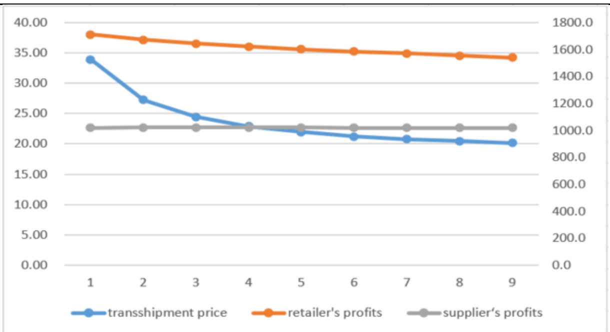

Figure 1. Sensitivity analysis for v_i

As shown in Table 4 and Fig. 2, the transshippment price declines with the increase of the marginal value, while the expected profits of retailers and the supplier increase.When the marginal value of the product is high, the transshipment price should be appropriately reduced, so as to enable the retailer to obtain higher expected profits.

5. Conclusion

This paper studies the coordination mechanism for a supply chain system composed of multiple retailers and a single supplier. According to the proposed coordination mechanism, buy-back contract and uses revenue sharing contract are use to realize the supply chain coordination. Under the proposed coordination mechanism framework, lateral transshipments are used among retailers while revenues caused by lateral transshipment are allocated by revenue-sharing contract. Meanwhile, buy-back contract is used to coordinate the retailer and supplier. The coordination conditions are discussed. A numerical example is used to demonstrate the effectiveness of the proposed coordination mechanism. In the example, we find that for the retailers and the supplier under completely symmetric conditions, the coordination mechanism can help them achieve Pareto improvement, but this improvement will not change as sharing revenue ratio changes. It also shows that when the retailers cost parameters are incompletely same, as the marginal value of retailer increases, the retailer's profits increases sharply and supplier's profits falls slightly.

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