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Novel Multi-Attribute Group-Decision Making Method with TOPSIS: A Fermatean Fuzzy Hypersoft Sets and Correlation Coefficients Approach

Murat KİRİŞCİ[1]
Istanbul University-Cerrahpaşa, Cerrahpaşa Medicine Faculty, Department of Biostatistics and Medical Informatics, Istanbul, Türkiye

Abstract. One goal of this research is to provide Fermatean fuzzy hypersoft sets and investigate their basic properties. The second goal of this research is to develop the concept and properties of the correlation coefficient and weighted correlation coefficient for the Fermatean fuzzy hypersoft set, as well as to introduce aggregation operators such as the Fermatean fuzzy hypersoft weighted average and Fermatean fuzzy hypersoft weighted geometric operators in the Fermatean fuzzy hypersoft set scenario. A prioritization technique for order preference by similarity to the ideal solution (TOPSIS) is presented using correlation coefficients and weighted correlation coefficients in a Fermatean fuzzy hypersoft set. A technique for solving the multi-attribute group decision-making problem is planned using the developed methodology. In addition, examples of medical decision-making are presented to demonstrate the importance and application of the developed methodology.

Keywords. Correlation coefficients, informational energy, Fermatean fuzzy hypersoft set, TOPSIS, group decision-making,

1. Introduction

In decision-making situations, tools such as aggregation operators and information measures are routinely employed. The correlation coefficients(KK) assessment of the amount of dependence between two sets may also be used to choose the optimal alternative. Using KKs, one may determine how strongly two variables are related. Because the information in various settings is typically unclear, ambiguous, and partial, numerous scholars have created KKs in fuzzy environments. Chiang and Lin [1] developed a strategy for KK of fuzzy sets(FS) in addition to providing the correlation for fuzzy information in line with traditional statistics. The KK of fuzzy information has been examined in [2] using a mathematical programming approximation, according to the standard notion of KKs. According to the findings of the FS theory, Atanassov [3] claimed that intuitionistic fuzzy set(IFS) results were more complete and precise. The IFS theory considers both membership(MD) and non-membership(ND) degrees, and it

1 Corresponding Author, Istanbul University-Cerrahpaşa, Cerrahpaşa Medicine Faculty, Istanbul, Türkiye; E-mail: mkirisci@hotmail.com.

demands that their sum be one or less than one. The IFS-derived KKs have a variety of applications, including decision-making(DM), cluster analysis, image processing, pattern recognition, and so on ([4]-[8]). Several DM problems utilizing Pythagorean fuzzy information have been described in the literature as a result of the PFS ([9]-[16]), which was designed to address an IFS issue.

Senapati and Yager [17] developed the concept of FFSs as an extension of the IFSs and PFSs. In an FFS, the cubic sum of an object's MD and ND is limited by 1. Consider 0.9+0.6>1, $(0.9)^2+(0.6)^2>1$, and $(0.9)^3+(0.6)^3<1$ as examples. That is because the total of the cubes of the MD and ND of FFSs is in the [0,1], FFSs give a more comprehensive view of FSs. When dealing with unclear data, FFSs are more adaptable and efficient than IFSs and PFSs. FFS is now playing an important role in a variety of disciplines since it is a strong notion for dealing with imprecise and unclear information in a Fermatean fuzzy environment ([18]-[22]).

The originality: Fuzzy KK, IF-KK, and PF-KK are examples of expansions to the classical KKs. The KKs' performance has increased as a result of these extensions. FFSs can handle ambiguity and partial information problems more effectively than IFSs and PFSs. In this study, first of all, the concept of the Fermatean fuzzy hypersoft set(FFHSS) is introduced and its basic properties are examined. Secondly, new KKs based on FFHSS were defined and the theoretical basis of these coefficients was demonstrated. The reason for using FFSs to define new correlation coefficients is that since the $MD^3 + ND^3 \leq 1$ requirement for an object is met, it is likely to cover more items than IFSs and PFSs. The multi-criteria group decision-making(MCGDM) algorithm was produced by combining the new KKs and the TOPSIS method, and an example for the selection of hip prosthesis materials is given to demonstrate the operability and reliability of the method. KKs given in previous studies and KKs defined in this study were compared.

2. Fermatean Fuzzy Hypersoft Sets

Definition 3.1. Let U be a universe of discourse, $P(U)$, and $k = \{k_1, k_2, \dots, k_n\}$ $(n \geq 1)$ be a set of attributes and set K_i be a set of corresponding sub-attributes of k_i, respectively, with $K_i \cap K_j = \varphi$ for $n \geq 1$ for each $i, j \in \{1,2,\dots,n\}$, and $i \neq j$. Assume $K_1 \times K_2 \times \dots \times K_n = \bar{A} = \{d_{1h} \times d_{2k} \times \dots \times d_{nl}\}$ be a collection of all intuitionistic fuzzy subsets over U. Then, the pair $(F, K_1 \times K_2 \times \dots \times K_n = \bar{A})$ is said to be FFHSS over U, and its mapping is defined as $F: K_1 \times K_2 \times \dots \times K_n = \bar{A} \to FFS^U$.

It is also defined as $(F, \bar{A}) = \left\{\left(\tilde{d}, F_{\bar{A}}(\tilde{d})\right): \tilde{d} \in \bar{A}, F_{\bar{A}}(\tilde{d}) \in FFS^U \in [0,1]\right\}$, where $F_{\bar{A}}(\tilde{d}) = \{\delta, \zeta_{F(\tilde{d})}(\delta), \eta_{F(\tilde{d})}(\delta): \delta \in U\}$, in which $\zeta_{F(\tilde{d})}(\delta)$ and $\eta_{F(\tilde{d})}(\delta)$ represent the MV and NV of the attributes such as $\zeta_{F(\tilde{d})}(\delta), \eta_{F(\tilde{d})}(\delta) \in [0,1]$, and $0 \leq \zeta_{F(\tilde{d})}(\delta)^3 + \eta_{F(\tilde{d})}(\delta)^3 \leq 1$.

Definition 3.2. Let $(F, \bar{A})$ and $(G, \bar{B})$ be two FFHSS over U.

(i.) For $\zeta_{F(\tilde{d})}(\delta) \leq \zeta_{G(\tilde{d})}(\delta)$ and $\eta_{F(\tilde{d})}(\delta) \leq \eta_{G(\tilde{d})}(\delta)$, $\delta \in U$, if $\bar{A} \subseteq \bar{B}$ and $F_{\bar{A}}(\tilde{d})(\delta) \subseteq G_{\bar{B}}(\tilde{d})(\delta)$ for all $\delta \in U$, then $F_{\bar{A}}(\tilde{d})$ is called an FFHS subset of $G_B(\tilde{d})$.

(ii.) If $\zeta_{F(\tilde{d})}(\delta) = 0$ and $\eta_{F(\tilde{d})}(\delta) = 1$ for all $\tilde{d} \in \bar{A}$ and $\delta \in U$, $(\emptyset, \bar{A}) = \{\tilde{d}, (\delta, (0,1)) : \delta \in U, \tilde{d} \in \bar{A}\}$ is called empty FFHSS and denoted by $\emptyset_{F(\tilde{d})}(\delta)$.

(iii.) If $\zeta_{F(\tilde{d})}(\delta) = 1$ and $\eta_{F(\tilde{d})}(\delta) = 0$ for all $\tilde{d} \in \bar{A}$ and $\delta \in U$, $(E, \bar{A}) = \{\tilde{d}, (\delta, (0,1)) : \delta \in U, \tilde{d} \in \bar{A}\}$ is called universal FFHSS and denoted by $E_{F(\tilde{d})}(\delta)$.

(iv.) If for all $\delta \in U$ and $\tilde{d} \in \bar{A}$, $\zeta_{F_{\bar{A}}(\tilde{d})}(\delta) = \zeta_{G_{\bar{B}}(\tilde{d})}(\delta)$ and $\eta_{F_{\bar{A}}(\tilde{d})}(\delta) = \eta_{G_{\bar{B}}(\tilde{d})}(\delta)$, then $(F, \bar{A})$ and $(G, \bar{B})$ is called equal FFHSSs.

(v.) Let $(F, \bar{A}) = \left\{\left(\tilde{d}, \left[\zeta_{F(\tilde{d})}(\delta), \eta_{F(\tilde{d})}(\delta)\right] : \delta \in U\right) : \tilde{d} \in \bar{A}\right\}$ be a FFHSS over U. $(F, \bar{A})^c = \left\{\left(\tilde{d}, \left[\eta_{F(\tilde{d})}(\delta), \zeta_{F(\tilde{d})}(\delta)\right] : \delta \in U\right) : \tilde{d} \in \bar{A}\right\}$ is called the complement of $(F, \bar{A})$.

Theorem 3.3. Let $(F, \bar{A})$, $(G, \bar{B})$, and $(H, \bar{C})$ be three FFHSS over U.

(i.) $(F, \bar{A}) \subseteq (E, \bar{A})$,

(ii.) $(\emptyset, \bar{A}) \subseteq (F, \bar{A})$,

(iii.) $(F, \bar{A}) \subseteq (G, \bar{B})$ and $(G, \bar{B}) \subseteq (H, \bar{C}) \Rightarrow (F, \bar{A}) \subseteq (H, \bar{C})$,

(iv.) $((F, \bar{A})^c)^c = (F, \bar{A})$; $(\emptyset, \bar{A})^c = (E, \bar{A})$; $(E, \bar{A})^c = (\emptyset, \bar{A})$.

3. New Correlation Coefficients

Definition 4.1. Let $(F, \bar{A})$, and $(G, \bar{B})$ be two FFHSS over U. Then

$$IE(F, \bar{A}) = \sum_{k=1}^{m} \sum_{i=1}^{n} \left(\left(\zeta_{F(\tilde{d}_k)}(\delta_i) \right)^6 + \left(\eta_{F(\tilde{d}_k)}(\delta_i) \right)^6 \right)$$
$$IE(G, \bar{B}) = \sum_{k=1}^{m} \sum_{i=1}^{n} \left(\left(\zeta_{G(\tilde{d}_k)}(\delta_i) \right)^6 + \left(\eta_{G(\tilde{d}_k)}(\delta_i) \right)^6 \right)$$

are called the *informational energies* of $(F, \bar{A})$, and $(G, \bar{B})$.

$$C[(F, \bar{A}), (G, \bar{B})] = \sum_{k=1}^{m} \sum_{i=1}^{n} \left(\left(\zeta_{F(\tilde{d}_k)}(\delta_i) \right)^3 . \left(\zeta_{G(\tilde{d}_k)}(\delta_i) \right)^3 + \left(\eta_{F(\tilde{d}_k)}(\delta_i) \right)^3 . \left(\eta_{G(\tilde{d}_k)}(\delta_i) \right)^3 \right)$$

is called the *correlation measure* between $(F, \bar{A})$, and $(G, \bar{B})$.

$$CC[(F, \bar{A}), (G, \bar{B})] = \frac{C[(F, \bar{A}), (G, \bar{B})]}{\sqrt{IE(F, \bar{A})} . \sqrt{IE(G, \bar{B})}}$$

$$= \frac{\sum_{k=1}^{m} \sum_{i=1}^{n} \left(\left(\zeta_{F(\tilde{d}_k)}(\delta_i) \right)^3 . \left(\zeta_{G(\tilde{d}_k)}(\delta_i) \right)^3 + \left(\eta_{F(\tilde{d}_k)}(\delta_i) \right)^3 . \left(\eta_{G(\tilde{d}_k)}(\delta_i) \right)^3 \right)}{\sqrt{\sum_{k=1}^{m} \sum_{i=1}^{n} \left(\left(\zeta_{F(\tilde{d}_k)}(\delta_i) \right)^6 + \left(\eta_{F(\tilde{d}_k)}(\delta_i) \right)^6 \right)} . \sqrt{\sum_{k=1}^{m} \sum_{i=1}^{n} \left(\left(\zeta_{G(\tilde{d}_k)}(\delta_i) \right)^6 + \left(\eta_{G(\tilde{d}_k)}(\delta_i) \right)^6 \right)}} \quad (1)$$

is called *KKs* between $(F, \bar{A})$, and $(G, \bar{B})$. Further, another a KKs formulas as follows:

$$CC_M[(F,\bar{A}),(G,\bar{B})] = \frac{C[(F,\bar{A}),(G,\bar{B})]}{max\{IE(F,\bar{A}),IE(G,\bar{B})\}}$$

$$= \frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta_{F(\tilde{a}_k)}(\delta_i)\right)^3.\left(\zeta_{G(\tilde{a}_k)}(\delta_i)\right)^3 + \left(\eta_{F(\tilde{a}_k)}(\delta_i)\right)^3.\left(\eta_{G(\tilde{a}_k)}(\delta_i)\right)^3\right)}{max\left\{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta_{F(\tilde{a}_k)}(\delta_i)\right)^6 + \left(\eta_{F(\tilde{a}_k)}(\delta_i)\right)^6\right),\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta_{G(\tilde{a}_k)}(\delta_i)\right)^6 + \left(\eta_{G(\tilde{a}_k)}(\delta_i)\right)^6\right)\right\}}. \quad (2)$$

Theorem 4.2. Let $(F,\bar{A})$, and $(G,\bar{B})$ be two FFHSS over U. For the Equation (1),

(i.) $0 \leq CC[(F,\bar{A}),(G,\bar{B})] \leq 1$,

(ii.) $CC[(F,\bar{A}),(G,\bar{B})] = CC[(G,\bar{B}),(F,\bar{A})]$,

(iii.) If $(F,\bar{A})$=$(G,\bar{B})$, that is, $\zeta_{F(\tilde{a}_k)}(\delta_i) = \zeta_{G(\tilde{a}_k)}(\delta_i)$ and $\eta_{F(\tilde{a}_k)}(\delta_i) = \eta_{G(\tilde{a}_k)}(\delta_i)$, $CC[(F,\bar{A}),(G,\bar{B})] = 1$.

The conditions in Theorem 4.2 also apply to Equation (2).

It is critical to examine the weights of FFHSS in practical applications. The choice may change when the decision-maker assigns different weights for each alternative in the universe of discourse. As a result, it is critical to plan the weight before making a selection. Let $\Omega = \{\Omega_1,\Omega_2,\dots,\Omega_m\}^T$ be a weight vector for experts such as $\Omega_k > 0$, $\sum_{k=1}^{m}\Omega_k = 1$, and $\gamma = \{\gamma_1,\gamma_2,\dots,\gamma_m\}^T$ be a weight vector for parameters such as $\gamma_k > 0$, $\sum_{k=1}^{m}\gamma_k = 1$. By extending Definition 4.4, we will be created the weighted KKs between FFHSSs.

Definition 4.3. Let $(F,\bar{A})$, and $(G,\bar{B})$ be two FFHSS over U. Then

$$IE_W(F,\bar{A}) = \sum_{k=1}^{m}\sum_{i=1}^{n}\Omega_k\left(\gamma_i\left(\zeta_{F(\tilde{a}_k)}(\delta_i)\right)^6 + \gamma_i\left(\eta_{F(\tilde{a}_k)}(\delta_i)\right)^6\right)$$

$$IE_W(G,\bar{B}) = \sum_{k=1}^{m}\sum_{i=1}^{n}\Omega_k\left(\gamma_i\left(\zeta_{G(\tilde{a}_k)}(\delta_i)\right)^6 + \gamma_i\left(\eta_{G(\tilde{a}_k)}(\delta_i)\right)^6\right)$$

are called the *informational energies* of $(F,\bar{A})$, and $(G,\bar{B})$.

$$C[(F,\bar{A}),(G,\bar{B})] = \sum_{k=1}^{m}\sum_{i=1}^{n}\Omega_k\left(\gamma_i\left[\left(\zeta_{F(\tilde{a}_k)}(\delta_i)\right)^3.\left(\zeta_{G(\tilde{a}_k)}(\delta_i)\right)^3\right] + \gamma_i\left[\left(\eta_{F(\tilde{a}_k)}(\delta_i)\right)^3.\left(\eta_{G(\tilde{a}_k)}(\delta_i)\right)^3\right]\right)$$

is called the *correlation measure* between $(F,\bar{A})$, and $(G,\bar{B})$.

$$CC_W[(F,\bar{A}),(G,\bar{B})] = \frac{C_W[(F,\bar{A}),(G,\bar{B})]}{\sqrt{IE_W(F,\bar{A})}.\sqrt{IE_W(G,\bar{B})}}$$

$$= \frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\Omega_k\left(\gamma_i\left[\left(\zeta_{F(\tilde{a}_k)}(\delta_i)\right)^3.\left(\zeta_{G(\tilde{a}_k)}(\delta_i)\right)^3\right] + \gamma_i\left[\left(\eta_{F(\tilde{a}_k)}(\delta_i)\right)^3.\left(\eta_{G(\tilde{a}_k)}(\delta_i)\right)^3\right]\right)}{\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\Omega_k\left(\gamma_i\left[\left(\zeta_{F(\tilde{a}_k)}(\delta_i)\right)^6 + \left(\eta_{F(\tilde{a}_k)}(\delta_i)\right)^6\right]\right)}.\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\Omega_k\left(\gamma_i\left[\left(\zeta_{G(\tilde{a}_k)}(\delta_i)\right)^6 + \left(\eta_{G(\tilde{a}_k)}(\delta_i)\right)^6\right]\right)}} \quad (3)$$

is called *KKs* between $(F,\bar{A})$, and $(G,\bar{B})$. Further, another a KKs formulas as follows:

$$CC_{MW}[(F,\bar{A}),(G,\bar{B})] = \frac{C_W[(F,\bar{A}),(G,\bar{B})]}{max\{IE_W(F,\bar{A}),IE_W(G,\bar{B})\}}$$

$$= \frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\Omega_k\left(\gamma_i\left[\left(\zeta_{F(\tilde{a}_k)}(\delta_i)\right)^3.\left(\zeta_{G(\tilde{a}_k)}(\delta_i)\right)^3\right] + \gamma_i\left[\left(\eta_{F(\tilde{a}_k)}(\delta_i)\right)^3.\left(\eta_{G(\tilde{a}_k)}(\delta_i)\right)^3\right]\right)}{max\left\{\sum_{k=1}^{m}\sum_{i=1}^{n}\Omega_k\left(\gamma_i\left[\left(\zeta_{F(\tilde{a}_k)}(\delta_i)\right)^6 + \left(\eta_{F(\tilde{a}_k)}(\delta_i)\right)^6\right]\right),\ \sum_{k=1}^{m}\sum_{i=1}^{n}\Omega_k\left(\gamma_i\left[\left(\zeta_{G(\tilde{a}_k)}(\delta_i)\right)^6 + \left(\eta_{G(\tilde{a}_k)}(\delta_i)\right)^6\right]\right)\right\}}. \quad (4)$$

Theorem 4.2. Let $(F,\bar{A})$, and $(G,\bar{B})$ be two FFHSS over U. For the Equation (3),

(i.) $0 \leq CC_W[(F,\bar{A}),(G,\bar{B})] \leq 1$,

(ii.) $CC_W[(F,\bar{A}),(G,\bar{B})] = CC_W[(G,\bar{B}),(F,\bar{A})]$,

(iii.) If $(F,\bar{A})$=$(G,\bar{B})$, that is, $\zeta_{F(\tilde{a}_k)}(\delta_i) = \zeta_{G(\tilde{a}_k)}(\delta_i)$ and $\eta_{F(\tilde{a}_k)}(\delta_i) = \eta_{G(\tilde{a}_k)}(\delta_i)$, $CC_W[(F,\bar{A}),(G,\bar{B})] = 1$.

The conditions in Theorem 4.2 also apply to Equation (4).

4. New Method

Let's consider a set of "s" alternatives such as $A = \{A^1, A^2, \dots, A^s\}$ for the evaluation of the set $E = \{E_1, E_2, \dots, E_n\}$ expert team with $\Omega = \{\Omega_1, \Omega_2, \dots, \Omega_m\}^T$ weights providing $\Omega_k > 0$, $\sum_{k=1}^{m} \Omega_k = 1$. Let $D = \{d_1, d_2, \dots, d_m\}$ specified as a set of attributes. Let $T = \{(t_{1p} \times t_{2p} \times \dots \times t_{mp})\}$, for all $p \in \{1,2,\dots,t\}$ be a collection od sub-attributes with $\gamma = \{\gamma_{1p}, \gamma_{2p}, \dots, \gamma_{mp}\}^T$ weights satisfying the conditions $\gamma_p > 0$, $\sum_{p=1}^{m} \gamma_p = 1$. The elements in the collection of sub-attributes are multi-valued; for the sake of convenience, the elements of T can be expressed as $T = \{\tilde{d}_\partial : \partial \in \{1,2,\dots,k\}\}$. The team of experts $\{E_i : i = 1,2,\dots,n\}$ evaluate the alternatives $\{A^{(z)} : z = 1,2,\dots,s\}$ based on the desired sub-attributes of the considered parameters $\{\tilde{d}_\partial : \partial = 1,2,\dots,k\}$ given in the form of FFHSNs such as $\left(T_{\tilde{d}_\partial}^{(z)}\right)_{n \times \partial} = \left(\zeta_{\tilde{d}_\partial}^{(z)}, \eta_{\tilde{d}_\partial}^{(z)}\right)_{n \times \partial}$, where $0 \leq \zeta_{\tilde{d}_\partial}^{(z)} + \eta_{\tilde{d}_\partial}^{(z)} \leq 1$ for all *i,j*.

Algorithm

Step 1: Create a matrix in the form of FFHSNs for each alternative.

Step 2: Normalize the collective information decision matrix by using the normalization procedure to turn the rating value of the cost-type parameters into benefit-type parameters.

$$h_{ij} = \begin{cases} T_{\tilde{d}_{ij}}^{c}, & for\ cost-type\ parameter \\ T_{\tilde{d}_{ij}}, & for\ benefit-type\ parameter \end{cases}$$

Step 3: Construct the weighted decision matrix for each alternative.

$$\bar{T}_{\tilde{d}_{ij}}^{(z)} = \gamma_i \Omega_i T_{\tilde{d}_{ij}}^{(z)} = \left(\sqrt[3]{1 - \left(\left(1 - \zeta_{\tilde{d}_{ij}}^{3}\right)^{\Omega_i} \right)^{\gamma_j}}, \left(\left(\eta_{\tilde{d}_{ij}}\right)^{\Omega_i} \right)^{\gamma_j} \right) = \left(\zeta_{\tilde{d}_{ij}}^{(z)}, \eta_{\tilde{d}_{ij}}^{(z)}\right) \tag{5}$$

Step 4: Find the indices $h_{ij} = argmax_z\left\{\theta_{ij}^{(z)}\right\}$ and $g_{ij} = argmin_z\left\{\theta_{ij}^{(z)}\right\}$ for each expert and determine the positive ideal solution and negative ideal solution.

$$L^{+} = \left(\zeta_{\tilde{d}_{ij}}^{+}, \eta_{\tilde{d}_{ij}}^{+}\right)_{n \times \partial} = \left(\bar{\zeta}_{\tilde{d}_{ij}}^{h_{ij}}, \bar{\eta}_{\tilde{d}_{ij}}^{h_{ij}}\right) \tag{6}$$

$$L^{-} = \left(\zeta_{\tilde{d}_{ij}}^{-}, \eta_{\tilde{d}_{ij}}^{-}\right)_{n \times \partial} = \left(\bar{\zeta}_{\tilde{d}_{ij}}^{g_{ij}}, \bar{\eta}_{\tilde{d}_{ij}}^{g_{ij}}\right) \tag{7}$$

Step 5: Calculate the KK between each alternative of weighted decision matrices. Compute the positive ideal solution and negative ideal solution.

$$p^{(z)} = CC\left(\bar{A}^{(z)}, L^{+}\right) = \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\zeta_{\tilde{a}_{ij}}^{(z)}.\zeta_{\tilde{a}_{ij}}^{+}+\eta_{\tilde{a}_{ij}}^{(z)}.\eta_{\tilde{a}_{ij}}^{+}\right)}{\sqrt[3]{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta_{\tilde{a}_{ij}}^{(z)}\right)^{3}+\left(\eta_{\tilde{a}_{ij}}^{(z)}\right)^{3}\right)}.\sqrt[3]{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta_{\tilde{a}_{ij}}^{+}\right)^{3}+\left(\eta_{\tilde{a}_{ij}}^{+}\right)^{3}\right)}} \tag{8}$$

$$q^{(z)} = CC\left(\bar{A}^{(z)}, L^{-}\right) = \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\zeta_{\tilde{a}_{ij}}^{(z)}.\zeta_{\tilde{a}_{ij}}^{-}+\eta_{\tilde{a}_{ij}}^{(z)}.\eta_{\tilde{a}_{ij}}^{-}\right)}{\sqrt[3]{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta_{\tilde{a}_{ij}}^{(z)}\right)^{3}+\left(\eta_{\tilde{a}_{ij}}^{(z)}\right)^{3}\right)}.\sqrt[3]{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\zeta_{\tilde{a}_{ij}}^{-}\right)^{3}+\left(\eta_{\tilde{a}_{ij}}^{-}\right)^{3}\right)}} \tag{9}$$

Step 6: Calculate the closeness coefficient for each alternative.

$$R^{(z)} = \frac{H\left(\bar{A}^{(z)}, L^{-}\right)}{H(\bar{A}^{(z)}, L^{+}) + H(\bar{A}^{(z)}, L^{-})} \tag{10}$$

where $H\left(\overline{A}^{(z)}, L^{-}\right) = 1 - q^{(z)}$ and $H\left(\overline{A}^{(z)}, L^{+}\right) = 1 - p^{(z)}$.

Step 7: Select the alternative with a maximum value of the closeness coefficient.

Step 8: Analyze the ranking of the alternatives.s

5. Numerical Example

In this part, we will continue the TOPSIS approach for FFHSS information based on KKs to construct a framework for solving DM concerns. Using the TOPSIS technique, we will be able to find the best potential options with the shortest and biggest distances to the positive ideal solution and negative ideal solution, respectively. By using rankings, the TOPSIS approach assures that the correlation measure can discriminate between positive and negative ideals. In general, researchers use the TOPSIS approach to determine proximity coefficients, unique distance forms, and similar measurements.

Let's assume that there are four orthopaedists ($O = \{O_1, O_2, O_3, O_4\}$) in the group that will decide on the selection of biomaterials. Orthopaedists with weights $(0.2,\ 0.3,\ 0.4,\ 0.1)^T$, including $A = \{A_1, A_2, A_3, A_4\}$ (vitallium(Co-Cr-Mo), stainless steel, high-density polyethylene, polymethylmethacrylate, titanium and titanium alloys) the set of biomedical materials, evaluate the grades of these material types. The group of orthopaedists decides the criteria for the choice of biomedical materials as $L = \{l_1, l_2, l_3\}$ (strength, resistance, tolerance). The sub-attributes corresponding to these attributes are: $l_1=\{d_{11}, d_{12}\}$={tensile strength, fatigue strength}, $l_2=\{d_{21}, d_{22}\}$={corrosion resistance, relative wear resistance}, : $l_3=\{d_{31}, d_{32}\}$={tissue tolerance, elasticity}. Let $T' = l_1 \times l_2 \times l_3$ be a set of sub-attributes:

$T' = l_1 \times l_2 \times l_3 = \{d_{11}, d_{12}\} \times \{d_{21}, d_{22}\} \times \{d_{31}, d_{32}\} = \{(d_{11}, d_{21}, d_{31}),$ $(d_{11}, d_{21}, d_{32}), (d_{11}, d_{22}, d_{31}), (d_{11}, d_{22}, d_{32}), (d_{12}, d_{21}, d_{31}), (d_{12}, d_{21}, d_{32}),$ $(d_{12}, d_{22}, d_{31}), (d_{12}, d_{22}, d_{32})\}$.

Let $T' = \{\tilde{d}_1, \tilde{d}_2, \tilde{d}_3, \tilde{d}_4, \tilde{d}_5, \tilde{d}_6, \tilde{d}_7, \tilde{d}_8\}$ be a set of all multi-attributes with weights $(0.12,\ 0.18,\ 0.1, 0.15,\ 0.05, 0.22, 0.08)^T$. Each orthopaedist will evaluate the ratings of biomedical materials in the form of FFHSNs for each sub-attribute of the considered parameters (Table 1-4). The developed method to find the best alternative is as follows:

Step 1: Create decision matrices for each alternative under defined multi-sub-attributes based on each decision-FFHSN maker's rating.

Step 2: Because all of the criteria are of beneficial kinds, they must be normalized.

Table 1. Decision Matrix for Vittalium(Co-Cr_Mo)

	$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{d}_3$	$\tilde{d}_4$	$\tilde{d}_5$	$\tilde{d}_6$	$\tilde{d}_7$	$\tilde{d}_8$
O_1	(0.33, 0.75)	(0.72, 0.29)	(0.64, 0.75)	(0.59, 0.32)	(0.25, 0.35)	(0.46, 0.54)	(0.52, 0.76)	(0.88, 0.32)
O_2	(0.59, 0.73)	(0.37, 0.64)	(0.25, 0.49)	(0.92, 0.27)	(0.34, 0.82)	(0.27, 0.46)	(0.73, 0.55)	(0.44, 0.65)
O_3	(0.75, 0.35)	(0.28, 0.61)	(0.17, 0.66)	(0.34, 0.48)	(0.46, 0.69)	(0.87, 0.47)	(0.61, 0.78)	(0.28, 0.53)
O_4	(0.83, 0.45)	(0.29, 0.84)	(0.25, 0.47)	(0.46, 0.68)	(0.63, 0.58)	(0.55, 0.69)	(0.47, 0.56)	(0.83, 0.37)

Table 2. Decision Matrix for Stainless Steel

	$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{d}_3$	$\tilde{d}_4$	$\tilde{d}_5$	$\tilde{d}_6$	$\tilde{d}_7$	$\tilde{d}_8$
O_1	(0.69, 0.64)	(0.32, 0.46)	(0.67, 0.55)	(0.33, 0.82)	(0.58, 0.52)	(0.46, 0.62)	(0.73, 0.61)	(0.47, 0.81)
O_2	(0.83, 0.55)	(0.73, 0.46)	(0.93, 0.19)	(0.75, 0.33)	(0.43, 0.65)	(0.95, 0.16)	(0.28, 0.67)	(0.31, 0.82)
O_3	(0.36, 0.72)	(0.48, 0.57)	(0.42, 0.78)	(0.35, 0.65)	(0.62, 0.65)	(0.37, 0.57)	(0.91, 0.18)	(0.68, 0.23)
O_4	(0.53, 0.54)	(0.69, 0.58)	(0.87, 0.25)	(0.86, 0.48)	(0.93, 0.25)	(0.27, 0.44)	(0.53, 0.66)	(0.67, 0.35)

Table 3. Decision Matrix for High-Density Polyethylene

	$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{d}_3$	$\tilde{d}_4$	$\tilde{d}_5$	$\tilde{d}_6$	$\tilde{d}_7$	$\tilde{d}_8$
O_1	(0.51, 0.71)	(0.83, 0.46)	(0.68, 0.45)	(0.46, 0.36)	(0.48, 0.82)	(0.28, 0.38)	(0.83, 0.44)	(0.75, 0.54)
O_2	(0.79, 0.52)	(0.77, 0.34)	(0.86, 0.22)	(0.55, 0.24)	(0.53, 0.68)	(0.72, 0.46)	(0.66, 0.59)	(0.61, 0.42)
O_3	(0.63, 0.78)	(0.45, 0.56)	(0.61, 0.63)	(0.65, 0.48)	(0.69, 0.54)	(0.77, 0.46)	(0.54, 0.77)	(0.46, 0.52)
O_4	(0.48, 0.67)	(0.89, 0.31)	(0.35, 0.54)	(0.56, 0.68)	(0.36, 0.55)	(0.77, 0.51)	(0.77, 0.39)	(0.34, 0.38)

Table 4. Decision Matrix for High-Density Polyethylene

	$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{d}_3$	$\tilde{d}_4$	$\tilde{d}_5$	$\tilde{d}_6$	$\tilde{d}_7$	$\tilde{d}_8$
O_1	(0.50, 0.69)	(0.79, 0.54)	(0.76, 0.45)	(0.43, 0.36)	(0.48, 0.92)	(0.26, 0.46)	(0.81, 0.44)	(0.72, 0.52)
O_2	(0.77, 0.45)	(0.63, 0.48)	(0.83, 0.55)	(0.57, 0.26)	(0.53, 0.69)	(0.70, 0.46)	(0.69, 0.63)	(0.61, 0.45)
O_3	(0.56, 0.45)	(0.44, 0.77)	(0.52, 0.67)	(0.35, 0.49)	(0.72, 0.61)	(0.68, 0.55)	(0.41, 0.88)	(0.58, 0.18)
O_4	(0.43, 0.67)	(0.18, 0.34)	(0.74, 0.45)	(0.56, 0.78)	(0.33, 0.54)	(0.77, 0.34)	(0.33, 0.58)	(0.27, 0.55)

Step 3: Using Equation (5), create a weighted decision matrix for each alternative $\bar{A}^{(z)} = \left(\bar{L}_{ij}^{(z)}\right)_{n \times \partial}$.

Step 4: Determine the positive ideal solution and negative ideal solution based on indices by using Equations (6) and (7).

Step 5: Compute the KK between $\bar{A}^{(z)}$ and positive ideal solution L^{+} by using the Equation 8, given $p^{(1)} = 0.97724$, $p^{(2)} = 0.96823$, $p^{(3)} = 0.96182$, $p^{(4)} = 0.96221$.

Compute the KK between $\bar{A}^{(z)}$ and negative ideal solution L^{-} by using the Equation 9, given $q^{(1)} = 0.95651$, $q^{(2)} = 0.96107$, $q^{(3)} = 0.96463$, $q^{(4)} = 0.96501$.

Step 6: Compute the closeness coefficient by using Equation 10, $R^{(1)} = 0.77498$, $R^{(2)} = 0.55478$, $R^{(3)} = 0.43854$, $R^{(4)} = 0.47487$.

Step 7: Choose the alternatives with maximum closeness coefficient $R^{(1)} = 0.77498$, so $\bar{A}^{(1)}$ is the best alternative.

Step 8: Analyzing the ranking of the alternatives, we can see $R^{(1)} > R^{(2)} > R^{(4)} > R^{(3)}$, so the ranking of the alternatives is $\bar{A}^{(1)} > \bar{A}^{(2)} > \bar{A}^{(4)} > \bar{A}^{(3)}$.

$$L^{+} = \left\{ \begin{matrix} (0.5987, 0.8799), (0.5295, 0.9167), (0.6019, 0.9426), (0.6249, 0.9784), (0.8194, 0.9704), \\ (0.4874, 0.9678), (0.7787, 0.9489), (0.6219, 0.9572)(0.7152, 0.9726), (0.5909, 0.9468), \\ (0.6875, 0.9436), (0.7567, 0.9289), (0.8977, 0.9710), (0.6459, 0.9533), (0.7965, 0.9899), \\ (0.6948, 0.9903), (0.7345, 0.9428), (0.6122, 0.9313), (0.7170, 0.9698), (0.8427, 0.9583), \\ (0.4957, 0.9115), (0.7639, 0.9388), (0.7709, 0.9617), (0.4783, 0.8874)(0.6523, 0.9905), \\ (0.4520, 0.9867), (0.6987, 0.9861), (0.7468, 0.9556), (0.8575, 0.9914), (0.4431, 0.9876), \\ (0.7784, 0.9961), (0.6447, 0.9808) \end{matrix} \right\}$$

$$L^{-} = \left\{ \begin{matrix} (0.7465, 0.9753), (0.6427, 0.9645), (0.7802, 0.9726), (0.6937, 0.9476), (0.8895, 0.9786), \\ (0.5778, 0.9629), (0.8219, 0.9648), (0.7725, 0.7762)\ (0.7359, 0.9576), (0.6968, 0.9681), \\ (0.8964, 0.9123), (0.8946, 0.9133), (0.8895, 0.9269), (0.8827, 0.9687), (0.4462, 0.9036), \\ (0.8471, 0.9445), (0.8578, 0.9075), (0.7197, 0.9313), (0.8892, 0.9426), (0.8167, 0.9365), \\ (0.8026, 0.9743), (0.6812, 0.9416), (0.8547, 0.9702), (0.8967, 0.9874)\ (0.9053, 0.9315), \\ (0.7285, 0.9862), (0.5028, 0.9748), (0.7438, 0.9859), (0.6618, 0.9772), (0.7614, 0.9839), \\ (0.6575, 0.9576), (0.7855, 0.9822) \end{matrix} \right\}$$

6. Conclusion

The concept of FFHSS has been used to handle problems with insufficient information, ambiguity, and inconsistency by taking the MD and ND of the sub-attributes of the examined attributes into account. We created the KK and WKK for FFHSS and exhibited their favourable properties. Similarly, an enhanced TOPSIS technique has been proposed based on the created correlation by taking into account the attribute set with its matching sub-attributes and decision-makers. We created correlation indices to discover positive ideal solution and negative ideal solution. The proximity coefficients were constructed based on the well-established TOPSIS approach for rating alternatives. A numerical demonstration of how to solve the MCGDM issue using the suggested TOPSIS approach has been provided. Finally, based on the findings, it is possible to infer that the suggested approach provides more stability and practicability for decision-makers during the decision-making process. Future studies will focus on providing ideas to various operators in the FFHSS environment about decision-making problems.

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