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# Potential Field Based Intelligent Vehicle Formation Control for Unstructured Roads

Shufeng WANG<sup>1</sup>, Shihao WANG, Xinkai WANG Shandong University of Science and Technology, Qingdao, Shandong, China

Abstract. In order to obtain environmental information safely and efficiently, a four-vehicle formation model is used as an example to study intelligent vehicle formation control under unstructured roads. Firstly, based on the formation distance and environmental constraints, a formation planning framework is proposed; secondly, a formation control model combining artificial potential field and virtual navigator is established by introducing a fixed-point potential field; Based on the elasticity theory and boundary potential field model, the virtual navigator R-value is improved, and then, the formation model stability is verified. Finally, simulation analysis is performed, and the results show that the established formation model can be formed stably and autonomously, and at the same time, formation maintenance and transformation can be realized.

Keywords. Intelligent vehicle, Formation control, Artificial potential field, Elasticity theory

### 1. Introduction

Multi-intelligent vehicle formation technology, as a key technology in the field of intelligent driving, can improve road access efficiency and fuel economy. The artificial potential field method [1] as a common method for formation control, has the advantages of good real-time performance and high system stability [2-3], but there is the plague of local minima. Therefore, Yin Lingyi et al [4] improved the artificial potential field method using the gray wolf algorithm to eliminate the hidden problem of local minima.

According to different application scenarios, it can be divided into structured and unstructured roads formation. At present, there are more studies on structured road formation, for example, Wang Shufeng et al [5] used the artificial potential field method combined with virtual navigator to study the formation control problem of highway. However, there are fewer studies related to unstructured road formation, and Du Guangze et al [6] implemented obstacle avoidance for wheeled unmanned vehicles in unstructured roads, lacking horizontal and vertical interactions to ensure the application of the model to multi-vehicle formations. Tang Chuancong [7] studied vehicle formations for off-road environments, but the formation is single and lacks the constraints of virtual boundaries, making it difficult to cope with complex environmental transformations. L et al [8] proposed multi-navigator submarine formation model to achieve autonomous formation change, but ignored the effect of environmental changes on formation change behavior.

<sup>&</sup>lt;sup>1</sup> Corresponding Author, Shufeng WANG, Shandong University of Science and Technology, Qingdao, Shandong, China; E-mail: shufengwang@sdust.edu.cn

Most of the aforementioned studies on unstructured roads are aimed at a single simple scenario, and ignore the influence of environmental changes on vehicle formation. Therefore, this paper combines the artificial potential field and virtual navigator,takes the vehicle formation in unstructured roads as the research object, introduces the fixed-point potential field and establishes the formation model; combines the elasticity theory and the boundary potential field model, improves the virtual navigator R-value and realizes the vehicle formation transformation behavior.

# 2. Potential Field based on Vehicle Formation Model

#### 2.1. Formation Behavior Analysis

Multi-vehicle formations are designed to address both travel safety issues and to enable vehicles to form and maintain formations during travel. Therefore, we propose a vehicle formation planning framework based on formation constraints, as shown in figure 1.



Figure 1. Formation planning framework

When the vehicle formation runs in an unstructured road, in order to maximize the sensing and obtaining the surrounding environment information, the speed and running direction among the vehicles in the formation should be kept consistent.

# 2.2. Formation Model based on Artificial Potential Fields and Virtual Navigators

The basic idea of artificial potential field method is to establish different potential fields to control the vehicles to produce motion. As shown in figure 2, for example, a four-vehicle formation, a virtual navigator is introduced in the artificial potential field to form an ideal formation with the radius of *R*.And vehicle *i*'s own potential field force is  $f_{vv}$ , to control the spacing with other vehicles, the expression is as follows.

$$f_{vv} = -\nabla U_{vv}(d_{ij})$$
$$U_{vv}(d_{ij}) = \begin{cases} \frac{1}{2}k_{vv}(d_{ij} - L) & d_{ij} \le S_r \\ 0 & d_{ij} > S_r \end{cases}$$
(1)

In the above equation:  $U_{vv}$  is the vehicle potential field;  $d_{ij}$  The actual distance between the number *i* car and the number *j* car;  $k_{vv}$  is a positive gain coefficient; *L* is the ideal formation distance.

The virtual pilot potential force is used to control the formation of formation, and the expression  $f_{vl}$  is as follows.

$$f_{vl} = -\nabla U_{vl} (d_{il})$$

$$U_{vl} (d_{il}) = \frac{1}{2} k_{vl} (d_{il} - R)^2$$
(2)

In the above equation:  $U_{vl}$  is the virtual pilot potential field;  $d_{il}$  is the actual distance between the number *i* car and the virtual navigator;  $k_{vl}$  is the positive gain coefficient of the virtual pilot; *R* is the radius of the circumscribed circle of the formation, in the structured roads,  $R=L/[2\sin(\pi/n)]$ , and *n* is the number of vehicles in the formation.

### 2.3. Determination of the Direction and the Position of Vehicles in the Formation



Figure 2. Formation shape and position map Figure 3. Geometric characteristic diagram of formation

In the traditional potential field, the potential force of the target point directly acts on the controlled object, when the number of controlled objects is too large, the magnitude and direction of the potential force of each individual will be different, making it difficult to determine the direction of the formation. This paper equates the movement direction of the virtual navigator to the running direction of the controlled formation. The expression of the gravitational force  $F_{at}$  of the target point of the virtual navigator is as follows.

$$F_{ot} = -\nabla(U_{ot}) = -\nabla(\frac{1}{2}k_{at} (q_g - q_v))$$
(3)

In the above equation:  $k_{at}$  is the gravitational gain coefficient;  $q_g$  is the spatial position of the target point;  $q_v$  is the spatial position of the virtual navigator.

As shown in figure.3, the circle inside the positive multi-deformation has the geometrical characteristic of indeterminate position, so this paper determines the position of the vehicle in the formation by introducing a fixed point potential field, and selects a certain point on the outer circle of the formation, the fixed point is the intersection point  $x^*$  of the line connecting the outer circle of the formation and the virtual pilot and the target point, its potential field is  $U_{oi}$ , set the formation forward direction horizontal to the right, according to the geometric principle, there must be a formation vehicle between the intersection point  $x^1$  and  $x^2$ , so the range of action of potential field  $U_{oi}$  is.

$$N = \{ x \mid ||x - x^*|| < R^* \}$$
(4)

Also from the geometric relationship in Figure 3 it can be concluded that.

$$R^* = L / \{2\cos[(\arcsin(L/2R))/2]\}$$
(5)

Therefore, the fixed-point potential field force  $f_{oi}$  is defined as follows.

$$f_{oi} = -\nabla U_{oi}(q_i)$$

$$U_{oi}(q_i) = \begin{cases} \frac{1}{2}k_{oi}(d_{oi} - L)^2 & \text{if } x_i \in N \\ K & \text{else} \end{cases}$$
(6)

In the above equation:  $d_{oi}$  is the actual distance between the intelligent vehicle and the fixed point  $x^*$ ;  $k_{oi}$  and K are constants,  $k_{oi} > 0$ .

# 3. Formation Transformation based on Elasticity Theory

#### 3.1. Improvement of Virtual Navigator and Road Boundary Potential Field Model

The role of the virtual navigator is to adapt to the formation changes and maintain the formation by changing the *R* value. In the road environment,  $R=L/[2\sin(\pi/n)]$ , when the number of vehicles *n* is fixed, the *R* value of the virtual navigator is proportional to the ideal formation distance *L*. Similar to the spring property, when the virtual navigator is applied to the unstructured road, the virtual navigator model is improved by combining the elasticity equation  $y=k\Delta x$ .

$$U_{vl} = \frac{1}{2} k_{vl} (d_{il} - R')^2$$

$$R' = F / K_v$$
(7)

In the above: R' is the radius of the outer circle of the formation in different environments;  $k_v$  is the elasticity coefficient of the virtual navigator's action boundary R; F is the boundary potential field force of the "drivable area" to which the virtual navigator is subjected.

Since there is no standard environmental boundary for unstructured roads, the value of F cannot be directly derived, and the environmental constraint of unstructured roads can be combined with the boundary potential field of structured roads to determine the travelable area, and then the value of F can be derived from the boundary potential field of the travelable area. Define the boundary potential field model of the travelable area of the unstructured road.

$$P_r(X) = A(X) \times \cos \left| (2\pi X) / L_w + 1 \right| / 2$$
(8)

In the above:  $L_w$  is the width of the "travelable area"; X is the variation parameter of the width of the "travelable area"; A(X) is the magnitude of the hazard potential energy of the "travelable area", and its expression is.

$$A(X) = 1/2 \left[ \left| \text{sgn}(X - X_1) \right| (1 - P_m) + P_m \right]$$
(9)

In the above:  $X_l$  is the transverse coordinate of the center point of "drivable area";  $P_m$  is the corresponding hazard potential energy, and its value range is [0 1].



Figure 4. Force diagram of virtual navigator

As shown in figure 4, when the formation change needs to reduce the formation distance, the radius of action R of the virtual navigator should be compressed by the external pressure; conversely, when the formation change needs to increase the formation distance, the radius R is compressed by the pulling force. Therefore, the boundary potential field model of the navigable region is improved as.

$$U_r(L') = k_r \times \cos\left(L_f - L'\right) / S_r \times \frac{\pi}{2}$$
(10)

In the above equation: where kr is the road boundary potential field gain coefficient; L' is the ideal formation distance of the vehicle formation under different boundaries;  $L_f$  is the formation distance before formation transformation;  $S_r$  is the vehicle perception range radius, which is also the maximum distance of the formation under non-road conditions.

The forces of the potential field at the boundary of the navigable area on the virtual pilot are.

$$F = -\nabla U_r \left( L' \right) \tag{11}$$

The potential field force at the boundary of the travelable area to which vehicle *i* is subjected is.

$$f_r = -\nabla U_r(q_i)$$

$$U_r(q_i) = 1/2 \times k_r \left| 1/Y_1^2 + 1/Y_2^2 \right|$$
(12)

Therefore, according to the above analysis, the virtual force  $F_{virtual}$  on the vehicle *i* is as follows.

$$F_{virtual} = \dot{q}_i = f_{vl} + \sum_{i=1}^4 f_{vv} + f_{oi} + f_r - b\dot{q}_i$$
(13)

In the above: b is the positive coefficient to achieve the damping effect of the system;  $q_i$  is the position of the number i car.

# 3.2. Formation Stability Analysis in Unstructured Roads

From the Lyapunov function combined with the virtual force  $F_{\text{virtual}}$  applied to the vehicle, the total energy of the formation model system can be derived as.

$$V = \sum_{i=1}^{4} \left[ U_{\nu l} \left( d_{il} \right) + \sum_{j=1, j \neq i}^{4} U_{\nu \nu} \left( d_{ij} \right) + U_{ot} \left( q_i \right) + U_r \left( q_i \right) + \frac{1}{2} \left\| \dot{q}_i \right\|^2 \right]$$
(14)

From the definition of the potential field function, we know that V is non-negative and the time derivative of V is obtained.

$$\dot{V} = \sum_{i=1}^{4} \dot{q}_{i}^{T} \left[ \nabla U_{\nu l} \left( d_{il} \right) + \sum_{j=1, j \neq i}^{4} \nabla U_{\nu \nu} \left( d_{ij} \right) + \nabla U_{r} \left( q_{i} \right) + \nabla U_{ol} \left( q_{i} \right) + \ddot{q}_{i} \right]$$
(15)

$$\ddot{q}_i = -\nabla U_{vl} \left( d_{il} \right) - \sum_{j=1, j \neq 1}^4 \nabla U_{vv} \left( d_{ij} \right) - \nabla U_{ot} \left( q_i \right) - \nabla U_r \left( q_i \right) - b \dot{q}_i$$
(16)

then it can be derived that.

$$\dot{V} = -\sum b \left\| \dot{q} \right\|^2 \tag{17}$$

It can be seen from the equation that V is always non-positive. And from LaSalle invariance principle, it follows that the equilibrium state, which is already close to stability and free of dissipation, will reach stability in this dissipation-free form.

# 4. Formation Path Simulation Test

In order to verify the rationality of the proposed formation model under unstructured roads, MATLAB is used for vehicle formation path simulation. Set the width of the environment before and after the formation transformation to 80m and 50m respectively, and the corresponding ideal distance between vehicles to 60m and 40m respectively, and set the virtual navigator to travel at a uniform speed of 50km/h with the initial position of [0, 0], and the initial state parameters of the formation are shown in table 1.

Table 1. State parameters of formation vehicles			
vehicle	Initial position /m	Potential parameter	value
vehicle (black)	[-35,35]	$\mathbf{k}_{\mathrm{vl}}$	2
vehicle (purple)	[-40,-35]	k <sub>vv</sub>	2
vehicle (red)	[-120,33]	k <sub>oi</sub>	2
vehicle (green)	[-110,-35]	k <sub>re</sub>	10

The simulation results are shown in figure 5, where four intelligent vehicles of different colors gradually form the ideal formation shape under the action of the formation model. In order to better analyze the simulation results, the path simulation is elaborated in this paper.



Figure 5. Schematic diagram of formation transformation

As shown in figure 5(a), when the formation starts, the initial position of each intelligent vehicle does not meet the ideal formation requirements. By the control role of the vehicle formation model, blue and black vehicles in the role of potential occasion force to the southeast direction, while the red and green vehicles to the northeast direction, red and black vehicles travel to 50 meters, the distance between the vehicles for 80m, at this time each vehicle to reach the ideal formation distance, the formation of

the ideal formation, and the ideal formation form to follow the virtual navigator continue to move forward.

As shown in figure 5(b), when the formation encounters a narrowing of the width of the "road" ahead, the "road" boundary potential field force changes the R-value length of the virtual navigator, and at the same time controls the vehicles to perform formation change behavior, finally making the distance between the vehicles reach 40m, forming a new ideal formation.



(b) Change of velocity along the Y axis

Figure 6. Changes of speed parameters during formation

Figure 6 shows the change curve of the velocity of the formation vehicle during the formation process. From figure 6, it can be seen that the vehicle's speed varies from 0 to 50 km/h during the formation process, and finally agrees with the speed of the virtual navigator. As the vehicle is limited by the maximum longitudinal as well as lateral acceleration of the vehicle, its velocity variation has some differences, but the variation trend is flat, which also meets the vehicle dynamics requirements on the basis of meeting the lane requirements.

From the above formation simulation results, it can be seen that the formation path planned by the control model proposed in this paper satisfies the vehicle dynamics constraints and safety constraints and meets the fleet formation objective requirements.

### 5. Conclusion

In this paper, the formation control of intelligent vehicles in unstructured roads is studied, and the specific formation behavior of vehicles is analyzed and the formation control model is established based on the intelligent body formation model of artificial potential field and virtual navigator algorithm and environmental characteristics. And the validity of the formation control model is verified by using the four-vehicle formation as a simulation example, and the main conclusions are as follows.

(1) Based on the formation target, a fixed-point potential field is introduced, and a new target potential field model is established to ensure the accuracy of the formation vehicle movement.

(2) Based on the application of formation transformation, the theory of flexible virtual navigator is proposed, and the "road" boundary potential field is set up to avoid the system instability caused by the increase of virtual navigator, ensure the safety of fleet driving, and improve the flexibility of formation application.

#### References

- Wang D Y, Wang P, Zhang X T, et al. An obstacle Avoidance Strategy for the Wave Glider Based on the Improved Artificial Potential Field and Collision Prediction Model. Ocean Engineering, 2020,vol.206,pp.107356.
- [2] Wu Tong, Wang Jie, Tian Bailing. Periodic event-triggered formation control for multi-UAV systems with collision avoidance. Chinese Journal of Aeronautics, 2022, vol.35, no.8.
- [3] Zhi Yongran, Liu Lei, Guan Bin, et al. Distributed robust adaptive formation control of fixed-wing UAVs with unknown uncertainties and disturbances. Aerospace Science and Technology, 2022, vol. 126.
- [4] YIN Lingyi, et al. Fusion of improved grey wolf optimization algorithm and artificial potential field method for path planning. Electronic Measurement Technology,2022,vol.45,no.03,pp.43-53.
- [5] Wang Shufeng, Zhang Junxin, et al. Research on intelligent vehicle overtaking path planning based on improved artificial potential field. Automotive Technology, 2018,no.3,pp.5-9.
- [6] Du Guangze, et al. Wheeled unmanned vehicle obstacle avoidance algorithm in unstructured road scenarios. Journal of Military Engineering, 2020, vol.41, no.10, pp.2096-2105.
- [7] Tang Chuancong. Research on cooperative control of intelligent networked vehicle queue considering following behavior. Chongqing University of Posts and Telecommunications, 2019.
- [8] Sabattini L, et al. Arbitrarily shaped formations of mobile robots: artificial potential fields and coordinate transformation. Autonomous Robots, 2011, vol.30,no.04,pp.385–397.