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Data Analysis for Lung Cancer: Fermatean Hesitant Fuzzy Sets Approach

Murat KİRİŞCİ[1]
Istanbul University-Cerrahpaşa, Cerrahpaşa Medicine Faculty, Department of Biostatistics and Medical Informatics, Istanbul, Türkiye

Abstract. The Fermatean fuzzy set concept, which was developed by merging Fermatean fuzzy sets with hesitant fuzzy sets, may be utilized in practice to ease the solution of complex multi-criteria decision-making (MCDM) issues. The idea of a Fermatean hesitant fuzzy set is introduced first, followed by the operations associated with this concept. Aggregation operations based on Fermatean hesitant fuzzy sets are provided, and their fundamental features are investigated. A novel MCDM approach obtained using operators has been developed to choose the best choice in practice. Finally, the efficiency of the recommended strategies was demonstrated using a lung cancer case study.

Keywords. Lung cancer, Fermatean hesitant fuzzy set, PHFWA operator, PHFWG operator, decision-making.

1. Introduction

Yager [1] created the q-step orthopair fuzzy set. The essential requirement in this theory is that the combined membership grade and non-membership grade should not be more than 1. Senapati and Yager [2] proposed and researched the Fermatean fuzzy set (FFS). [3] defines and examines the properties of Fermatean arithmetic means, division, and subtraction, which are new FFS transactions. Senapati and Yager[2] have adopted the TOPSIS technique to FFS, which is often employed in multi-criteria decision-making (MCDM) settings. Senapati and Yager [3] expanded on their work by looking at a variety of other operations, such as subtraction, division, and FF arithmetic mean operations, as well as employing the FF weighted product model to solve MCDM difficulties. [4] built more FFS aggregation operators and analyzed the properties associated with these operators. In this work, Donghai et al. [5] introduce the concept of FF linguistic word sets. The operations, scoring, and accuracy functions for these sets were supplied. [6] provides a new measure for FF linguistic word sets' similarity. The new metric is a cross between the Euclidean distance and the cosine similarity measures. [4] defines new weighted aggregated operators applicable to FFSs. Shahzadi and Akram [7] created new aggregated operators and suggested a unique FFSS decision support method. In [8], [9], Hamacher-type operators for Fermatean fuzzy numbers (FFN) are investigated. In [10], the ELECTRE I technique is created utilizing FFSs and the group DM process, in which several people participate at the same time.

1 Corresponding Author, Istanbul University-Cerrahpaşa, Cerrahpaşa Medicine Faculty, Istanbul, Türkiye; E-mail: mkirisci@hotmail.com.

FFS research has recently grown in the literature ([11]-[13], [14], [15], [16]). Despite all of their possible answers, these theories have limitations. Examples of these constraints are how to express the membership function in each individual object and flaws in thinking about the parametrization tool. During the analysis, these limits make it difficult for decision-makers to make effective conclusions.

Torra [17] expanded the FS concept to the hesitant fuzzy set (HFS) concept. This new set of FS may handle scenarios in which the difficulty in constructing the membership degree (MD) does not arise from a margin of error or a specific probability distribution of the likely values, but rather from hesitancy among a few numerous options [18]. As a result, as compared to the FS and its other generalizations, the HFS can more exactly reflect people's reluctance to express their preferences for items. IFS and HFS were then integrated to create a new HFS known as the intuitionistic hesitant fuzzy set (IHFS) [19]. The main idea is to create a case in which, instead of an individual MD and a non-membership degree (ND), humans hesitate between a group of MD and ND and need to represent such a reluctance. The concept of dual HFS was developed and given various features in [20]. The HIVIFS technique was offered as an extension of the dual IVHFS approach [21]. The concept of IHFS was used in [22] to group DM issues using fuzzy cross-entropy. Khan et al. [23] introduced the Pythagorean HFS (PFHS). PHFS compensates when the total of its MDs is less than 1.

Early cancer detection using the theories of FS and its expansions has recently become the most focused issue among researchers. Furthermore, as evidenced by various research, the employment of ideas based on FS and its expansion in the field of medicine is becoming increasingly important nowadays. In the medical profession, researchers often apply these theories' approaches to diagnose cancer and cancer types. Due to the unreliability of Computed Tomography (CT) scans in the diagnosis of lung cancer, specialists disagree on the best course of action. FS-type decision-making systems that can detect these and comparable diseases earlier and more correctly, as well as deliver more dependable outcomes, have become a viable alternative. To overcome the lung cancer screening problem, Liao et al. [24] devised a technique based on double normalization-based multiple clustering approaches. The fuzzy Delphi technique was used in this study to determine the key parameters for lung cancer screening. Unsupervised Deep Fuzzy C-Means Clustering Network (UDFCMN) is suggested in [25] as a new deep learning and clustering model that combines Deep Belief Network (DBN) and Fuzzy C-Means (FCM) to cluster lung cancer patients from lung CT scans. [26] provides a customized similarity metric for attribute selection based on a fuzzy rough fast reduct technique. The suggested technique was tested on a random forest classifier with leukemia, lung, and ovarian cancer gene expression datasets. Ghosh et al. [27] developed a unique intuitionistic fuzzy set for feature extraction from a microarray gene expression dataset using a rough multigranulation approximation. Using an intuitionistic fuzzy soft set-based similarity measure, the suggested approach discovers cancer-mediating human biomarkers. The fuzzy VIKOR approach was first proposed [28] to use sequencing methods to determine the severity of lung cancer patients with a malignant nature and a high death rate. Treatment might be recommended for the patient based on this sequence.

In this work, a novel HFS dubbed the Fermatean hesitant fuzzy set (FHFS) will be presented and its features will be examined. A new scoring function (FHN) will be constructed for comparing Fermatean hesitant fuzzy numbers. In addition, aggregation operators associated with FHS will be investigated, and the MCDM technique associated with FHFS will be introduced. A medical DM case will be investigated to demonstrate how the technique works.

2. New Hesitant Fuzzy Sets

Let us first give some necessary definitions:

Definition 1. Let U be a fixed set. Therefore, the FFS, F over U can be written as: $F = \{(a, m_F(a), n_F(a)\ ): a \in U\}$, where $m_F, n_F: U \to [0,1]$ and the condition $0 \leq m_F^3(a) + n_F^3(a) \leq 1$ is holds.

Definition 2. The FHFS can be given as a set $T = \{(a,\ t_T(a)): a \in U\}$, where $t_T(a)$ denoted the set of some values in the [0,1], that is MD of $a \in U$ to T.

From now on, the Hesitant fuzzy number (HFN) will be denoted as $t = t_T(a)$.

Definition 3. The Pythagorean HFS can be given as a set $P_T = \{(a, m_{t_T}(a), n_{t_T}(a)): a \in U\}$, where $(m_{t_T}(a), n_{t_T}(a))$ are functions from U to unit interval, denoting a possible degree of MD, ND of factor $a \in U$, $\forall\, t_{P_T} \in m_{t_T}(a)$, $\exists\, t_{P_T}{}' \in n_{t_T}(a)$, such that condition $0 \leq h_F^3(a) + h_F'^3(a) \leq 1$, and , $\forall\, t_{P_T}{}' \in m_{t_T}(a)$, $\exists\, t_{P_T} \in n_{t_T}(a)$, such that condition $0 \leq h_F^3(a) + h_F'^3(a) \leq 1$.

Now let's define the new hesitant fuzzy set and examine its properties.

Definition 4. The FHFS can be given as a set $F_T = \left\{\left(a, m_{F_T}(a), n_{F_T}(a)\right): a \in U\right\}$, where

(i.) For each element $a \in U$, $m_{F_T}(a), n_{F_T}(a): U \to [0,1]$, representing a probably MD, ND of factor $a \in U$ in F_T, respectively.

(ii.) $\forall\, t_{F_T}(a) \in m_{F_T}(a), \exists\, t'_{F_T} \in n_{F_T}(a)$, such that $0\ \leq t_{F_T}^3(a) + t'^3_{F_T}(a) \leq 1$,

(iii.) $\forall\, t'_{F_T}(a) \in n_{F_T}(a)$, $\exists\, t_{F_T}(a) \in m_{F_T}(a)$, such that $0 \leq t_{F_T}^3(a) + t'^3_{F_T}(a) \leq 1$.

The set of all factors bound up with FHFSs will be shown by FHFS(U). If U has $\left(a, m_{F_T}(a), n_{F_T}(a)\right)$, then it is called a FHFN and denoted by $\tilde{t} = \{m_{\tilde{t}}, n_{\tilde{t}}\}$.

If $m_{F_T}(a), n_{F_T}(a)$ have one factor, then the FHFS turns into a FFS. Furthermore, if we take ND as $\{0\}$, then FHFS turns into an HFS.

$$B_{F_t}(a) = \bigcup_{t_{F_T}(a) \in m_{F_T}(a),\, t'_{F_T}(a) \in n_{F_T}(a)} \sqrt[3]{1 - t_{F_T}^3 - t'^3_{F_T}}$$

is called a indeterminacy degree of a to F_T, where $1 - t_{F_T}^3 - t'^3_{F_T} \geq 0$ with for any FHFS $F_T = \left\{\left(a, m_{F_T}(a), n_{F_T}(a)\right): a \in U\right\}$.

Definition 5. Let $\tilde{t} = \{m_{\tilde{t}}, n_{\tilde{t}}\}$, $\widetilde{t_1} = \{m_{\widetilde{t_1}}, n_{\widetilde{t_1}}\}$, $\widetilde{t_2} = \{m_{\widetilde{t_2}}, n_{\widetilde{t_2}}\}$ be three FHSS and α>0. Therefore, the basic operations of FHFS as follows:

(i.) $\widetilde{t_1} \cup \widetilde{t_2} = \left(max\left(m_{\widetilde{t_1}}, m_{\widetilde{t_2}}\right), min\left(n_{\widetilde{t_1}}, n_{\widetilde{t_2}}\right)\right)$,

(ii.) $\widetilde{t_1} \cap \widetilde{t_2} = \left(min\left(m_{\widetilde{t_1}}, m_{\widetilde{t_2}}\right), max\left(n_{\widetilde{t_1}}, n_{\widetilde{t_2}}\right)\right)$,

(iii.) $\widetilde{t^c} = \{n_{\tilde{t}}, m_{\tilde{t}}\}$, (iv.) $\tilde{t_1} \oplus \tilde{t_2} = \left(\cup_{t_{\tilde{t_1}} \in m_{\tilde{t_1}},\ t_{\tilde{t_2}} \in m_{\tilde{t_2}}} \left\{\sqrt[3]{t_{\tilde{t_1}}^{\ 3} + t_{\tilde{t_2}}^{\ 3} - t_{\tilde{t_1}}^{\ 3}.t_{\tilde{t_2}}^{\ 3}}\right\},\ \cup_{t_{\tilde{t_1}}' \in n_{\tilde{t_1}},\ t_{\tilde{t_2}}' \in n_{\tilde{t_2}}} \{t_{\tilde{t_1}}',\ t_{\tilde{t_2}}'\}\right)$

(v.) $\tilde{t_1} \otimes \tilde{t_2} = \left(\cup_{t_{\tilde{t_1}} \in m_{\tilde{t_1}},\ t_{\tilde{t_2}} \in m_{\tilde{t_2}}} \{t_{\tilde{t_1}},\ t_{\tilde{t_2}}\},\ \cup_{t_{\tilde{t_1}}' \in n_{\tilde{t_1}},\ t_{\tilde{t_2}}' \in n_{\tilde{t_2}}} \left\{\sqrt[3]{t_{\tilde{t_1}}'^{3} + t_{\tilde{t_2}}'^{3} - t_{\tilde{t_1}}'^{3}.t_{\tilde{t_2}}'^{3}}\right\}\right)$

(vi.) $\alpha\tilde{t} = \left(\cup_{t_{\tilde{t}} \in m_{t_{\tilde{t}}}} \left\{\sqrt[3]{1-(1-(t_{\tilde{t}})^3)^{\alpha}}\right\},\ \cup_{t_{\tilde{t}}' \in n_{t_{\tilde{t}}}} \{(t_{\tilde{t}}')^{\alpha}\}\right),\ \alpha > 0,$

(vii.) $\tilde{t}^{\alpha} = \left(\cup_{t_{\tilde{t}} \in m_{t_{\tilde{t}}}} \{(t_{\tilde{t}})^{\alpha}\}, \cup_{t_{\tilde{t}}' \in n_{t_{\tilde{t}}'}} \left\{\sqrt[3]{1-(1-(t_{\tilde{t}}')^3)^{\alpha}}\right\}\right),\ \alpha > 0.$

Theorem 6. Let $\tilde{t} = \{m_{\tilde{t}}, n_{\tilde{t}}\}$, $\tilde{t_1} = \{m_{\tilde{t_1}}, n_{\tilde{t_1}}\}$, $\tilde{t_2} = \{m_{\tilde{t_2}}, n_{\tilde{t_2}}\}$ be three FHSS and α, α_1, α_2>0.

(i.) $\tilde{t_1} \oplus \tilde{t_2} = \tilde{t_2} \oplus \tilde{t_1}$,

(ii.) $\tilde{t_1} \otimes \tilde{t_2} = \tilde{t_2} \otimes \tilde{t_1}$,

(iii.) $\alpha(\tilde{t_1} \oplus \tilde{t_2}) = (\alpha\tilde{t_1} \oplus \alpha\tilde{t_2})$,

(iv.) $(\alpha_1 + \alpha_2)\,\tilde{t} = \alpha_1\,\tilde{t} + \alpha_2\,\tilde{t}$,

(v.) $(\tilde{t_1} \otimes \tilde{t_2})^{\alpha} = \tilde{t_1}^{\alpha} \otimes \tilde{t_2}^{\alpha}$,

(vi.) $\tilde{t}^{\alpha_1+\alpha_2} = \tilde{t}^{\alpha_1} \otimes \tilde{t}^{\alpha_2}$.

Definition 7. Let $\tilde{t} = \{m_{\tilde{t}}, n_{\tilde{t}}\}$ be a FHFS. Then,

$$S(\tilde{t}) = \frac{1}{E_{t_{\tilde{t}}} \in m_{\tilde{t}}} \sum_{t_{\tilde{t}} \in m_{\tilde{t}}} t_{\tilde{t}}^{\ 3} - \frac{1}{E_{t_{\tilde{t}}'} \in n_{\tilde{t}}} \sum_{t_{\tilde{t}}' \in m_{\tilde{t}}} t_{\tilde{t}}'^{3}, \qquad S(\tilde{t}) \in [-1,1]$$

is called a score function of $\tilde{t}$, where $E_{t_{\tilde{t}}}$ and $E_{t_{\tilde{t}}'}$ denote the number of elements $m_{\tilde{t}}$ and $n_{\tilde{t}}$, respectively.

Let $\tilde{t_1} = \{m_{\tilde{t_1}}, n_{\tilde{t_1}}\}$, $\tilde{t_2} = \{m_{\tilde{t_2}}, n_{\tilde{t_2}}\}$ be two FHSS. Then,

(i.) If $S(\tilde{t_1}) < S(\tilde{t_2})$, then $\tilde{t_1} < \tilde{t_2}$,

(ii.) If $S(\tilde{t_1}) > S(\tilde{t_2})$, then $\tilde{t_1} > \tilde{t_2}$,

(iii.) If $S(\tilde{t_1}) = S(\tilde{t_2})$, then $\tilde{t_1} \sim \tilde{t_2}$.

Definition 8 Let $\tilde{t} = \{m_{\tilde{t}}, n_{\tilde{t}}\}$ be a FHFS. Then,

$$A(\tilde{t}) = \frac{1}{E_{t_{\tilde{t}}} \in m_{\tilde{t}}} \sum_{t_{\tilde{t}} \in m_{\tilde{t}}} t_{\tilde{t}}{}^{3} + \frac{1}{E_{t_{\tilde{t}}'} \in n_{\tilde{t}}} \sum_{t_{\tilde{t}}' \in m_{\tilde{t}}} t_{\tilde{t}}'^{3}$$

is called a accuracy function of $\tilde{t}$.
Let $\widetilde{t_1} = \{m_{\widetilde{t_1}}, n_{\widetilde{t_1}}\}$, $\widetilde{t_2} = \{m_{\widetilde{t_2}}, n_{\widetilde{t_2}}\}$ be two FHSS. Then,

(i.) If $S(\widetilde{t_1}) < S(\widetilde{t_2})$, then $\widetilde{t_1} < \widetilde{t_2}$,

(ii.) If $S(\widetilde{t_1}) > S(\widetilde{t_2})$, then $\widetilde{t_1} > \widetilde{t_2}$,

(iii.) If $S(\widetilde{t_1}) = S(\widetilde{t_2})$, then $\widetilde{t_1} \sim \widetilde{t_2}$.

(a) If $A(\widetilde{t_1}) < A(\widetilde{t_2})$, then $\widetilde{t_1} < \widetilde{t_2}$,

(b) If $A(\widetilde{t_1}) > A(\widetilde{t_2})$, then $\widetilde{t_1} > \widetilde{t_2}$,

(c) If $A(\widetilde{t_1}) = A(\widetilde{t_2})$, then $\widetilde{t_1} = \widetilde{t_2}$

Now, we will give aggregation operators based on FHFSs.

Definition 9. Let $\tilde{t}_i = \{m_{\tilde{t}_i}, n_{\tilde{t}_i}\}$ $(1 \le i \le n)$ be a FHFS. Then the FHF weighted average (FHFWA) operator is a function $\text{FHFWA}: \text{FHFN}^n \to \text{FHFN}$,

$$FHFWA(\tilde{t}_1, \tilde{t}_2, \dots, \tilde{t}_n) = \omega_1 \tilde{t}_1 \oplus \omega_2 \tilde{t}_2 \oplus \dots \oplus \omega_n \tilde{t}_n$$

$$= \left(\bigcup_{t_{\tilde{t}_1} \in m_{\tilde{t}_1},\ t_{\tilde{t}_2} \in m_{\tilde{t}_2}, \dots, t_{\tilde{t}_n} \in m_{\tilde{t}_n}} \sqrt[3]{1 - \prod_{i=1}^{n} \left(1 - t_{t_{\tilde{t}_i}}^{3}\right)^{\omega_i}},\ \bigcup_{t_{\tilde{t}_1}' \in n_{\tilde{t}_1},\ t_{\tilde{t}_2}' \in n_{\tilde{t}_2}, \dots, t_{\tilde{t}_n}' \in n_{\tilde{t}_n}} \prod_{i=1}^{n} \left(t_{t_{\tilde{t}_i}}'\right)^{\omega_i} \right)$$

where ω_i is a weight vector of $\tilde{t}_i$ with $\omega_i \ge 0$, $\sum_{i=1}^{n} \omega_i = 1$.

Definition 10 Let $\tilde{t}_i = \{m_{\tilde{t}_i}, n_{\tilde{t}_i}\}$ $(1 \le i \le n)$ be a FHFS and ω_i be a weight vector of $\tilde{t}_i$. Then the FHF weighted geometric (FHFWG) operator is a function $\text{FHFWG}: \text{FHFN}^n \to \text{FHFN}$,

$$FHFWG(\tilde{t}_1, \tilde{t}_2, \dots, \tilde{t}_n) = \omega_1 \tilde{t}_1 \otimes \omega_2 \tilde{t}_2 \otimes \dots \otimes \omega_n \tilde{t}_n$$

$$= \left(\bigcup_{t_{\tilde{t}_1} \in m_{\tilde{t}_1},\ t_{\tilde{t}_2} \in m_{\tilde{t}_2}, \dots, t_{\tilde{t}_n} \in m_{\tilde{t}_n}} \prod_{i=1}^{n} \left(t_{t_{\tilde{t}_i}}\right)^{\omega_i},\ \bigcup_{t_{\tilde{t}_1}' \in n_{\tilde{t}_1},\ t_{\tilde{t}_2}' \in n_{\tilde{t}_2}, \dots, t_{\tilde{t}_n}' \in n_{\tilde{t}_n}} \sqrt[3]{1 - \prod_{i=1}^{n} \left(1 - t_{t_{\tilde{t}_i}}'^{3}\right)^{\omega_i}} \right)$$

3. New Method

3.1 Scenario

A decision matrix whose entries provide the evaluation info of all options with regard to an attribute can be employed to demonstrate a MCDM issue. We construct a FH decision matrix, the components of which are FHFNs that provide not only the knowledge that the U_i satisfies the attributes of O_j, but also the info that the U_i does not compensate the attributes O_j.

Consider a MCDM scenario with anonymity and a discrete set of alternatives $U = \{U_1, U_2, \dots, U_m\}$. Let U be the attribute-containing discussion universe. Take the set of all attributes $O = \{O_1, O_2, \dots, O_n\}$. To assess the performance of the i-th alternative U_i under the j-th attribute O_j, the expert must give not only info indicating that the alternative U_i fulfills the attribute O_j but also info indicating that the alternative U_i does not compensate the attribute O_j. This two-part info can be stated by m_{ij}, n_{ij} which indicate the MDs in the attribute O_j, then the performance of the alternative U_i under the attribute O_j can be stated by an FHFN $\tilde{t}_{ij} = \{m_{ij}, n_{ij}\}$ with the condition that for all $t_{ij} \in m_{ij}$, $\exists t'_{ij} \in< n_{ij}$ such that $0 \leq (t_{ij})^2 + (t'_{ij})^2 \leq 1$, and for all $t_{ij} \in n_{ij}$, $\exists t'_{ij} \in m_{ij}$ such that $0 \leq (t_{ij})^2 + (t'_{ij})^2 \leq 1$. The FHF decision matrix T can be given as:

$$T = \begin{bmatrix} \tilde{t}_{11} & \tilde{t}_{12} & \dots & \tilde{t}_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{t}_{m1} & \tilde{t}_{m2} & \dots & \tilde{t}_{mn} \end{bmatrix}$$

Given that each attribute has a variable level of relevance, $\omega = \{\omega_1, \omega_2, \dots, \omega_n\}^T$ where $0 \leq \omega_j \leq 1$, $\sum_{j=1}^{n} \omega_j = 1$, represents the weight vector of all the attributes provided by the decision-makers. In general, decision-makers must assess the relative value of the traits. As a result of the complexity and ambiguity of real DM situations as well as the essentially subjective character of human thought, knowledge concerning attribute weights is typically insufficient. For $i \neq j$, the following forms of attribute weight information may be supplied, and it is assumed that the decision-makers supply this data,

(1) A weak ranking: $\{\omega_i \geq \omega_j\}$;
(2) A strict ranking: $\{\omega_i - \omega_j \geq \beta_i (> 0)\}$;
(3) A ranking with multiples: $\{\omega_i \geq \beta_i \omega_j\}$, $0 \leq \beta_i \leq 1$;
(4) An interval form: $\{\lambda_i \leq \omega_i \leq \lambda_i + \beta_i\}$, $0 \leq \lambda_i \leq \lambda_i + \beta_i \leq 1$;
(5) A ranking of differences: $\{\omega_i - \omega_j \geq \omega_k - \omega_l\}$, $for\ j \neq k \neq l$.

The FHF distance between PHFNs $\tilde{t}_{ij}$ and $\tilde{t}_{kl}$ defined as:

$$d(\tilde{t}_{ij}, \tilde{t}_{kj}) = \frac{1}{2}\left[\frac{1}{l}\sum_{\lambda=1}^{l}\left[\left(h_{ij}^{\beta(\lambda)}\right)^2 - \left(h_{kj}^{\beta(\lambda)}\right)^2\right] + \frac{1}{l}\sum_{\lambda=1}^{l}\left[\left(h'^{\beta(\lambda)}_{ij}\right)^2 - \left(h'^{\beta(\lambda)}_{kj}\right)^2\right]\right]$$

Then, we have

$$D_{ij}(\omega) = \sum_{k=1}^{m} \omega_j d(\tilde{t}_{ij}, \tilde{t}_{kj})$$

$$D_j(\omega) = \sum_{k=1}^{m} D_{ij}(\omega) = \sum_{i=1}^{m}\sum_{k=1}^{m} \omega_j \left(\frac{1}{2}\frac{1}{l}\sum_{\lambda=1}^{l}\left[\left(h_{ij}^{\beta(\lambda)}\right)^2 - \left(h_{kj}^{\beta(\lambda)}\right)^2\right] + \frac{1}{l}\sum_{\lambda=1}^{l}\left[\left(h_{ij}'^{\beta(\lambda)}\right)^2 - \left(h_{kj}'^{\beta(\lambda)}\right)^2\right] \right)$$

For normalizing

$$\omega_j = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m}\left(\frac{1}{l}\sum_{\lambda=1}^{l}\left[\left(h_{ij}^{\beta(\lambda)}\right)^2 - \left(h_{kj}^{\beta(\lambda)}\right)^2\right] + \frac{1}{l}\sum_{\lambda=1}^{l}\left[\left(h_{ij}'^{\beta(\lambda)}\right)^2 - \left(h_{kj}'^{\beta(\lambda)}\right)^2\right]\right)}{\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{k=1}^{m}\left(\frac{1}{2}\frac{1}{l}\sum_{\lambda=1}^{l}\left[\left(h_{ij}^{\beta(\lambda)}\right)^2 - \left(h_{kj}^{\beta(\lambda)}\right)^2\right] + \frac{1}{l}\sum_{\lambda=1}^{l}\left[\left(h_{ij}'^{\beta(\lambda)}\right)^2 - \left(h_{kj}'^{\beta(\lambda)}\right)^2\right]\right)\right]} \quad (1)$$

There are instances where knowledge of the weight vector is not entirely unknown but is just half unknown. For these situations, we build the constrained optimization model shown below using the set of known weight information:

$$(M1)\begin{cases} maxD(\omega) = \sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{k=1}^{m}\omega_j\left(\frac{1}{l}\sum_{\lambda=1}^{l}\left[\left(h_{ij}^{\beta(\lambda)}\right)^2 - \left(h_{kj}^{\beta(\lambda)}\right)^2\right] + \frac{1}{l}\sum_{\lambda=1}^{l}\left[\left(h_{ij}'^{\beta(\lambda)}\right)^2 - \left(h_{kj}'^{\beta(\lambda)}\right)^2\right]\right)\right] \\ s.t.\ \ \omega \in \Delta,\ \omega_j \geq 0, \quad j = 1,2,\ldots,n, \quad \sum_{j=1}^{n}\omega_j = 1 \end{cases}$$

where Δ is a collection of constraints that the weight value j must meet to comply with real-world needs.

3.2 Algorithm

Step 1: Create the $C = (\tilde{t}_{ij})_{m\times n}$-matrices for PHF decisions. The FHF decision matrix can be transformed into the normalized FHF decision matrix $D_N = (\mu_{ij})_{m\times n}$ if the attribute has two categories, such as cost and benefit attributes, where

$$\mu_{ij} = \begin{cases} \tilde{t}_{ij}, & if\ the\ attributes\ is\ of\ benefit\ type \\ {\tilde{t}_{ij}}^{c}, & if\ the\ attributes\ is\ of\ cost\ type \end{cases}$$

Step 2: Equation (1) can be used to obtain the attribute weights if the info about the attribute weights is fully unknown; if the info about the attribute weights is only partially known, the attribute weights are gained by solving the model (M1).

Step 3: Using the developed aggregation operators, compute the FHFN $\tilde{t}_i$ for the alternatives U_i.

Step 4: Calculate the scores and the level of accuracy for each overall value $\tilde{t}_i$ using the scoring function equation.

Step 5: Rank the alternatives O_j ; choice the best one.

4. Lung Cancer Analysis

This section has been written to verify the suggested approach.

The possible results of diagnosis are $O = \{O_1, O_2, O_3, O_4\}$ as disease-free, benign tumors, pneumonia, and lung cancer, respectively. Four criteria of great significance according to the patient's conditions for differentiating lung cancer together with CT images can be viewed as four criteria for diagnosis, which are recorded $A = \{A_1, A_2, A_3, A_4\}$ as CT diagnosis report, occupational exposure, history of malignant tumors, family history of lung cancer. For i=1,2,3,4 and j=1,2,3,4, according to their expertise and experience, two physicians should supply the FHFNs as the evaluation values $\tilde{t}_{ij}$. Assuming that the attribute weight info is only partially available, the known weight info is as follows:

$$\Delta = \left\{0.14 \le \omega_1 \le 0.19,\ 0.18 \le \omega_2 \le 0.21,\ 0.33 \le \omega_3 \le 0.37,\ 0.35 \le \omega_4 \le 0.38,\ \sum_{j=1}^{4} \omega_j = 1\right\}$$

Step 1: Normalize the decision matrix.

Table 1: FHFS Decision Matrix

	O1	O2	O3	O4
U1	((0.8,0.7), (0.6,0.5))	((0.3,0.5), (0.7,0.9))	((0.6), (0.6,0.7,0.8))	((0.8,0.9), (0.4,0.5))
U2	((0.6,0.5, 0.7), (0.8))	((0.5,0.6), (0.6,0.7))	((0.8,0.9), (0.4,0.5))	((0.7,0.8), (0.4,0.6))
U3	((0.6,0.7), (0.6,0.8, 0.9))	((0.6,0.7, 0.9), (0.3,0.4))	((0.5,0.7), (0.5,0.6))	((0.4,0.5), (0.7,0.9))
U4	((0.9), (0.3,0.4,0.6))	((0.6,0.7), (0.6))	((0.5,0.6,0.4), (0.7,0.8))	((0.6,0.5), (0.8))

Table 2: FHFN Decision Matrix

	O1	O2	O3	O4
U1	((0.8,0.8,0.7), (0.6,0.6,0.5))	((0.3,0.3,0.5), (0.7,0.7,0.9))	((0.6,0.6,0.6), (0.6,0.7,0.8))	((0.8,0.8,0.9), (0.4,0.4,0.5))
U2	((0.6,0.5, 0.7), (0.8,0.80.8))	((0.5,0.5,0.6), (0.6,0.6,0.7))	((0.8,0.8,0.9), (0.4,0.4,0.5))	((0.7,0.7,0.8), (0.4,0.4,0.6))
U3	((0.6,0.6,0.7), (0.6,0.8, 0.9))	((0.6,0.7, 0.9), (0.3,0.3,0.4))	((0.5,0.5,0.7), (0.5,0.5,0.6))	((0.4,0.4,0.5), (0.7,0.7,0.9))
U4	((0.9,0.9,0.9), (0.3,0.4,0.6))	((0.6,0.6,0.7), (0.6,0.6,0.6))	((0.5,0.6,0.4), (0.7,0.7,0.8))	((0.6,0.6,0.5), (0.8,0.8,0.8))

Step 2: Use the model (M1) to create the

$$(M1)\begin{cases} maxD(\omega) = 4.056\omega_1 + 4.193\omega_2 + 0.3648\omega_3 + 0.4324\omega_4 \\ s.t.\ \omega \in \Delta,\ \omega_j \ge 0,\ \ j = 1,2,3,4 \end{cases}$$

single objective model. We obtain the ideal weight vector $(0.14, 0.18, 0.33, 0.35)^T$ by solving this model.

Step 3: To get the overall values $\tilde{t}_i$ of the alternatives O_i, use the decision-making information provided in matrix $C = \left(\tilde{t}_{ij}\right)_{m\times n}$ and the FHFWA operator.

Step 4: Score function values: $S(\tilde{t}_1)$ = -0.0361; $S(\tilde{t}_2)$ = 0.3498; $S(\tilde{t}_3)$ = 0.0643; $S(\tilde{t}_4)$ =0.4037.

Step 5: Rank the options according to the score values. Then, $O_4 > O_2 > O_3 > O_1$.

The same ranking will be reached when trading with FHFWG in Step 3

5. Conclusion

In this study, the FFS and the HFS were combined to create an FHFS. The operations and comparison methods were provided for FHFNs. The FHFWA and FHFWG operators were proposed to aggregate the FHFNs provided by the expert in order to address the MCDM challenges under the FHFS environment. To solve the MCDM issues in various scenarios, an MCDM technique is constructed and integrated with the provided operators. The numerical example based on lung cancer is then given to illustrate the uses and benefits of the suggested methodologies.

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