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Gold or BTC:The Best Trading Strategy

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Abstract. This paper focuses on the Trading strategy issues of gold, US dollar and bitcoin in the financial markets.We are supposed to deal with the C problem of 2022 Mathematical Contest In Modeling. To solve this problem, we firstly build a VMD(CEEMD)-LSTM-AdaBoost-SVM model to predict the price of gold and BTC. Based on the forecast data, we build the optimal investment dynamic planning problem by using Floyd algorithm to find the maximum profit route of daily optimal trading strategy from September 11, 2016 to September 11, 2021. Our assets went from an initial $1,000 to $52,048 with an annual profit margin of 219.58%. Then, we performed sensitivity analysis and error analysis of the developed model. Finally, we improved and extended the existing model by building the Measure-VaR multi-stage portfolio model and solve it by using the PSO algorithm, the theoretical results is good.

Keywords. Signal Decomposition; LSTM; AdaBoost; Floyd algorithm; PSO algorithm

1. Introduction

Financial Management has always been a topic of concern. How to develop a suitable trade strategy is a very delicate matter.In 1952, Markowit[1] introduced the portfolio theory, which transformed the qualitative analysis of return and risk into quantitative analysis, and was of transitional significance. After that, people started trading strategies based on quantitative analysis in a very diverse way. At the same time, gold and bitcoin, two very common investment options in today's market, are chosen by different people in different ways due to their different characteristics. Based on quantitative analysis, it is very meaningful to provide investors with more valuable and personalized references in a complex financial market.

Now, let's have a discussion on trade strategies based on a real situation, assuming that today is September 11, 2016 and we have $1,000 as initial capital to solve for the best trade strategy when we reach September 10, 2021.

2. Price Forecasting Model

2.1. Problem Analysis

When we are on a date before September 10, 2021, we need to make a reasonable choice based on the historical prices available since we do not know the actual gold and

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bitcoin closing prices on that day. A structured choice is that we first predict the gold and bitcoin prices from the historical data. Then make an investment decision by using the data obtained from the forecast.

First, we make a prediction. Due to gold, bitcoin's price curve is subject to many instabilities. We first decompose it into different frequency bands by performing signal decomposition. For the gold price curve, we use VMD to decompose it; for the bitcoin price curve, we use CEEMDAN combined with principal component analysis to decompose it. For the decomposed signal curves, we use LSTM algorithm to fit the past data to predict the future data if the curve fluctuates significantly, and we use AdaBoost algorithm to fit the past data to predict the future data if the curve changes smoothly.Finally, we use the SVM algorithm to reconstruct the signal to get the corresponding predicted price data.

2.2. Data Description

2.2.1. Missing Value Interpolation

The title gives the price data for gold and bitcoin from September 11, 2016 to September 10, 2021. Here we first examine the data and we can find that there are some missing data in gold. Considering that only the data involved in the title can be used, we use the mean interpolation method to fill in the missing data in gold. The mean interpolation method is based on yesterday's and tomorrow's data to determine the data of the day, which effectively reduces the impact of uncontrollable factors on the missing values. The formula is as follows:

$$gprice_j = \frac{gprice_{j-1} + gprice_{j+1}}{2} \tag{1}$$

Where $gprice_j$ denotes the closed price of gold on day j.Based on this, we can get the price curves of gold and bitcoin respectively.

2.2.2. Decomposition of Signals

After the interpolation is complete, we will act in line with the logic of the day based on the actual situation of the day we are in.The first thing we naturally do is to forecast the future data.Now we only have closed price data for gold and bitcoin.And due to the complexity of price formation factors, closed prices do not exhibit a simple linear forecast. The first thing we naturally do is to forecast the future data.

Here we first decompose the price curve into signals and then analyze the forecasts for different frequencies.

Common signal decomposition methods include: EMD, EEMD, CEEMDAN, and VMD.Taking gold as an example, according to the analysis of the spectrum graph we find that the spectrum graph effect of VMD has the smallest frequency mixing phenomenon, which indicates that the signal decomposition effect of VMD is the best, so we choose the VMD algorithm to decompose the price curve of gold, the specific steps are as follows.VMD aims to find several eigenmodular functions to satisfy that the bandwidth sum of IMFs is minimal.

At the end of the iteration, we can obtain an IMF component in figure 1. In turn, we can find the signal decomposition of the gold price curve and the corresponding spectrogram.Combining the pictures, we can see that VMD divides the gold price curve into 4 segments, where IMF 1 represents the high frequency band, IMF 2 represents the medium frequency band, IMF3 represents the low frequency band, and RES represents the trend band.Among them, IMF 1 and IMF 2 exhibit small long-term fluctuations around the median, IMF 3 exhibits large periodic fluctuations, and RES has a good trend fit with the actual gold price curve.

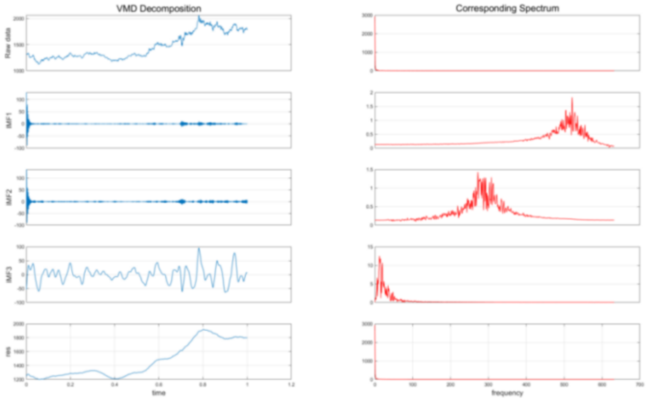

Figure 1. VMD Analysis Chart of Gold

2.3. Gold Forecasting Model

Let's still start with gold as an example, and here we assume that we go back to September 11, 2016 and start trading behavior day by day. Obviously, our trade strategy is to forecast the future data first, and then based on the forecast data, we can develop a suitable investment strategy. But we do not have enough data to support us to build a suitable forecasting model in the first few days, and arbitrarily forecasting will have bad effects instead. Therefore, we plan not to build a forecasting model for the first 27 days, i.e., from September 11, 2016 to October 2, 2016.

2.3.1. LSTM Predictive Model

We now start with October 3, 2016, at which point we have a certain amount of data, so we can make relevant predictions based on this data, including the historical data for the signal decomposition mentioned in 4.1. IMF 1 and IMF 2 exhibit frequent small fluctuations around the median. Traditional models to predict this feature of the array are very complex. The LSTM network is a recursive model that learns and remembers long-term information about sequences, which is very effective for data with sequential characteristics, and it is able to mine the data for temporal as well as semantic information. Using this ability, we can effectively process and predict data with this characteristic in high as well as low frequency bands.[2] The main calculation principle of LSTM is shown in Figure 2.

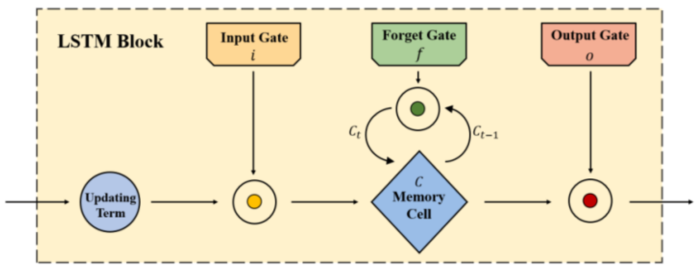

Figure 2. LSTM Basic Cell Structure Diagram

The main core of LSTM is the three memory cells of *input*, *forget*, and *output*. Each memory cell is controlled by a gate, which is recorded as 1 if it is saved and 0 if it is not. These three gates control the input of information, the saving of information, and the output of information respectively.

$$i_t = \sigma(W_{ix}x_t + W_{ih}h_{t-1}) \tag{2}$$

$$f_t = \sigma(W_{fx}x_t + W_{fh}h_{t-1}) \tag{3}$$

$$o_t = \sigma(W_{ox}x_t + W_{oh}h_{t-1}) \tag{4}$$

$$c_t = f_t \cdot c_{t-1} + i_t \cdot \tanh(W_{cx}x_t + W_{ch}h_{t-1}) \tag{5}$$

$$h_t = c_t \cdot o_t \tag{6}$$

In the above table, x denotes the input vector, h denotes the output vector, and the matrix W denotes the parameters to be trained. σ denotes the sigmoid[3] nonlinear function and $\tanh$ [4]denotes the hyperbolic tangent function.Next we present the overall steps for the implementation of the LSTM algorithm through MATLAB.

We find that the LSTM model performs well for both the high and medium frequency bands due to the availability of sufficient training samples. Figure 3 shows the results of one of the high frequency band model runs, and we can intuitively feel that the fitting results are very good.

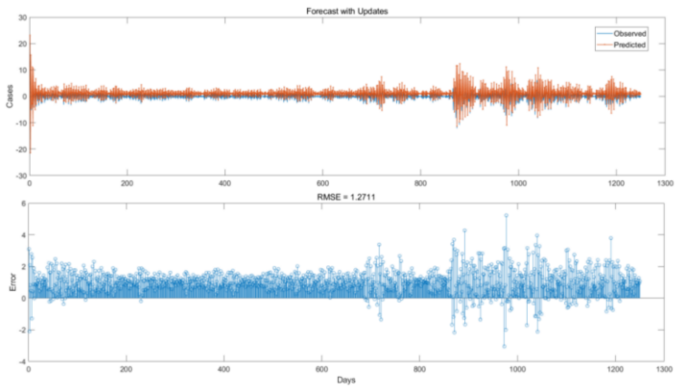

Figure 3. Gold Forecast Result Chart for High Frequency Band

2.3.2. AdaBoost Predictive Model

Next, we analyze and forecast IMF 3 and the trend term. Since the performance of the data is relatively stable compared to IMF 1 and IMF 2, and considering that the trend of gold price movement is affected by many factors, we choose the AdaBoost model to forecast these two sets of data.The full name of AdaBoost is Adaptive boosting, and the idea of the algorithm is that multiple basic classifiers are trained by continuously updating their weights for different sample points only, and then combining the wake-regression models with different weights to form a strong regression model. The self-adaptability of model lies in the fact that the samples that are wrong in the previous basic classifier are enhanced and the weighted samples are used here to train the next basic classifier. At the same time, a new weak classifier is added in each round until the maximum iteration probability or some predefined sufficiently small error.[5] The main calculation principle of AdaBoost is shown in figure 4. And figure 5 shows the forecast result of the trend items of the gold that calculated by AdaBoost, which is very consistent with reality.

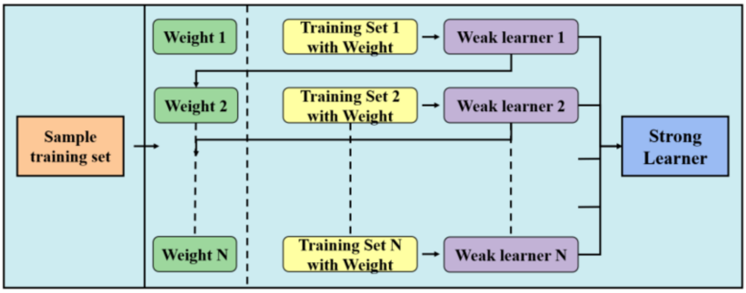

Figure 4. AdaBoost Algorithm Flow Chart

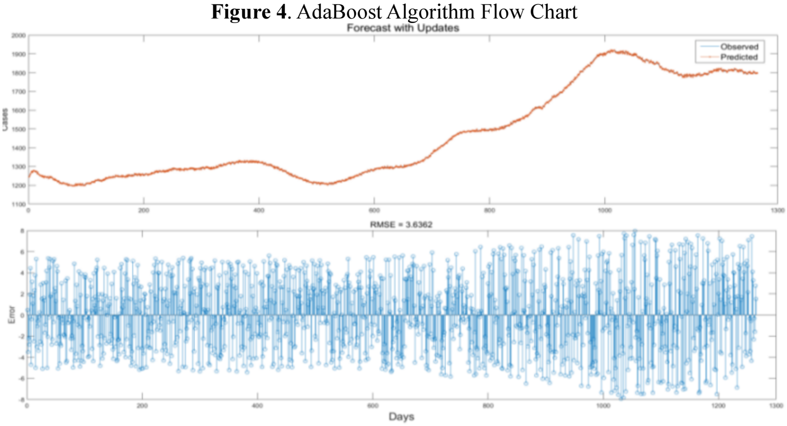

Figure 5. Gold Forecast Result Chart for Trend Items

2.3.3. Support Vector Machine(SVM) Signal Reconfiguration

After obtaining the data for each signal, we use Support Vector Machine(SVM) to reconfigure the signal into the predicted gold price values. Here we apply the support vector regression machine to analyze the data.

As shown in the figure 6, the above models predict the gold price very well.

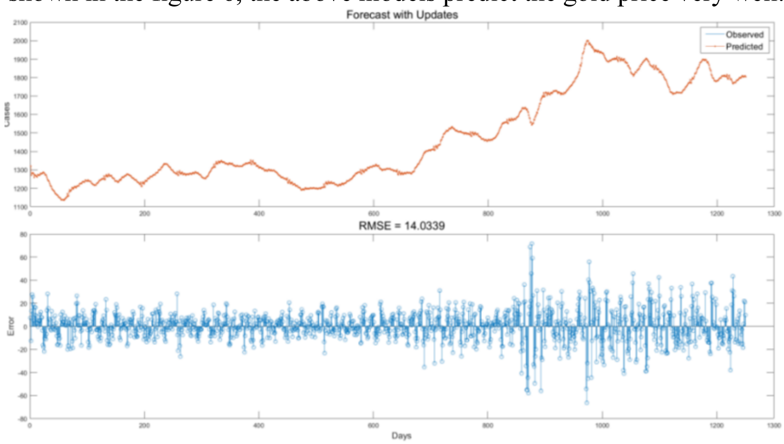

Figure 6. Gold Forecast Result Chart

2.4. BTC Forecasting Model

Naturally we refer to the gold prediction model for bitcoin, but we soon found the problem: after using VMD to decompose the signal of bitcoin price, the prediction effect of LSTM prediction model and AdaBoost prediction model significantly decreases, which in turn leads to a decrease in the accuracy of the final prediction results. We analyzed the BTC price curve and found that the BTC curve fluctuated more significantly than the gold price curve, which made it more difficult to analyze the data after the VMD signal decomposition. So, we select CEEMDAN to perform signal decomposition on the BTC price curve. Then, we used principal component analysis to select the main components.In the next step, we use the LSTM prediction model with the AdaBoost prediction model again to segment the predictions. Finally, we use Support Vector Machine(SVM) signal reconfiguration. In this section we focus on the models and algorithms that were not mentioned in detail in the previous paper.

2.4.1. CEEMADAN Signal Reconfiguration

The CEEMDAN method is to add white noise to the residuals every time the first-order IMF component is completed and find the mean value of the IMF component at that time, and iterate through the process, which is more suitable for data with significant volatility like BTC than VMD. CEEMDAN decomposition not only inherits the advantages of EEMD, but also has the completeness and improves the signal reconstruction accuracy again, and the problem of different number of decomposition under different noise can be solved by setting the relevant parameters to choose the optimal signal-to-noise ratio.

2.4.2. Principal Component Analysis(PCA)

After obtaining the decomposed 11 sets of signal information, we then need to analyze the decomposed data. If we analyze the data one by one, it will certainly increase the complexity of the prediction model. Therefore, we first use principal component analysis to reduce the dimension, and here we select the first 10 items as the object of dimension reduction, and the RES is listed separately. Principal component analysis transforms highly correlated variables into several new variables that are independent or uncorrelated with each other to explain most of the variation in the information and is used to explain comprehensive indicators of the information. It is based on the principle of optimal synthesis and simplification of multivariate cross-sectional data tables while ensuring the least possible loss of data information, i.e., dimension reduction of the high-dimensional variable space. Figure 7 shows the result of PCA.

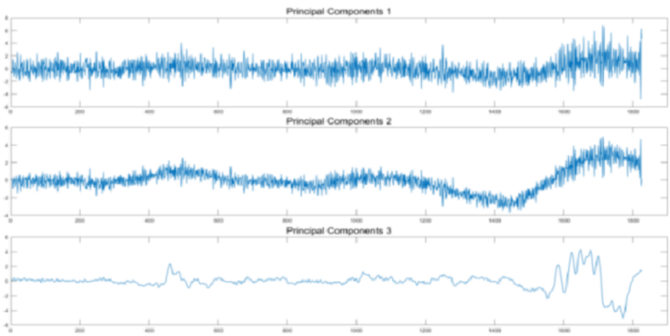

Figure 7. Principal Component Signal Graph

2.4.3. LSTM-AdaBoost-SVM Combined Predictive Models

After obtaining the results of the three principal component analyses, we can find that the principal component 1 and principal component 2 plots exhibit frequent small fluctuations, so we use the LSTM model to predict them; while the principal component 3 plot is relatively smooth with the trend term, so we use the AdaBoost model to predict them. Eventually, we then use the SVM model for signal integration to get the final results. We can find that the RMSE of Bitcoin is about 1070 and the MAPE is about 0.08. From the fitting results, we can see that BTC is more volatile and disorderly and more difficult to predict compared to gold. However, in general, the large value of RMSE is mainly due to the larger actual value of bitcoin's late rise, while MAPE is around 0.08, indicating that the model still has good fitting results. Therefore, this model still has a good prediction accuracy. Figure 8 shows forecast result of BTC.

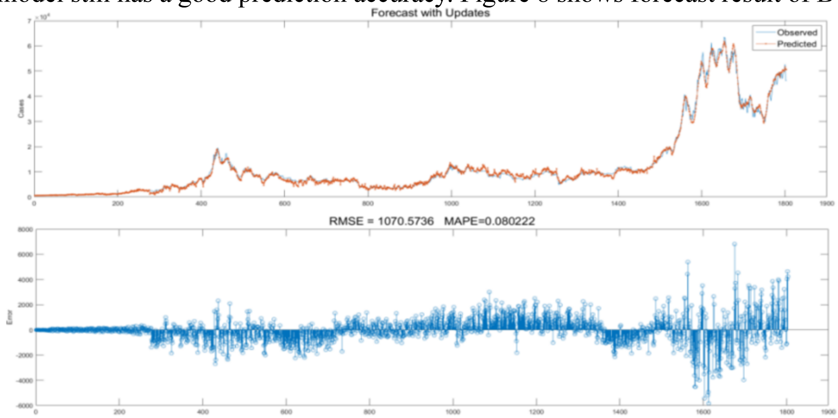

Figure 8. BTC Forecast Result Chart

3. Trade Decision Planning Model

3.1. Problem Analysis

After obtaining the forecasted price curves, we use the forecasted data as an important measure for our weekly decisions. We use Monday of each week as the starting point and Sunday as the ending point to construct dynamic programming models about USD, Gold, and Bitcoin. We use Floyd algorithm to find the investment path with the highest increase in a week, and at the end of the week, we keep looping until we get the result.

3.2. Investment Strategy without Forecasting Model

When we started our investment strategy from September 11, 2016 to October 2, 2016, we were unable to effectively predict the price of gold and bitcoin due to the lack of data from previous periods. At this time, we make insurance investments based mainly on the investor's personality. We assume three types of risky investment population, each based on their investment propensity for the following financial products fixed investment behavior.

High risk investors: invest \$30 per Bitcoin trading day and \$15 per gold trading day in the first three weeks.

Medium risk investors: invest \$15 per Bitcoin trading day and \$25 per gold trading day in the first three weeks.

Low risk investors: invest \$10 per Bitcoin trading day and \$20 per gold trading day in the first three weeks.

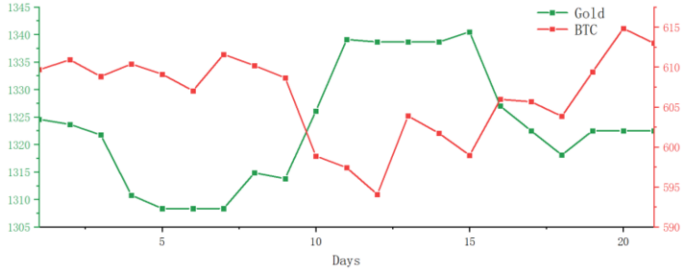

Figure 9. Price Graph for the First 3 Weeks

Table 1. Distribution of Property by Population

After 3 Weeks	Theoretical Input			Actual Changes		
Property	Gold	USD	BTC	Gold	USD	BTC
High Risk	225	145	630	223	145	624
Medium Risk	375	310	315	371	310	312
Low Risk	300	490	210	297	490	208

The change in the data for the first three weeks is shown in figure 9. It is not significant and there was almost no increase. At the same time, every investor lost money in the first three weeks because of the commission that needs to be borne for each trade. High-risk investors lost \$8, medium-risk investors lost \$7, and low-risk investors lost \$5. Although the difference between the amounts lost by each investor is small, when the capital is large enough, the amount of loss will be multiplied. Obviously, the high-risk investors will bear more losses and the low-risk investors bear smaller losses.The distribution of property for the three types of investors is in table 1.

3.3. Weekly Investment Planning Model Based on Floyd Algorithm

We take a high-risk investor as an example of an investment strategy with a prediction model case. In the prediction model, we predict the gold/bitcoin price curve at the end of each week for a full week of the following week. When we consider the forecast data to be real and reliable, we can then use it as information to inform potential decisions during the week.

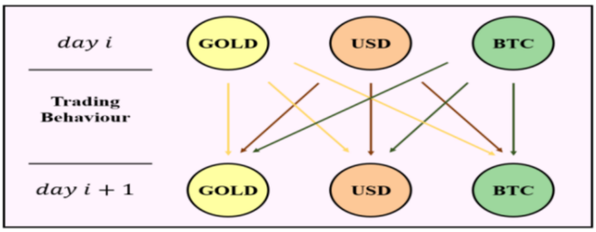

Figure 10. Diagram of Trade Behaviour

As shows in figure 10, there are always nine different potential trading behaviors from one day of the week to the next, as shown in the chart above, we use the letters *G*, *U*, and *B* (each product's acronym) in two-by-two combinations with themselves, which in turn represent the trading behavior of investors from day i to day $i+1$, e.g. *GG* would represent gold on day i to still be gold on day $i+1$.

Table 2. Investor Behaviour Analysis Form

<table>
<tr><td>Day i</td><td colspan="3">G</td><td colspan="3">U</td><td colspan="3">B</td></tr>
<tr><td>Day i+1</td><td>G</td><td>U</td><td>B</td><td>G</td><td>U</td><td>B</td><td>G</td><td>U</td><td>B</td></tr>
<tr><td>Value (normal)</td><td>p_{gi}</td><td>$p_{gi} \cdot c_g$</td><td>$p_{gi} \cdot c_g \cdot c_b$</td><td>$c_g$</td><td>1</td><td>$c_b$</td><td>$p_{bi} \cdot c_g \cdot c_b$</td><td>$p_{bi} \cdot c_b$</td><td>$p_{bi}$</td></tr>
<tr><td>Value (GCD)</td><td>1</td><td>/</td><td>/</td><td>/</td><td>1</td><td>c_b</td><td>/</td><td>$p_{bi} \cdot c_b$</td><td>p_{bi}</td></tr>
</table>

*GCD means "Gold Closing Day"

In table 2, p_{gi}, p_{bi} denote the profit of gold, bitcoin on day i respectively (or in negative form if there is a loss). c_g, c_b are the fees of gold, bitcoin respectively. It can be found that there are 32 paths in two days. This means that there are 37 paths in a week. If we set each point on Monday as the starting point and each point on the weekend as the end point, this problem can be transformed into a path problem, and we can build a dynamic programming model to find the extreme value of each starting point to each end point through Floyd's algorithm, and then determine the path of the maximum profit end point for each starting point. The specific steps are as follows:

We first introduce the most basic Floyd algorithm. Based on the theoretical basis of Floyd algorithm, we select one node in Monday as the starting point and three nodes in Sunday as the end points in turn, calculate the optimal paths from the starting point to the three end points respectively, and compare the magnitude of the three values. The maximum path is determined as the optimal investment path of the week for this starting point, and the optimal end point of this week is taken as the starting point of the next week, and so on. In turn, we can get the optimal investment path for each of the three starting nodes every Monday. Finally, we then cycle through the weeks, we realized the automatic planning from the 4th week to the last week by MATLAB programming. This allows us to make weekly optimal decisions, and some of the results are shown in table 3:

Table 3. Decision-Making Diagram

Data	Decision	Data	Decision	···	Data	Decision
2016/11/5	B—>U	2016/12/20	U—>B	···	2021/6/15	B—>U
2016/11/13	U—>B	2016/12/21	B—>U	···	2021/6/29	U—>B
2016/12/2	B—>U	2016/12/22	U—>B	···	2021/7/5	B—>U
2016/12/5	U—>B	2016/12/23	B—>U	···	2021/7/19	U—>B
2016/12/17	B—>U	2016/12/24	U—>B	···	2021/9/10	B—>U

When the decision is made until September 11, 2021, the capital changes from an initial $1,000 to $52,048. The annual profit rate is 219.581%.In a normal money management situation, it is obviously too high. However, considering that bitcoin's 5-year rate of increase is as high as 73.5897 times, the annual rate of increase is 236.25%. Combined with the investment strategy we can see that high risk investments tend to invest in bitcoin rather than gold, and it is reasonable to achieve such a profit return, which also side by side demonstrates the accuracy of our prediction model. At the same time, combined with the investment strategy under the no-prediction model at the beginning, we can find that risk and profit go hand in hand, and taking higher risk often has the possibility of gaining higher profit.

4. Improvements and Extensions of Trading Model[6]

In the above process, we made predictions and decisions by considering only the gold and bitcoin closing prices, and did not consider risk indicators, although we also achieved good results (including a very high profit amount in the end). However, considering that in real life prices are influenced by more factors and that Bitcoin cannot always maintain an extremely high growth trend. We improve and extend the original model to create a Measure-VaR multi-stage portfolio model that takes into account transaction costs to make it more adaptable to real-life situations.

4.1. Model Building

We have two risky assets (Gold and Bitcoin) and one risk-free asset (Cash), and the investor invests in T stages from time 0 to the end of time T. The proportional portfolio at stage t is denoted by P, and the assets are reallocated at the beginning of each stage, for which we have[7].

4.1.1. Parameters of the Model

r_t^{cash}: The rate of return of the US dollar in phase t (we found from the official website that the weekly rate of the US dollar is about 0.05%);

$r_t^{bitcoin}$: The maximum weekly return of bitcoin in phase t;

r_t^{gold}: Maximum weekly return of gold in phase t;

W_t: Total assets owned at the end of phase t;

T: Total number of investment phases;

$VaR_\alpha(t)$: Value-at-risk metric at confidence level α for stage t;

N: Arbitrarily selected date number (Mark September 11, 2016 as 0, and so on);

4.1.2. Decision Variables

x_t: The proportion of dollars that continue to be held at the start of allocation in period t;

y_t: The percentage of bitcoins bought at the beginning of the distribution in phase t ;

z_t : The percentage of gold bought at the start of allocation in phase t .

4.1.3. Binding Conditions

- Risk constraints:

There are many ways to measure risk in a portfolio, such as variance, lower half variance, etc., and our team uses VaR [8][9] to measure risk, After consulting the information, the formula is as follows:

$$VaR_{\alpha}(t)=\Phi^{-}(\alpha)\sigma_t-R_t \tag{7}$$

where σ_t^2 denotes the variance of the portfolio at stage t.; R_t denotes the expected return of the portfolio P at stage t; $\Phi(x)$ is the distribution function of the standard normal distribution $N(0,1)$ and $\Phi^{-1}(a)$ is the lower α quantile of the standard normal distribution. Here we consider the value-at-risk not to exceed some given level of risk ω , i.e.

$$VaR_{\alpha}(t)=\Phi^{-}(\alpha)\sigma_t-R_t\leq\omega \tag{8}$$

where σ_t^2 and R_t are calculated as follows:

$$\sigma_t^2=\sigma_{bitcoin}^2y_t^2+\sigma_{gold}^2z_t^2+2y_tz_tCov(B,G) \tag{9}$$

$$R_t=r_t^{cash}x_t+r_t^{bitcoin}y_t+r_t^{gold}z_t \tag{10}$$

We take the confidence level $\alpha=0.95$ and according to the normal distribution correspondence table we can find $\Phi^{-}(\alpha)=1.96$. In addition, we can calculate according to the weekly maximum returns of bitcoin and gold for 261 weeks that:

$$\sigma_{bitcoin}^2=1.25\%,\quad \sigma_{gold}^2=0.03\%,\quad Cov(B,G)=\rho_{BG}\sigma_{bitcoin}\sigma_{gold}=0.19\% \tag{11}$$

where $\sigma_{bitcoin}^2$ denotes the variance of bitcoin assets; σ_{gold}^2 denotes the variance of gold assets; ρ_{BG} denotes the correlation coefficient between bitcoin assets and gold assets; and $Cov(B,G)$ denotes the covariance between bitcoin assets and gold assets.

The sum of the three assets to be invested in each new phase is 1, i.e.

$$x_t+y_t+z_t=1 \tag{12}$$

We place bounded constraints on the proportion of investments for the three assets, i.e.

$$\mathrm{m_t}\leq x_t,y_t,z_t\leq M_t \tag{13}$$

where m_t and M_t denote the lower and upper limits of the stage t investment ratio, respectively.

4.1.4. Objective function

The total assets at the end moment of week t and the total assets at the end moment of week $\mathrm{t}-1$ are related as follows:

$$W_t = W_{t-1}[(1+r_t^{cash})x_t + \alpha_{bitcoin}^2(1+r_t^{bitcoin})y_t + \alpha_{gold}^2(1+r_t^{gold})z_t] \tag{14}$$

Our ultimate goal is to:

$$\mathrm{MAX}(\mathrm{Measure_T}) \tag{15}$$

In summary our model can be integrated as:

$$\mathrm{P_0}:\begin{cases} MAX \quad f = \mathrm{Measure_T} \\ W_t = W_{t-1}[(1+r_t^{cash})x_t + \alpha_{bitcoin}^2(1+r_t^{bitcoin})y_t + \alpha_{gold}^2(1+r_t^{gold})z_t], \\ VaR_\alpha(t) = \Phi^-(\alpha)\sigma_t - R_t \le \omega \\ x_t + y_t + z_t = 1 \\ \mathrm{m_t} \le x_t, y_t, z_t \le M_t \\ \mathrm{Measure_T} = ln(W_T), \gamma = 0 \\ \mathrm{Measure_T} = \dfrac{(W_T)^\gamma}{\gamma}, \gamma \ne 0 \\ t = 1, 2, \cdots\cdots T, \quad W_0 = 1000 \end{cases} \tag{16}$$

$$\mathrm{P_1}:\begin{cases} MAX \quad f = \mathrm{Measure_T} - \delta\sum_{\mathrm{t=1}}^{\mathrm{T}}(\max\{\Phi^-(\alpha)\sigma_t - R_t - \omega, \ 0\})^2 \\ W_t = W_{t-1}[(1+r_t^{cash})x_t + \alpha_{bitcoin}^2(1+r_t^{bitcoin})y_t + \alpha_{gold}^2(1+r_t^{gold})z_t], \\ x_t + y_t + z_t = 1 \\ \mathrm{m_t} \le x_t, y_t, z_t \le M_t \\ \mathrm{Measure_T} = ln(W_T), \gamma = 0 \\ \mathrm{Measure_T} = \dfrac{(W_T)^\gamma}{\gamma}, \gamma \ne 0 \\ t = 1, 2, \cdots\cdots T, \quad W_0 = 1000 \end{cases} \tag{17}$$

γ refers to the degree of investor preference for investment risk, the smaller the value of γ , the more sensitive investors are to investment risk; this paper only considers the case of $\gamma = 1$, and uses the outer point semi-penalty function method to put the inequality constraint in the P_0 model into the objective function to construct a penalty function, generating a series of optimization problems containing equation constraints and bounded constraints as follows P_1 to solve, so as to obtain a solution to the original problem P_0.

Set the initial value of penalty factor δ to 10, and transform model P_0 into model P_1 by iterating $\delta = \delta + i * 10$ to get a series with being positive and $\delta_t \to +\infty$.

4.2. Model Solving

4.2.1. Introduction to the particle swarm algorithm

Particle Swarm Optimization (PSO) is a new evolutionary algorithm proposed by Kennedy[10] et al. The PSO algorithm is a kind of evolutionary algorithm, similar to other intelligent optimization algorithms, which generates a group of particles at random and finds the optimal solution by continuously updating the velocity and position of the particles, and evaluates the objective function of the model through the fitness function. The objective function of the model is evaluated by the fitness function.

4.2.2. Problems to be solved

What is established in the paper is a single-objective model for maximizing terminal wealth at a given level of risk ω. In the algorithm solution process, MAX in the objective function is a nonsmooth function, and the actual calculation requires first relaxing the original feasible region into a convex region, so that the convex region contains the original feasible region, and then linearizing the nonsmooth function in the extended convex region. In the multi-stage portfolio solution algorithm, the wealth transfer between stages is done by allocating the funds owned at the beginning of the first stage to several different assets, and using the wealth obtained at the end of the first stage as the initial wealth of the second stage, and reallocating it in the second stage. The wealth obtained at the end of the first stage is used as the initial wealth in the second stage and is reallocated in the second stage.

4.2.3. Algorithm steps

Step1:Randomly generate the initial particle positions and velocities, set the maximum number of iterations T_{max} $(T_{max}=120)$. Judge the second constraint, if it is satisfied, continue Step 2, if not, reinitialize, because the position of the initialized particles in the particle swarm algorithm has a certain influence on the search for the global optimal solution, we make the initialized particles as close as possible to the region where the feasible solution is located through this judgment.

Step 2: Evaluating the fitness values of the particles in turn according to the constructed penalty functions.

Step 3:According to the following method to determine the individual optimal solution $\mathrm{P}_i(t)$ of the current generation and the population optimal solution $\mathrm{P}_g(t)$ of the current generation, the objective function of this paper is the maximization problem, then the individual extreme value of the i rd particle is determined by the following equation.

$$\mathrm{P}_i^t=\begin{cases}\mathrm{P}_i^{t-1}\ ,\ fitness\ (\mathrm{X}_i^t)<\ fitness\ (\mathrm{P}_i^{t-1}),\\ \mathrm{X}_i^t\ ,\ fitness\ (\mathrm{X}_i^t)>\ fitness\ (\mathrm{P}_i^{t-1}),\end{cases}\tag{18}$$

The best position of the group is determined by the following equation:

$$f(P_g^t)=max\{f(P_0^t),\ f(P_1^t),\ \cdots,\ f(P_m^t)\},P_g^t\in\{P_0^t,\ P_1^t,\ \cdots,\ P_m^t\}$$
$$当t=0时，\ P_i(t)=X_i(t)\tag{19}$$

Step 4:The position and velocity of the i st particle are updated according to the following equation

$$V_i(t+1)=\omega V_i(t)+c_1r_1(P_j-X_i(t))+c_2r_2(P_g-X_i(t)),$$
$$X_i(t+1)=X_i(t)+V_i(t+1)\tag{20}$$

where the inertia weight ω is taken according to the following formula

$$\omega=\frac{\omega_{\max}-t*(\omega_{\max}-\omega_{\min})}{t_{\max}}\tag{21}$$

$\omega_{\max}$ and $\omega_{\min}$ are the maximum and minimum values of ω respectively, which generally take the values 0.9 and 0.4, t is the current number of iterations and $t_{\max}$ is the maximum number of iterations.

Step 5:If the termination condition set by the algorithm is satisfied, the iteration is stopped and the optimal solution of the required solution is output; otherwise, the next iteration is performed.

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