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An Optimization Model of Dam Water Resources Dynamic Scheduling

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Abstract. Since the water resources of Glen Canyon and Hoover Dam are not properly allocated and the drought conditions in the Colorado River Basin are gradually increasing, it is important to develop a reasonable water resources allocation plan for the present and future. A multi-dam joint dynamic operation model (MJDM) is introduced to study the dynamic water supply and power supply problem of the dams. Based on the dynamic simulation of combined operation of dams for water and electricity supply, we develop a multi-objective optimization model to solve the competitive scheduling problem of water and hydropower resources and provide a reasonable dynamic water resource allocation plan decision. We use simulated annealing algorithm(SA) to find out our water and electricity power supply allocation strategies and economic trend based on different scheduling periods. Finally, a sensitivity analysis is performed on the parameters related to initial water level and flow rate in the model.

Keywords: Multi-objective Optimization Model, MJDM, Simulated Annealing (SA).

1. Introduction

For hundreds of years, people around the world have been spending large amounts of money and manpower to build dams and reservoirs on rivers to manage water resources for irrigation, power generation, water supply, and navigation. Hoover Dam and Glen Canyon Dam in the United States are representative ones, which are key projects in the integrated development of water resources in the Colorado River. Below these dams, the Colorado River, originally the deepest and fastest flowing river in the United States, now moves slowly, like a tamed beast. However, since human economic development destroys the natural environment and climate changes, the hydrologic impacts will affect reservoir management, resulting in varying degrees of impact on drinking water sources, flood risk, irrigation, and hydropower generation [1].

In our research, we analyze the length of time to meet the demand with a fixed water supply and the additional water supply to meet the demand over time after considering water evaporation and water loss. Then, we establish model to handle the conflicting interests of general water use and electricity production water use. Finally, we analyze the different impacts of different conditions on the model, which shows the feasibility and advantages of our model.

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2. Establishment of Multi-dams Dynamic Operation Model

2.1. Water Capacity Analysis of the Single Dam

For the water capacity of the lake, we conclude that the relationship between water capacity and water level is

$$V=\sum_{i=1}^{n} S_i(h_i+h_{i+1}+h_{i+2})/3 \tag{1}$$

V is the total volume(m^3), S_i is the projection area(m^2) of the underwater terrain triangle surface on the water surface, h_i, h_{i+1} and h_{i+2} are distance(m) of the underwater triangle vertexes to the water surface, and n is the number of triangle grids.

Figure 1 shows a simulation of a realistic dam. On the basis of this model, the reservoir capacity is determined. In the reference, Cone method is a common method, whose formula is:

$$CA(X_i)=\sum_{i=0}^{n}\frac{1}{3}\left(A_i+A_{i+1}+\sqrt{A_i\times A_{i+1}}\right)\times \Delta L_i \tag{2}$$

Where: V is the reservoir capacity; A_i is the area of the i-th section; ΔL_i is the gap between the i_{th} section and the $(i+1)$ th section.

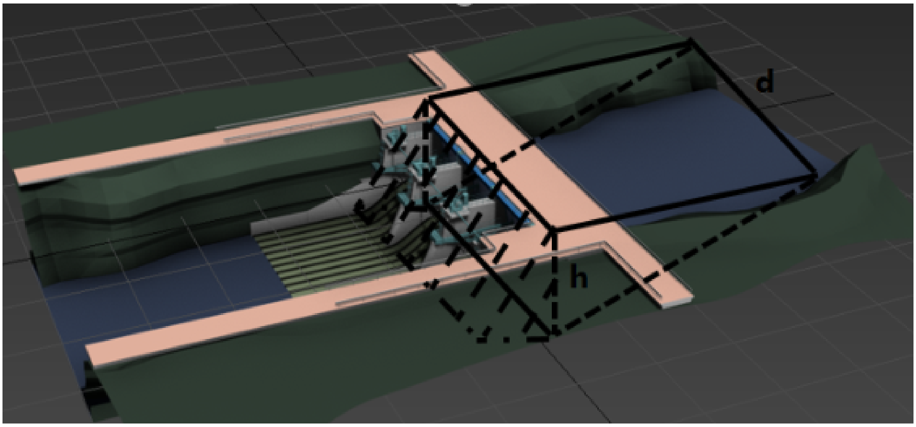

Figure 1. A simulation of a dam with 3ds MAX

2.2. Hydropower Calculation Model of the Dam

2.2.1 Velocity Analysis of Water Flow.

When calculating the dam power generation, one of the key variables is the average velocity of the water as it reaches the dam. We assume that the flow at Lake Powell and Lake Mead is a kind of channel flow, and then we use Manning's formula, one kind of empirical formulas, to calculate cross-sectional average velocity:

$$v = \frac{k}{n} R_h^{\frac{2}{3}} S^{\frac{1}{2}} \tag{3}$$

Where:

- k is a conversion factor between SI and English units;
- n cross-sectional average velocity;
- R_h is the hydraulic radius$(L; f_t, m)$;
- S is the slope of the hydraulic grade line or the linear hydraulic head loss

After briefly analyzing, we can easily get the idea that the average velocity of the water reaching the dam analyzed by this formula is only related to the river slope S and the hydraulic radius R_h.

2.2.2 Water Flow Analysis.

We simplify the analysis by using the average volume flow rate per unit time Q, which is calculated as follows:

$$Q = \rho \cdot L \cdot v \cdot t \tag{4}$$

Where, ρ is the density of water; L is the width of the river; v is the speed of water flow; t is the time.

2.2.3 Dam Hydropower Conversion Analysis.

Moreover, the available power of the dam can be measured by the flow rate, the acceleration of gravity, the height of descent and the density of the water. The formula can be shown as:

$$E = \eta \rho g h Q \tag{5}$$

Where, η is the dimensionless efficiency of the turbine; ρ is the density of water in kilograms per cubic meter; g is the density of water in kilograms per cubic meter.

2.3. Two-level Dams Analysis

Because the operation of Glen Canyon Dam will have an impact on Hoover Dam downstream, we establish hydrodynamic to analyze the two-level dams as shown in figure2. In order to simplify the treatment of this continuous process, we use a discrete approach to establish the difference equation to process it.

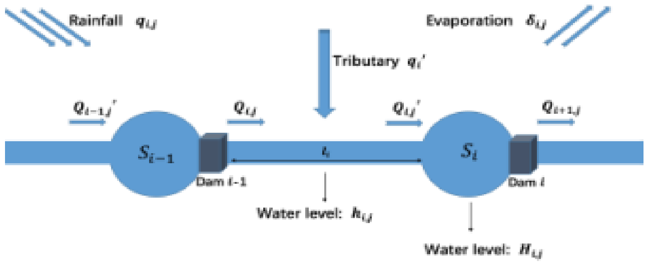

Figure 2. Multi-dam Model

2.3.1 Equations in the River Without the Dam.

Assuming that what we are studying are the hydrodynamic equations over a time period T. According to the law of conservation of flow and taking the rainfall and drought scenarios into consideration, the following equation can be derived:

$$Q_{in} + q_{rain} - q_{evaporation} - q_{drawn} - Q_{out} = \rho \Delta V \tag{6}$$

$$(q_{ij} - \delta_{ij} - \xi) l_i L + Q_{ij} - Q_{ij}' = l_i L \frac{\Delta h_i}{\Delta t} \tag{7}$$

where: $\frac{\Delta h_i}{\Delta t} = \frac{h_{i(j+1)} - hij}{T}$ .What's more, based on the conservation of energy, we can derive the following equation:

$$W_{gravity} + W_{resis\tan ce} = \Delta T_{water} + \Delta T_{rain} + \Delta T_{evaporation} \tag{8}$$

$$\rho L h_{ij} v T g \Delta y_i - \rho k h_{ij} L v T l_{\mathrm{i}} = \frac{1}{2} \rho (q_{ij} - \delta_{ij} - \xi) l_i L v^2 T + \frac{1}{2} \rho h l h_{ij} \Delta (v^2) T \tag{9}$$

2.3.2 Equations in the Dam Region.

Same as the analysis in 5.3.1, we can derive the formula,

$$(q_{ij} - \delta_{ij} - \xi) l_i L + Q'_{ij} - Q_{(i+1)j} = S \frac{\Delta H_i}{\Delta t} \tag{10}$$

where: $\frac{\Delta H_i}{\Delta t} = \frac{H_{i(j+1)} - Hij}{T}$.

According looking up the Hydraulics textbook,we establish the formula as follows:

$$Q_{(i+1)j} = A_i (H_{ij} - H_{0i})^{\frac{3}{2}} \tag{11}$$

Where, A_i is the outflow coefficient,determined by the width b_i of the gate hole and the relative gate opening θ of the dam. By reviewing the information,it was concluded that

$$A_i = 2.7674\theta b_i \tag{12}$$

2.3.3 Recursive Formula.

Concluding the equations above, the recurrence formulas of the river with dam systems can be obtained.

$$\begin{cases} Q'_{ij} = \dfrac{-\frac{1}{2}gLl_i + \sqrt{\frac{1}{4}g^2L^2l_i^2 - 4\left(\frac{1}{2}gLQ_{ij}l_i - Q_{ij}{}^2 + 2kh_{ij}{}^2L^2l_i - 2h_{ij}{}^2L^2g\Delta y_i\right)}}{2} \\ Q_{(i+1)j} = A_i(H_{ij} - H_{0i})^{\frac{3}{2}} \\ h_{i(j+1)} = \dfrac{(q_{ij} - \delta_{ij} - \xi)l_iL + Q_{ij} - Q'_{ij}}{l_iL}T + h_{ij} \\ H_{i(j+1)} = \dfrac{(q_{ij} - \delta_{ij} - \xi)S_i + Q'_{ij} - A_i(H_{ij} - H_{0i})^{\frac{3}{2}}}{S_i}T + H_{ij} \end{cases} \tag{13}$$

2.4. Results

2.4.1 Results description.

First, the initial water level of the model is set as described in the question, and the water levels are P and Q respectively. By using MATLAB software simulations, the results of the month-by-month variation of the water level of the dam for the year 2021 were obtained by iterating the difference equation by varying the rainfall-drought coefficient and the water demand by month in figure 3.

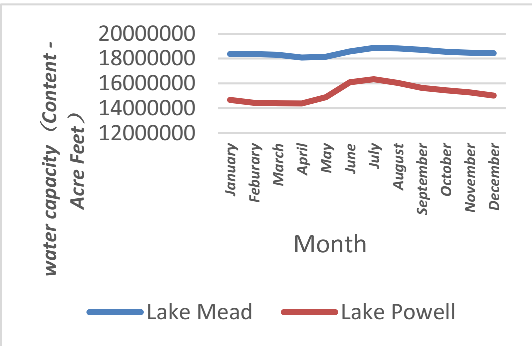

Figure 3. Month-by-month change of water level

The simulation results of the year-to-year variation of the reservoir capacity are as follows figure 4.

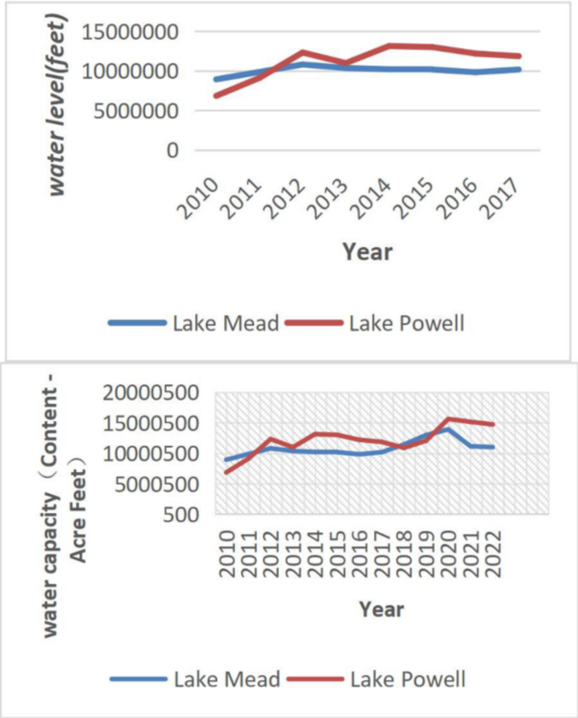

Figure 4. Year-by-year and year-to-year change of reservoir capacity

The month-to-month variation of water supply and electricity supply in reservoirs can be derived from the relationship between pumping coefficients and time as follows figure 5.

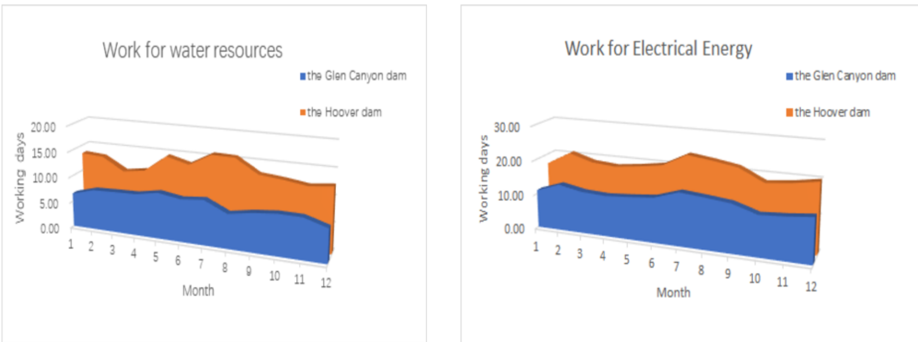

Figure 5. Month-to-month variation of water supply and electricity supply

2.4.2 Results Analysis.

Our analysis of the reservoir capacity of dam can clearly find that if the current weather has been continued, the system will face long-term operation of the collapse of the situation as shown in figure 7. We introduce an additional water source coefficient ν_{ij}. By changing the value of the coefficient, we simulate system operation and observe the dam water level changes until the system can be stable long-term operation, that is, we need to figure 6.

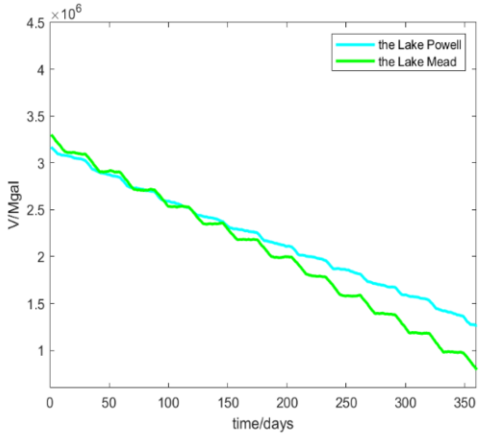

Figure 6. Water supply time

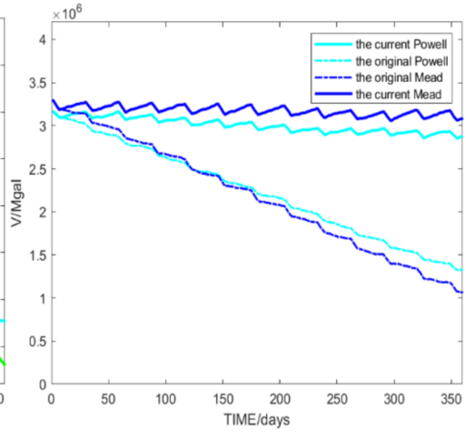

Figure 7. Electricity supply time

2.4.3 Analysis of the Gulf of California.

By using MATLAB simulation to run the water flow system, we use equation (12), equation (14) and the parameters to derive the river distance along the river, and height after the Hoover Dam. Via the iterative analysis, we conclude that after the upstream water supply to the five states have been completed:

(1) no parameter ν_{ij} is introduced when the region's water resources trend: figure 6.

(2) the parameter ν_{ij} is introduced when the region's water resources trend: figure 7.

3. Dam Water Scheduling Programming Model

Optimal scheduling of hydroelectric power stations is a strongly constrained, nonlinear, multi-stage combinatorial optimization problem whose optimal solution is a research problem with practical value. Therefore, the establishment of a reasonable water scheduling scheme for dams can help local governments maximize the social benefits. In the dam water scheduling, the most important aspect is to address the competing interests of water availability for general (agricultural, industrial, residential) usage and electricity production.[2]

3.1. The Objective

Dams perform two main functions: electricity generation and water supply. First analyzing the power generation, the electricity generated by the hydroelectric power plant is shipped to various industries in each state, bringing huge economic benefits and reducing the pressure of other non-renewable sources of electricity generation. We let the power resource during the time period Δt be E.

$$E=\sum_{i=1}^{n}\sum_{j=1}^{T}\eta_{ij}*\rho*g*h_{ij}*Q_{ij} \tag{14}$$

where: η_j denotes the coefficient for the dispatch period.

According to the analysis already available earlier, we divide the electricity resources into three categories: E_1、E_2、E_3. Different electricity resources create different benefits, here we simplify the treatment. Eventually we establish the target of benefits regarding hydropower as

$$X_e=\varepsilon_1E_1+\varepsilon_2E_2+\varepsilon_3E_3 \tag{15}$$

In addition to creating a huge source of electricity, dams also provide abundant water resources for the city which can satisfy a large amount of water for industry and agriculture as well as for domestic use.

$$I_w=\sum_{i=1}^{n}\sum_{j=1}^{T}\xi_{ij}\cdot\Delta t \tag{16}$$

In the same analysis, we divide the water resources into three destinations: I_{w1}、I_{w2}、I_{w3}. At the same time, different water resources create different benefits. Eventually we establish the target of benefits regarding water resources as

$$X_I=w_1I_{w1}+w_2I_{w2}+w_3I_{w3} \tag{17}$$

First, we analyzed the profit relationship by analyzing the difference in price between a ton of water that can generate 1.8 kWh of electricity under normal conditions and a ton of water that can be used directly as a general water use to arrive at a base ratio, and then we analyzed the ecological impacts of dam water use for general water use and general electricity use to vary the base ratio to arrive at the final weights α_1、α_2.

$$X=\alpha_1\frac{X_e-X_{e\min}}{X_{e\max}-X_{e\min}}+\alpha_2\frac{X_I-X_{I\min}}{X_{\mathrm{Imax}}-X_{\mathrm{Imin}}} \tag{18}$$

3.2. The Optimization Model

All in all, in order to make social benefit as much as possible while accommodating the constraint conditions, we need to develop water resources optimized dispatching scheme as follows,

$$\max X=X=\alpha_1\frac{X_e-X_{e\min}}{X_{e\max}-X_{e\min}}+\alpha_2\frac{X_I-X_{I\min}}{X_{\mathrm{Imax}}-X_{\mathrm{Imin}}} \tag{19}$$

$$s.t.\begin{cases} V_{\min_i} \leqslant CA_{it} \leqslant \begin{cases} V_{\max_{1i}} & (flood) \\ V_{\max_{2i}} & (non-flood) \end{cases} \\ Q_{\min_i} \leqslant Q_{it} \leqslant Q_{\max_i} \\ Q_{in} + q_{rain} - q_{evaporation} - q_{drawn} - Q_{out} = \rho \Delta V \\ DE_t \leqslant \sum_{i=1}^{n} \theta_i D_{it} \\ DF_t \leqslant \sum_{i=1}^{n} \gamma_i F_{it} \end{cases} \tag{20}$$

3.3. Model Solving

3.3.1 Simulated Annealing Algorithm.

The simulated annealing algorithm is derived from the solid annealing principle and is a probability-based algorithm.[3] The simulated annealing algorithm is an optimization algorithm that can effectively avoid falling into a serial structure of local minima and eventually converge to a global optimum by giving the search process a time-varying and eventually zero probabilistic burstiness.

3.3.2 Results.

The calculation can be derived from the scheduling allocation for each month of the year and the corresponding benefits are shown in figure 8, and the scheduling scheme for each year and the corresponding benefits are shown in figure 8.

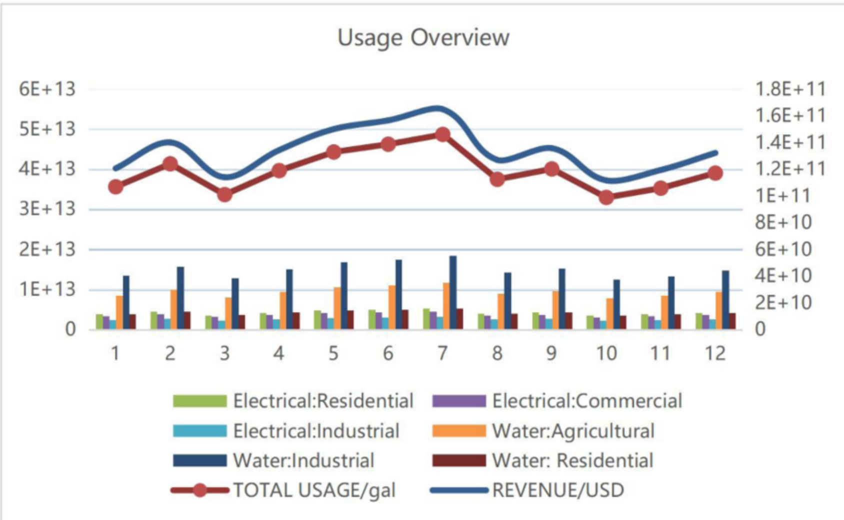

Figure 8. Usage of water resources in 12 months

For the allocation of water and electricity dispatch for different states, the partial results of our scenario are shown in figure 9.

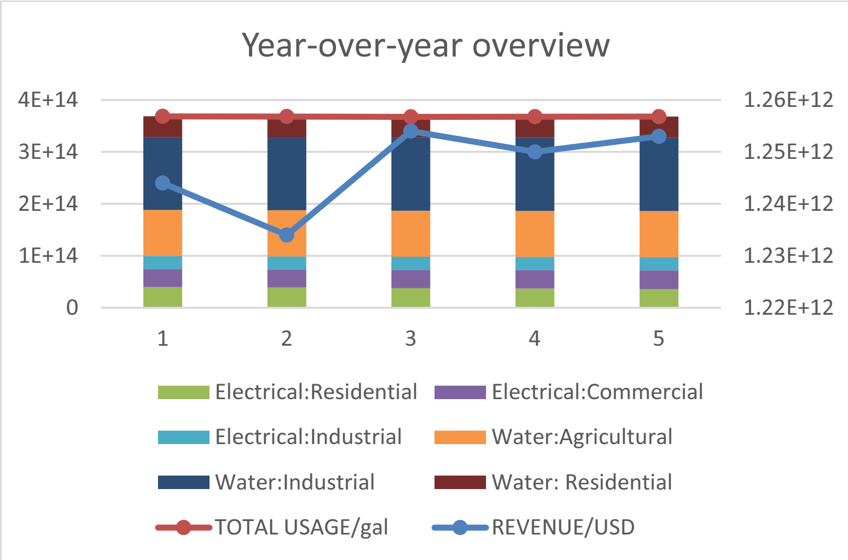

Figure 9. Usage of water resources in 5 years

Similarly, the results after the analysis of the planning are shown in figure 10. Since the penalty for electricity is stronger, it leads to a significant increase in the proportion of dispatch of water allocation compared to when no penalty is made.

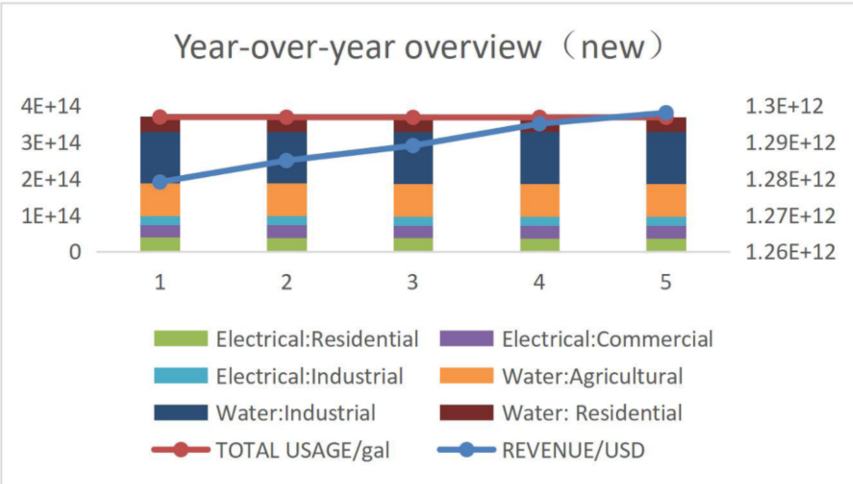

Figure 10. Usage of water resources in 5 years with penalty

4. Sensitivity Analysis

In our model, we cannot obtain the real value of the inputs Q_i, $H_{i,0}$, $h_{i,0}$ and parameter α with the data we have. These inputs or parameters may influence the result of our calculation, so we implement a sensitive analysis to test the robustness of our model.

Table 1. Sensitivity Analysis

	-5%	-3%	-1%	0%	1%	3%	5%
$H_{i,0}$	2.9582%	2.9789%	3.1263%	0%	3.1251%	3.1234%	2.9879%
$h_{i,0}$	3.6342%	2.5691%	1.1263%	0%	3.2281%	3.7389%	5.2578%
Q_i	3.2378%	3.2298%	3.4612%	0%	3.2330%	3.3570%	3.1342%

All the percentage presented on the table 1 represents the maximum fluctuation among H_i. Therefore, H_i is not sensitive to the value of Q_i , $h_{i,0}$ and $H_{i,0}$.

5. Conclusion

Our paper presents a dynamic water scheduling solution for Glen Canyon and Hoover dams in the U.S. states. For the joint dynamic operation of dams to supply water and electricity, we establish a difference equation to solve the problem. For the water supply scheduling problem, we establish a multi-objective planning model to solve the problem, and analyze the conflict between the two by considering the ecological and profit aspects between water supply and electricity. The problem of insufficient water resources is considered by introducing the irreplaceable classification to penalize the resource efficiency and thus derive a new scheduling scheme.

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