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Asteroid Mining Model Based on Global Equity

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Abstract By defining global equity and other mathematical methods, this paper constructs an equity distribution system for the asteroid mining industry which uses AHP and weight-based Topsis to construct a global equity evaluation model. This model obtains the equity coefficients of 117 countries, which can provide a thorough guiding reference in the distribution of benefits to some degree. Then, the concept of the relative deprivation coefficient is added to the model to adjust the equity of distribution. The model assigns different priorities to countries in the asteroid mining industry, speeding up mining efficiency while improving equity. What's more, we also combine the models to construct a standard world framework to show the change of global equity coefficient in a century in the phenomenon of ideal asteroid mining. Based on the model we formulate, we conduct an experiment and visualization to show the worldwide change of equity coefficients.

Keywords. Analytic Hierarchy Process (AHP), weight-based Topsis, relative deprivation coefficient, global equity coefficient.

1. Introduction

Asteroid mining is a resource-based industry filled with potential. Human beings can safely bring their valuable resources back to the earth to relieve the tension of the earth's resources. The issue of equity distribution of benefits has been considered to be solved nowadays to predict and guide a harmony and balance development of the outer field. Prior to our work, there have been some discussions on the topic of specific resource mining[1,2]. In the process of improving the international legal order for the distribution of rights and interests in the development of outer space resources in the future, the international community may consider: Under the guidance of the concept of "a community with a shared future for mankind[3-6]", by establishing the right to preferential use of outer space resources, all countries will be relieved Disputes over the initial allocation of outer space resources; on the other hand, the parallel development system can be introduced and promote the sharing of non-monetary benefits.

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In this paper, we put forward a kind of global equity model based on the previous dialectical thinking, which take every country's comprehensive caliber into consideration and provide a quantified evaluation mechanism to distribute resources and opportunities from asteroid mining.

2. Global Equity Evaluation Model (TOPSIS with weights)

Global equity is complicated and hard to quantify—combined with Maslow. A.H.'s Hierarchy of Human Needs theory, resources and opportunities can be embodied into physiological security needs, social needs and self-actualization needs.

2.1 Construct the Analytic Hierarchy Model

According to the research, the comprehensive evaluation system of different levels is determined and constructed, including target level and criterion level. The index system of the model framework is shown in figure 1.

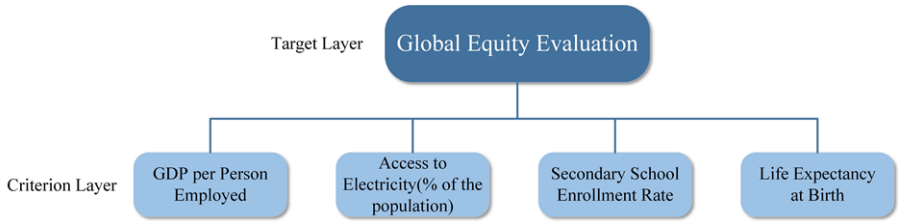

Figure 1. Hierarchical Model Diagram

2.2 Construct the Comparison Matrix

Firstly, construct a pairwise comparison matrix between factors, compare the four elements of the criterion layer pairwise, and obtain the MC comparison matrix in the following table 1 according to the data of the World Bank and experts.

C1: GDP per person employed;
C2: Life expectancy at birth;
C3: Secondary school enrollment rate;
C4: Access to electricity (% of the population).

Table 1. MC Comparison Matrix

M	C1	C2	C3	C4
C1	1	1/3	1/2	3
C2	3	1	1	3
C3	2	1	1	3
C4	1/3	1/3	1/3	1

Solving for M-C eigenvalues, then the maximum eigenvalue of the MC comparison matrix is obtained as 4.1179. Expressing the largest eigenvalue as λmax, we get the largest eigenvalue λmax=4.1179.

2.3 Consistency Check

n is the order of the matrix, and find the corresponding average random consistency index

RI. Calculate the consistency index CI and the Consistency Ratio:

$$CR = \frac{CI}{RI} = \frac{\frac{\lambda_{max} - n}{n-1}}{RI} \tag{1}$$

which obtains $CR = 0.0442 < 0.1$, and passes the consistency test.

2.4 Conclusions and Analysis

When the normal AHP is chosen for practical problems, a single method is generally used to obtain weights, and different ways may lead to divergence (figure 2).

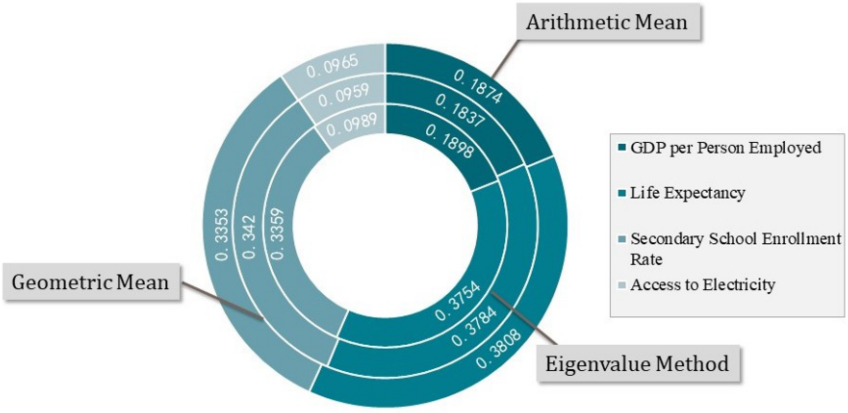

Figure 2. Weight distribution diagram

2.5 Standardized processing

To eliminate the influence of different indicator dimensions, it is necessary to standardize the data. After processing and cleaning the data for missing values, this paper retains the data of 117 countries as the analysis object. The original matrix composed of the four positive evaluation indicators is:

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ \vdots & \vdots & \vdots & \vdots \\ x_{117,1} & x_{117,2} & x_{117,3} & x_{117,4} \end{bmatrix} \tag{2}$$

Mark the normalized matrix as Z, and each element in Z has the following form:

$$Z_{ij} = \frac{X_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}} \tag{3}$$

2.6 Normalize processing and calculate the score.

After the original matrix (3) is calculated by the normalization formula (5), a normalized matrix with 117 rows and four columns is obtained.

Define the distance of i $(i = 1,2,\ldots,117)$ countries from the maximum value:

$$D_i^+ = \sqrt{\sum_{j=1}^{4} \omega_j \left(z_j^+ - z_{ij}\right)^2} \tag{4}$$

Define the distance of i $(i = 1,2,\ldots,117)$ countries from the minimum:

$$D_i^- = \sqrt{\sum_{j=1}^{4} \omega_j \left(z_j^- - z_{ij}\right)^2} \tag{5}$$

Compute the normalized score for the country i:

$$S_i = \frac{D_i^-}{D_i^+ + D_i^-} \tag{6}$$

$0 \le S_i \le 1$, and the larger S_i is, the larger D_i^+ is, that is, the closer to the maximum value. Otherwise, the further away from the maximum value.

2.7 Conclusions and Analysis

The results of the matrix obtained by MATLAB are shown in the appendix. The final score of each country, the equity coefficient, is shown in the following figure 3:

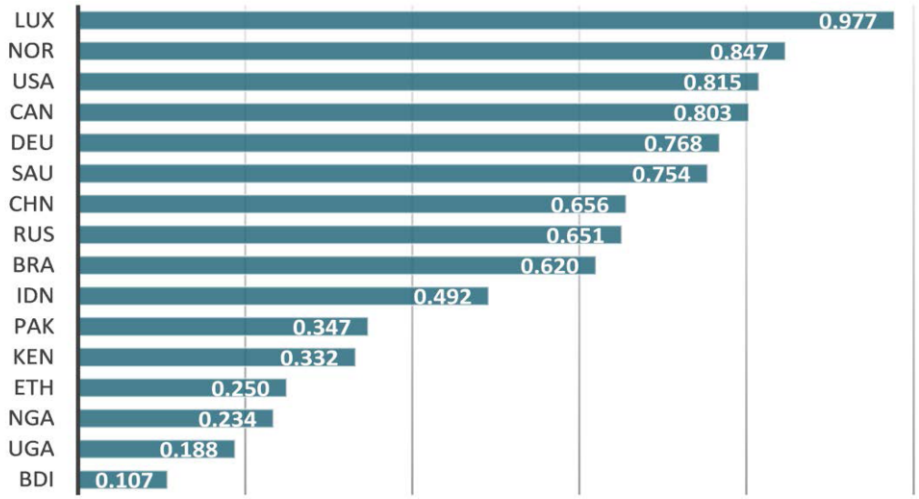

Figure 3. The Equity coefficients for each country

3. Global Asteroid Mining Priority Model

We use the Hierarchical Clustering Method to divide countries' priorities in asteroid mining into five levels: Priority mining, Second priority mining, Allowable mining, Pending mining, Planned mining. And assume that the mining volume of each country in the target period is Q_w. (figure 4)

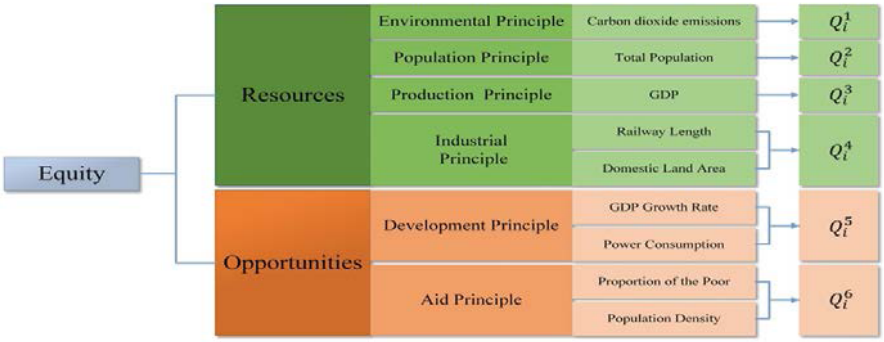

Figure 4. Equity Framework

3.1 Resource-based Definition of Equity

Carbon dioxide emissions are essential indexes of global environmental protection. We take carbon dioxide emissions as a negative indicator of ecological principles and use the following estimation formula:

$$Q_i^1 = Q_w * \frac{C_i}{\sum C_i} * \lambda_i^t \tag{7}$$

In equation 7, λ_i^t is the rate of change of carbon emissions in country i:

$$\lambda_i^t = \frac{C_i^{t-1}}{C_i^t} \tag{8}$$

C_i^t represents the carbon emissions of country i in year t. When the carbon emission of the country changes compared with last year, the obtained mineral ration will also change accordingly. We use the population ratio as an indicator of the population principle to get:

$$Q_i^2 = Q_w * \frac{pop_i}{\sum pop_i} \tag{9}$$

where pop_i represents the population of country i. Similarly, take GDP as an indicator of the production principle to get the formula:

$$Q_i^3 = Q_w * \frac{gdp_i}{\sum gdp_i} \tag{10}$$

where pop_i represents the gross domestic product of country i. As the index of the geographical principle, the formula is obtained:

$$Q_i^4 = Q_w * \frac{L_i \frac{1}{A_i}}{\sum L_i \frac{1}{A_i}} + \theta_w^{hpe} \sum pop_i^{ht} - \sum q_i^{ht} \tag{11}$$

In the equation 11, θ_w^{hpe} represents the historical cumulative capita mining volume in the world, $\sum pop_i^{ht}$ represents the historical cumulative population of country i, and q_i^{ht} represents the actual mining volume of country i in year t. We define the ratio of road length L to land area A as a reference for a country's industrialization level.

3.2 Opportunity-based Definition of Equity

We combine GDP growth rate and power consumption to comprehensively investigate and define the country's development principles formula:

$$Q_i^5 = Q_w \left(\frac{gdpG_i}{\sum gdpG_i} + \frac{cos_i}{\sum cos_i} \right) \tag{12}$$

In equation 12, $gdpG_i$ is the country's GDP growth rate, and cos_i is the country's electricity consumption. To enhance global equity and the living standards of the people worldwide, the formula is as follows:

$$Q_i^6 = \begin{cases} Q_w * \left[\frac{pov_i}{\sum pov_i} * k + \frac{den_i}{\sum den_i} \right], & K > 1 \\ Q_w * \left[\frac{pov_i}{\sum pov_i} * \frac{1}{k} + \frac{den_i}{\sum den_i} \right], & K < 1 \end{cases} \tag{13}$$

When the proportion of the poor in a country is too high, we will surge the aid intensity of the country accordingly. The aid index is K, and the size of K is affected by the specific value of a country's GDP and the GDP accumulated in the world:

$$K = \frac{gdp_i}{pop_i} * \frac{\sum pop_i}{\sum gdp_i} \tag{14}$$

3.3 Comprehensive Weighted Quota Estimation Based on Relative Deprivation Coefficients

We introduce Runciman's definition of the concept of Relative deprivation and the calculation method of the Relative Deprivation Coefficient proposed by $Yizhaki$, assuming that the group X is $X = (x1, x2, \ldots, xm)$, m is the number of samples, x_r and x_s are the individuals r and s from the X group. When the $x_r < x_s$ occurs, the individuals will have a sense of relative deprivation. If there are $n+$ individuals larger than x_s in group X, their proportion in X group is λ_+, and the mean of these individuals larger than x_s is $\bar{x}_+$, the sample mean of X group is x^-. The formula for calculating the standardised relative deprivation coefficient is:

$$D(x, x_s) = \sum_{i=1}^{m} \frac{d_X(x_r, x_s)}{m\bar{x}} = \frac{(n_+ * \bar{x}_+ - n_+ * x_s)}{m\bar{x}} = \frac{\lambda_+(\bar{x}_+ - x_s)}{\bar{x}} \tag{15}$$

This paper selects a joint probability function that meets the above definition, and the weight distribution function is:

$$p = f(\delta) = \frac{1}{1 + e^{\delta}};\ p_k = \frac{f(\delta_k)}{\sum_{k=1}^{6} f(\delta_k)} \tag{16}$$

- p_k represents the weight corresponding to the k principle, and pk satisfies $\sum_1^6 p_k$=1.
- δ_k represents the relative deprivation coefficient corresponding to the k principle, and the mapping relationship of the weight distribution function $p = f(\delta)$ satisfies $dp/dp = 0$. The value range $p_k \subset [0,1]$, the definition domain $\delta_k \subset [0, \infty)$.

The weights of the six principles of equity in each country can be estimated. The

difference between this method and single-principle allocation is that: under a single principle, the principle selection method is actually "0 or 1" (that is, the weight of a particular direction is 0, and the importance of the other tenets is 0) In conclusion, the final quota estimate for country i under the comprehensive weighting scheme is

$$Q_i = \sum_{k=1}^{6} Q_i^k p_k \tag{17}$$

4. Investment-Equity coefficient simulation experiment

The global equity index evaluation model is based on Maslow's human needs. The investment quota allocation model is based on comprehensive national strength, which we need to link to explore how global investors affect the global equity coefficient after they get benefits.

4.1 Standard World Model

To ensure equity, we need to include part of the proceeds in the "Equity Guarantee" to improve the benefits of countries with low ore quotas to balance the World Equity Coefficient. The country that obtains the higher percentage will pay more proportion of taxes. The formula is as follows:

$$T_i = t * \frac{Q_i}{\sum_{i=1}^{10} Q_i} * \Phi_i(V) \tag{18}$$

T_i is the equity security tax that should be charged by each country, $\Phi_i(V)$ is the income obtained by each country through the asteroid mining industry, and t is the tax rate coefficient. The formula for calculating the distribution of equity guarantee funds is inversely proportional to the equity coefficient of each country:

$$G_i = 1 - \frac{S_i}{\sum_{i=1}^{10} S_i} * \Phi_i(V) \tag{19}$$

G_i is the equity guarantee fund allocated by each country, and S_i is the equity coefficient of each country.

4.2 Equity Coefficient Change Simulation

Assuming that every trillion of funds can increase the equity coefficient of 100 million people β%:

$$\beta = 5e^{-5S_i} \tag{20}$$

e is the base of the natural logarithm, and s_i is the fairness index of country i. Simulation of the world standard model can get the following fairness index change graph.

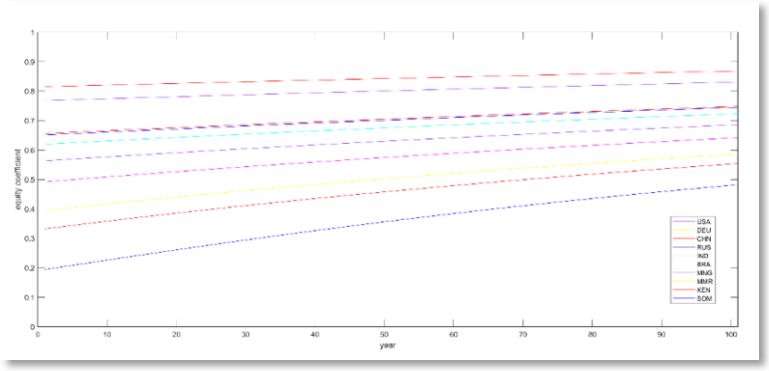

Figure 5. Changes in the Centennial Equity Index of Ten Countries

We simulate the evolution of the Standard World Model's equity index over the next hundred years. Obviously, under the maintenance of the equity guarantee fund, all the national equity indices in the Standard World Model are rising steadily, and the lower the starting point, the more pronounced the increase in the equity indices of countries (figure 5).

Acknowledgments

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References

[1] Anderson SW, Christensen K, LaManna J. The development of natural resources in outer space[J]. Journal of Energy & Natural Resources Law, 2019, 37: 227-258.
[2] Su J. Legality of unilateral exploitation of space resources under international law[J]. International and Comparative Law Quarterly, 2017, 66: 991-1008.
[3] Liu T. A nonlinear multigrid method for inverse problem in the multiphase porous media flow[J]. Applied Mathematics and Computation, 2018, 320: 271-281.
[4] Liu T, Xue R, Liu C, Qi Y. A regularization homotopy strategy for the constrained parameter inversion of partial differential equations[J]. Entropy, 2021, 23: 1480.
[5] Liu T. A multigrid-homotopy method for nonlinear inverse problems[J]. Computers & Mathematics with Applications, 2020, 79(06):1706-1717.
[6] Liu T. A wavelet multiscale-homotopy method for the parameter identification problem of partial differential equations[J]. Computers & Mathematics with Applications, 2016, 71: 1519-1523.