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Bayesian Model Updating with Adaptive Importance Sampling Using Gaussian Mixture: Case Study for Dynamic Analyses of 1-Story Moment Frame with Viscous Damper

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Abstract. Bayesian model updating provides a powerful and comprehensive framework for engineers to assimilate up-to-date observation data into models based on probability theory and significantly reduces model uncertainties. By integrating the concept of population Monte Carlo within the cross-entropy method, a novel adaptive importance sampling (AIS) algorithm is recently proposed to conduct robust and fast model updating using Gaussian mixture. This algorithm has been proved to enable constructing an importance sampling density (ISD) that mimics the target posterior density and is adopted in this paper to tackle a seismic analyses problem of 1-story moment frame with viscous damper. Results showcase that the distributions of parameters can be successfully updated using the algorithm with low computational cost. The updated results can also be further leveraged to guide the seismic safety assessment.

Keywords. Bayesian updating, adaptive importance sampling, Gaussian mixture, dynamic analyses, viscous damper; seismic safety

1. Introduction

Model updating has always been a topic of great concern because any theoretical or empirical model built for a civil engineering system is subject to uncertainty--that is, it cannot accurately predict the system response. The purpose of model updating is to reduce the model uncertainty using measurements of the actual response so that the model can more plausibly reflect the real characteristics of the system. Model updating can be handled as a Bayesian updating problem where the posterior is obtained by combining the prior which represents the engineer's empirical judgement and the likelihood which describes the observations and test data. With the maturity of sensor technology and the improvement of computing power, Bayesian model updating has been widely applied in structural identification [1], performance prediction [2], reliability assessment [3] and structural health monitoring [4].

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Consider an engineering model with n involved parameters represented by the $n\times 1$ convex $\boldsymbol{X} = [X_1; X_2; \ldots; X_n]$, the learning process of Bayesian model updating is derived by Bayes' theorem,

$$p(\boldsymbol{x}) = c_{\mathrm{E}} \pi(\boldsymbol{x}) L(\boldsymbol{x}) \tag{1}$$

where $\pi(\boldsymbol{x})$ and $p(\boldsymbol{x})$ denote the prior and updated (posterior) joint probability density function (PDF) of $\boldsymbol{X}$, respectively. They are associated through a normalized constant c_{E} and the so-called likelihood function $L(\boldsymbol{x})$ which is proportional with the probability of observation conditioned on $\boldsymbol{X} = \boldsymbol{x}$. Since the determination of c_{E} involves n-dimensional integrals, $p(\boldsymbol{x})$ is difficult to estimate analytically in most cases and the posterior samples are usually generated numerically. Markov chain Monte Carlo (MCMC) sampling technique constitutes a series of Bayesian updating methods, such as adaptive Metropolis-Hastings method [5], transitional MCMC method [6] and subset simulation-based BUS (Bayesian updating with structural reliability methods) method [7]. These methods take advantage of MCMC technique that it enables generating samples from arbitrary distributions. The inherit limitations of MCMC, including burn-in period, sample correlation and large coefficient of variance (COV) of estimators, however, are still not overcome. As a competing method, particle filter is mainly based on importance sampling (IS) technique and has been remarkably successful in tackling dynamic Bayesian updating problems. To protect from sample impoverishment of conventional particle filters such as sampling importance resampling filter [8], a rejuvenation step is added through the integration with MCMC [9]. Therefore, the shortcomings of MCMC have also been brought into particle filters. IS shows many advantages compare with MCMC, the only pity is that inappropriate choice of the ISD will cause significant bias of results. A novel AIS algorithm is proposed recently to provide a scheme of constructing the ISD resembling the posterior density using Gaussian mixture [10]. This method shows great potential in achieving robust and fast Bayesian updating.

Based on the dynamic analyses of a 1-story moment frame subjected to the 50% JR Takatori record from the Kobe 1995 earthquake, the foregoing AIS algorithm is used to update the structural parameters using the maximum roof displacement. Moreover, the updated results reflect the deviation of realistic situation and people's experience judgement, thus can give a guidance for the further seismic safety assessment.

2. Bayesian Model Updating with Adaptive Importance Sampling Using Gaussian Mixture

IS is the most fundamental variance reduction technique to improve sampling efficiency, which involves choosing an ISD that favors important samples. The construction of ISD is crucial to the success of this technique and is often the difficulty when using it. The optimal ISD g_{opt} that enables the variance of IS estimator reduce to zero is not available practically. When Bayesian updating is integrated with IS, the optimal ISD is exactly the posterior density $p(\boldsymbol{x})$. Toward the goal of constructing an ISD resembling $p(\boldsymbol{x})$, the novel AIS algorithm firstly searches for the local maxima and constructs a Gaussian mixture located at them. The initial Gaussian mixture is adaptively revised to approach $p(\boldsymbol{x})$ based on the combination of population Monte Carlo and cross-entropy. The detailed algorithm is summarized as Algorithm 1 [10].

Algorithm 1. Bayesian model updating with AIS using Gaussian mixture.

1. Transform the random vector $\boldsymbol{X}$ into a standard normal vector $\boldsymbol{U}$ using an isoprobabilistic transformation $\boldsymbol{T}$;
2. Search for the local maxima of $\varphi(\boldsymbol{u})L(\boldsymbol{T}^{-1}(\boldsymbol{u}))$ and construct the initial ISD using Gaussian mixture located at them;
3. Generate N_1 samples from the initial ISD. Estimate the covariance matrix of samples drawn from each component Gaussian density and calculate the normalized effective sample size $\bar{N}_{\text{eff}}$. Set a prescribed threshold value $\bar{N}_{\text{thr}}$ and turn to step 8 if $\bar{N}_{\text{eff}} \geq \bar{N}_{\text{thr}}$, otherwise turn to step 4;
4. Select out the population of important samples. Construct the ISD using Gaussian mixture located at the local maxima and these important samples with an enlarged version of the corresponding estimated covariance matrix.
5. Generate N_1 samples from the ISD in step 4 and calculate $\bar{N}_{\text{eff}}$. Turn to step 8 if $\bar{N}_{\text{eff}} \geq \bar{N}_{\text{thr}}$, otherwise turn to step 6;
6. Update the parameters (i.e., mean vectors, covariance matrixes and relative weights) of the Gaussian mixture based on cross-entropy method and then a new ISD is formed;
7. Generate N_1 samples from the ISD in step 6 and calculate $\bar{N}_{\text{eff}}$. Return to step 6 if $\bar{N}_{\text{eff}} < \bar{N}_{\text{thr}}$, otherwise continue to step 8;
8. Take the current Gaussian mixture as the final ISD;
9. Generate additional (N_2-N_1) samples from the final ISD. Transform these samples back to original space using $\boldsymbol{T}^{-1}$ and utilize the weighted samples to approximate the posterior distribution.

3. Case Study

The algorithm described in section 2 is adopted to update the parameters of a 1-story structure modeled as a moment frame based on the results of dynamic analyses. The frame model was posted by Sarven Akcelyan and Prof. Dimitrios G. Lignos [11]. As shown in figure 1, one viscous damper is installed in the single-story moment frame with 5m bay width and 3m story height. The beam is considered to be rigid and its weight is a deterministic constant 1000kN. Based on Opensees, columns and the beam are modeled with elastic beam-column elements, and the damping link is modeled with a *twoNodeLink* element. Moreover, the *ViscousDamper* material is used to model the viscous damper. The units of the Opensees model are mm, kN and seconds. Three parameters K_d, C_d and a_d are considered as random variables, where K_d denotes the axial stiffness, C_d the damping coefficient and a_d the exponent. They are mutually independent a prior and the prior distribution of $\boldsymbol{X} = [K_\text{d}; C_\text{d}; a_\text{d}]$ is listed in table 1. Therefore, the isoprobabilistic transformation between $\boldsymbol{X}$ and $\boldsymbol{U}$ can be written as $\boldsymbol{U} = \boldsymbol{T}(\boldsymbol{X}) = \{[\ln(K_\text{d})-3.2]/0.5; [\ln(C_\text{d})-3.0]/0.8; \Phi^{-1}[(a_\text{d}-0.3)/0.1]\}$, where Φ is the cumulative distribution function (CDF) of the standard normal distribution.

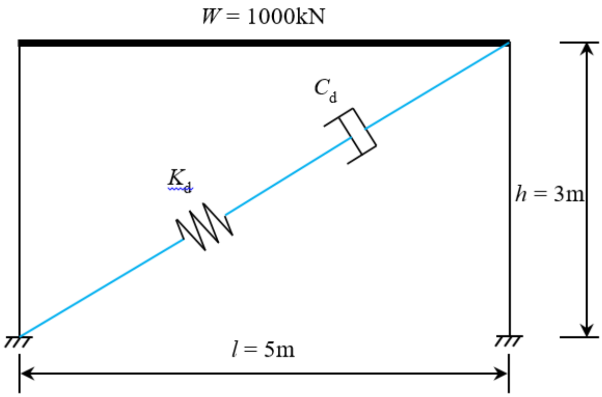


Figure 1. Schematic representation of the 1-story moment frame with a viscous damper.

Table 1. Prior distribution of ***X***.

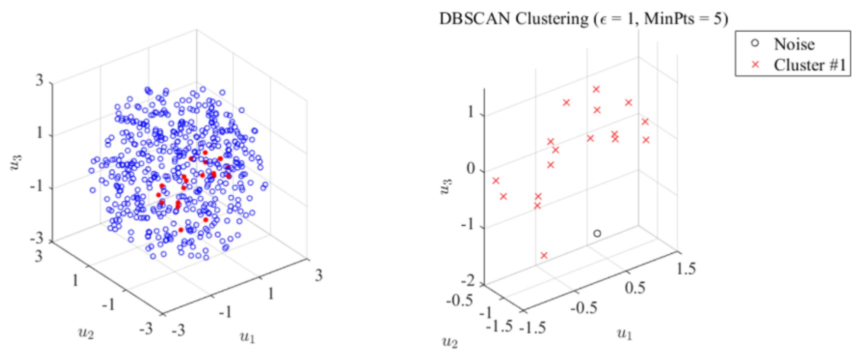
Random variables	Distribution type	Distribution parameters
K_d	Lognormal	$LN(3.2, 0.5)$
C_d	Lognormal	$LN(3.0, 0.8)$
a_d	Uniform	$U(0.3,0.4)$

Suppose the structure suffers from a small earthquake and the maximum roof displacement is recorded by the displacement sensors. This measurement can be used to update the parameters of the viscous damper. Since the small earthquake does no damage to the structure, these parameters do not change after the earthquake and the updated results are available for further seismic safety assessments. In this example, the 50% JR Takatori record is used to simulate the small earthquake and the measured maximum roof displacement is assumed $\tilde{\Delta} = 50\text{mm}$. Assume that the prediction error $\varepsilon = \Delta\left(\boldsymbol{T}^{-1}(\boldsymbol{u})\right) - \tilde{\Delta}$ follows a Gaussian distribution with mean 0 and standard deviation 5, the likelihood function is thus,

$$L\left(\boldsymbol{T}^{-1}(\boldsymbol{u})\right) \propto \exp\left\{-\frac{\left[\Delta\left(\boldsymbol{T}^{-1}(\boldsymbol{u})\right)-\tilde{\Delta}\right]^2}{2\times 5^2}\right\} \tag{2}$$

The proportional constant in Eq. (2) is set as 1. Toward the goal of searching for the local maxima of posterior density, 500 samples of $\boldsymbol{U}$ are uniformly generated inside the three-dimensional sphere with the origin as the center and radius of 3. Samples with the top 20 largest values of $\varphi(\boldsymbol{u})L(\boldsymbol{T}^{-1}(\boldsymbol{u}))$ are selected out and then are clustered using a modified DBSCAN (Density-Based Spatial Clustering of Application with Noise). As indicted in figure 2(b), the 20 samples are clustered into one cluster with one sample distinguished as a noise. Neglecting the noise, the mass center of the cluster [-0.161; -0.913; 0.079] is selected as the initial point of local optimization algorithm. The result of SQP (Sequential Quadric Programming) algorithm indicates that the local maximum point is very close to the initial point. Therefore, the initial ISD is constructed using a single Gaussian density,

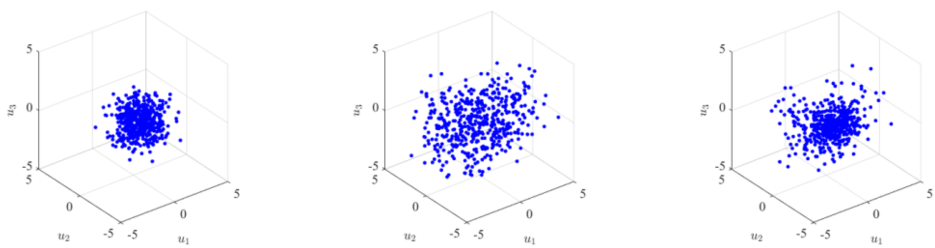
$$g(\boldsymbol{u}) = N\left(\boldsymbol{u} \mid [-0.161; -0.913; 0.079], \mathrm{diag}(1,1,1)\right) \tag{3}$$



(a) (b)

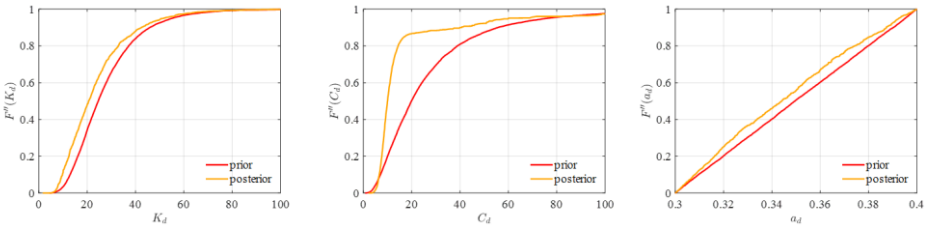
Figure 2. Searching for the local maxima of posterior density: (a) 500 samples and samples with the top 20 largest values of $\varphi(\boldsymbol{u})L(\boldsymbol{T}^{-1}(\boldsymbol{u}))$; (b) clustering result of the 20 samples

The initial ISD presented in Eq. (3) roughly covers the significant region of posterior density $p(\boldsymbol{x})$. One adaptively revises it to form an ISD that can more accurately delineate the shape of $p(\boldsymbol{x})$ by conducting steps 3-8 in Algorithm 1. The detailed procedure is omitted to avoid tedious descriptions. Take N_1=500, N_2=2000, $\bar{N}_{\mathrm{thr}} = 0.5$. As a result, a mixture of 15 Gaussian densities is chosen as the final ISD, whose parameters have been updated only once based on cross-entropy method. Figure 3 presents the details of this adaptive procedure and figure 4 presents the comparison of the prior and posterior marginal CDF of K_d, C_d and a_d. It can be observed that the measurement has an impact on the distributions of all three parameters. The damping coefficient is the most sensitive to the observation, while the other two are less influenced. According to the updated results, C_d is smaller than expected, which indicates that the structure is more likely to undergo large roof displacement and requires stabilization measures to enhance its seismic safety.



(a) (b) (c)

Figure 3. Constructing the ISD that resembling $p(\boldsymbol{x})$: (a) 500 samples generated from the initial single Gaussian density ($\bar{N}_{\mathrm{eff}} \approx 0.25$); (b) 500 samples generated from the initial mixture of 15 Gaussian densities (before using cross-entropy method, $\bar{N}_{\mathrm{eff}} \approx 0.19$); (c) 500 samples generated from the final of 15 Gaussian densities (after using once cross-entropy method, $\bar{N}_{\mathrm{eff}} \approx 0.56$)



(a) (b) (c)

Figure 4. Comparison of the prior and posterior marginal CDF of (a) K_d; (b) C_d; (c) a_d.

4. Conclusion

Based on the observed maximum roof displacement of a 1-story structure subjected to the 50% JR Takatori record, this paper successfully achieves Bayesian updating of three parameters of the viscous damper using a novel AIS algorithm. Results showcase that the algorithm can be applied into the dynamic analyses problem and update the distributions of three parameters at a computational cost of about 3500 model evaluations. The damping coefficient C_d is the most sensitive metric affected by the measured maximum roof displacement and the posterior marginal CDF indicates the large probability that it is smaller than expected, which informs engineers to take measures of stabilization.

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