

# Correlation Fitted of Loess Microscopic Parameter Statistics with Shear Strength $C$ and $\varphi$

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**Abstract.** The microstructure of loess mainly reflects the size distribution, orientation, contact mode and pore distribution of particles. The six original loess (low clayey silt loess) on the same site in Lanzhou is selected by the direct shear test, high power electron microscope and energy spectrum test. To simulated calculation the cohesion ( $C$ ) and internal friction angle ( $\varphi$ ), used by the parameters data, including to the mineral element (Fe/Al/Si, Al/Si, K) ratio, microstructure parameters (Long axis of the particles, long axis' skewness, kurtosis and orientation degree. The results indicated that the ( $\varphi$ ) value was linearly correlated with the mean value of the Angle, and the correlation coefficient of the fitting formula was 0.82. The cohesion  $C$  value was linearly correlated with the kurtosis, Skewness, Fe/Si, Al/Si and K/Al of the long axis, and the maximum correlation coefficient after fitting was 0.99. The fitting formula of microcosmic parameters and shear strength can provide a new research method for predicting the shear strength of loess samples.

**Keywords.** Loess, micro-parameters, shear strength, energy spectrum test, high power electron microscope, the cohesion ( $C$ ), internal friction angle ( $\varphi$ )

## 1. Introduction

The microstructure of loess has a very important influence on engineering geology. In the microscopic scanning electron microscope (SEM) image, the loess particles vary in size and shape. The microstructure of loess mainly reflects the size distribution, orientation, contact mode and pore distribution of particles. Because of the difference of geological environment, climate and age, the microstructure characteristics of loess are complex and diverse [1,2]. Soil minerals are mainly quartz, feldspar, illite kaolinite, montmorillonite, carbonate and a small amount of mica minerals [3]. The types and contents of mineral elements in the soil are different, and the stress – strain relationship of soil mechanics is also different. There are many articles on the study of loess and mechanical properties by SEM. Based on SEM micro-image, the effects of loess particle size, contact mode, cementing material and compactness on shear strength and deformation characteristics are analyzed. In addition, scholars comprehensively analyzed the difference of loess micro-characteristic parameters and the influence of

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soil particle cementation on mineral composition and mechanical properties [4-5]. It is very necessary and important to study the mechanical properties of loess and the quantitative analysis of the microstructure of loess [6-10].

It is very necessary and important to study the mechanical properties of loess by the methods of extracting micro-structure characteristics and quantifying microstructure parameters [11-22]. However, the factors of soil Micro-parameters (particle shape, porosity, mineral composition, etc) are not considered enough. In this paper, the relationship between the microstructure parameters and mineral element content of loess and shear strength index is established based on the SEM image, which provides a basis for further study of loess mechanical properties and engineering geological application.

## 2. Direct Shear Test of Loess in Lanzhou New Area

The undisturbed soil was collected every 50 meters in the undisturbed soil field near the primary school of Shanzhidun in Lanzhou new district. The soil sample is yellow and uniform in texture. The physical properties of the samples obtained from the geotechnical tests are shown in table 1. In this experiment, SDJ-II direct shear tester is used. Under the condition of the speed servo control 4/min, respectively for each sampling point on soil sample 50 kpa, 100 kpa, 200 kpa, 300 kpa, 400 kPa method to the load of soil samples in direct shear test, the maximum shear strength and the relationship between the load stress is shown in figure 1.

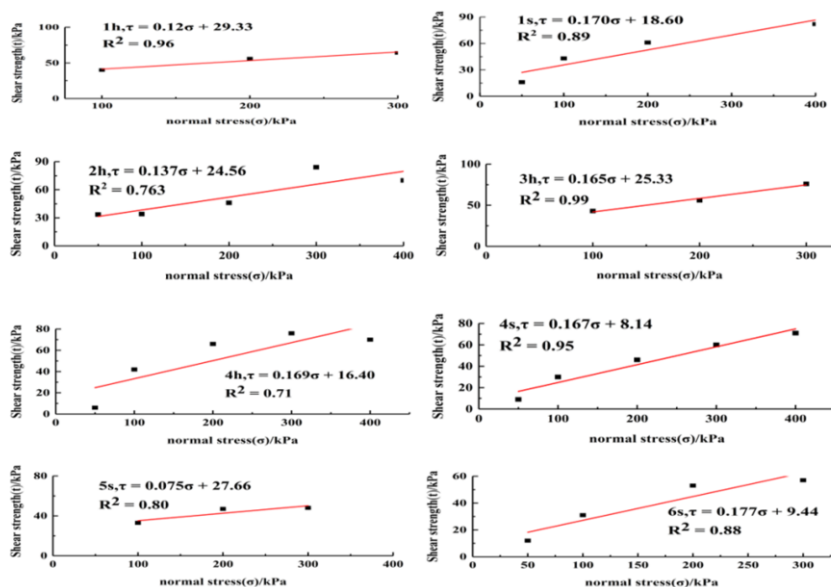


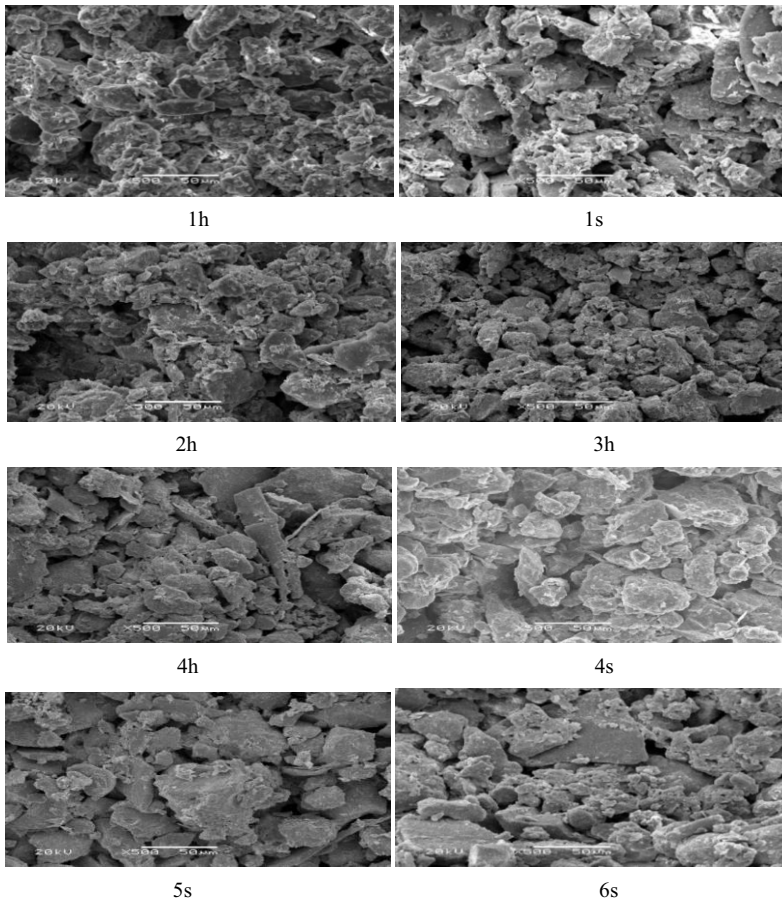
Figure 1. The relationship between shear stress and normal stress.

As shown in figure 1, the linear slope of six soil samples is the tangent value of  $\varphi$  and the constant term is the value of  $C$ . According to table 1 and figure 1, the

maximum value of  $\phi$  in 6s can up to 10 degrees, the minimum value of  $\phi$  in 5s is 4.3 degrees, the Maximum C is 1h, 29.33 kPa and the minimum value is 4s, 8.10 kPa.

### 3. Microstructure Characteristics and Parameters

Six undisturbed soil samples from different locations were selected and scanned with a kyky2800 electron microscope to obtain an SEM image magnified 500 times, as shown in figure 2. From the SEM images, it can be seen that the morphological distribution, particle size distribution and contact of different sampling positions are different. For example, 1h, 1s, 2h, 3h of skeleton particle surface and pore filling cementing material; while 4s and 5s of single particle size is relatively uniform, clear edges and fresh, powder, clay and cementing filling material less.



**Figure 2.** SEM microscopic diagram of loess samples in Lanzhou new area.

The microstructure parameters were extracted by the Kyky-2800 image processing program and the MTALB software image processing editing program was used to extract the microstructure parameters. The microstructural parameters include the

particle size of long axis and short axis, the particle angle (the angle between long axis and horizontal line), the kurtosis and skewness of long axis, the coefficient of grinding circle, and other data such as table 1.

**Table 1.** Microcosmic parameters, C,  $\phi$  and mineral element relative contents of loess in the New Area.

The sample number	c/kPa	$\phi/^\circ$	The water content %	The density of natural/g.cm <sup>3</sup>	Roundness coefficient	The Angle mean	The long axis of skewness	Long axis average	Long axis kurtosis	e/Si	I/Si	/Al
1h	29.33	6.84	2.01	1.33	1.05	88.79	5.01	34.71	5.80	0.13	0.36	0.93
1s	18.60	9.60	2.01	1.33	1.06	92.17	5.17	33.97	6.96	0.18	0.32	0.50
2h	25.70	7.24	3.3	1.27	0.90	87.02	4.28	25.89	6.29	0.12	0.37	0.26
3h	25.33	9.37	3.9	1.3	1.14	89.60	6.76	68.75	6.48	0.40	0.54	0.29
4h	16.40	9.60	3.5	1.37	1.03	90.53	5.48	45.35	11.10	0.09	0.33	0.24
4s	8.10	9.50	3.5	1.37	1.01	90.31	5.31	36.98	6.09	0.14	0.28	0.20
5s	27.66	4.30	6.5	1.29	0.87	84.14	8.31	112.09	6.67	0.26	0.35	1.01
6s	9.40	10.0	1.8	1.35	1.07	89.60	5.95	42.47	6.11	0.18	0.38	0.28

The calculation methods of the grinding coefficient for the kurtosis and skewness of the long axis are as follows:

Using skewness and kurtosis is beneficial to the analysis of particle parameters (long axis, short axis), distribution rules and structural characteristics. The skewness is the cubic of the particle dispersion and then divided by the standard deviation, as shown in formula (1); describes the measure of the skew and extent of the data distribution. When the skewness is less than 0, it is called left skewness, and when it is greater than 0, it is called right skewness. The larger the absolute value, the larger the offset. The long-axis particle kurtosis is the cube of the fourth-order deviation of the particle and then divided by the standard deviation, as shown in formula (2); the particle kurtosis describes the concentration trend of the particle size distribution. The larger the kurtosis, the higher and thinner the distribution; the smaller the kurtosis, Distribution is short and fatter.

$$Skewness = \frac{\sum_{i=1}^N (x_i - \bar{x})^3 F_i}{N\sigma^3} \quad (1)$$

$$Kurtosis = \frac{\sum_{i=1}^N (x_i - \bar{x})^4 F_i}{N\sigma^4} \quad (2)$$

$\bar{x}$  is the average value,  $\sigma$  is the standard deviation, N is the total number of particles,  $x_i$  is the *i*th particle, and  $F_i$  is the frequency of the long axis of the *i*th particle.

According to the formula (2), using the software of MATLAB and SPSS to process statistics, the long axis of particles is converted in  $\mu\text{m}$ , and the average value of the angle is in degrees. so as to obtain the particle long axis frequency distribution, as shown in figure 3; Long axis skewness and kurtosis square, as shown in figure 4.

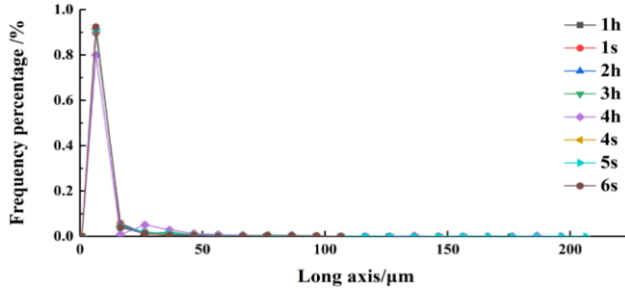


Figure 3. Frequency distribution of particles on the long axis.

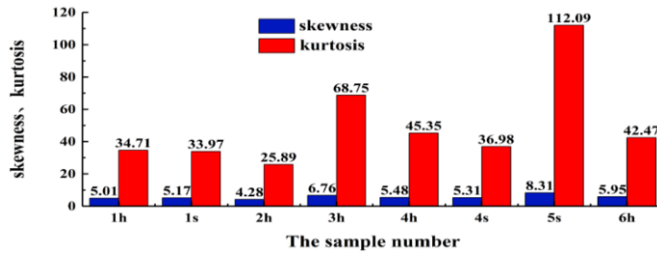


Figure 4. Long axis skewness and kurtosis of loess particles in the New Area.

According to figure 3 of the skewness and kurtosis values of the long axis of particles, it is found that the length of the long axis of particles is mostly concentrated at about 6.5 μm; the average length of the long axis is between 8 and 10 μm, and the majority of the long axis of particles is 0 < d ≤ 20 μm. According to the National Standard for Soil Classification (GBJ145-90) [16], it should be classified as fine-grained soil. All the long axis skewness is greater than 0, belonging to the right-skewed distribution. The kurtosis of the samples is greater than the normal distribution of 3, showing a peak distribution. The skewness and kurtosis of the sample 5s were the largest, 8.31 and 112.09 respectively, the sample 2s was the smallest, 3.77 and 17.96 respectively.

#### 4. Fitting analysis of microstructure parameters with c and φ values

In order to quantitatively analyze the related factors affecting the shear strength parameters C and φ of loess, multivariate regression analysis is used to study the relationship between the microstructure parameters and C, φ, and mineral content and C, φ values of loess in Lanzhou New District respectively.

Multiple regression analysis is a statistical method that studies the correlation between multiple variables, fits the model equations, and makes a variance analysis of the fitted model errors. In order to study the correlation between the random variables  $x_1, x_2, \dots, x_m$ , and the dependent variable y. The computational formula is the model equation (3).

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_m x_m + \varepsilon \tag{3}$$

$y$  is a regression equation,  $x_1, x_2, \dots, x_m$  is a random variable, and  $\beta_0, \beta_1 \dots \beta_m$  is a regression coefficient.  $\varepsilon$  is a random error.

The model equation (4) is established by the least square method to calculate the coefficient  $\beta_0, \beta_1 \dots \beta_m$ . Due to the error between the model fitting value and the actual measurement, anova F test is required to test whether the fitted equation is correlated and whether it can be used to predict the actual situation. If the variance significance is satisfied, the established regression equation has practical significance and can be used to predict actual data, as shown in the references [15,20] for details [15,20]. Due to the large calculation amount of multiple regression analysis, this paper uses SPSS software for regression analysis.

#### 4.1. Fitting of Internal Friction Angle

##### 4.1.1. Fitting of Single Factor

In order to study the relationship between the angle mean and the phi value, the angle mean and the internal friction angle of the Loess in the new area are regressive fitted. The results are as follows: the fitting regression equation is  $\varphi = 0.736\alpha - 57.20$   $R^2 = 0.82$ .

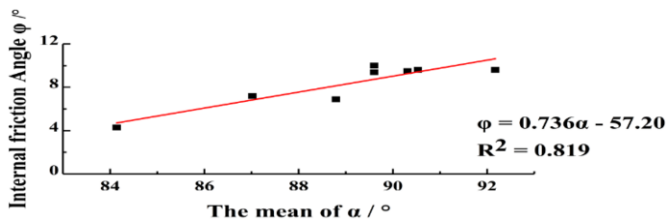


Figure 5.  $\varphi$ - $\alpha$  mean fitting.

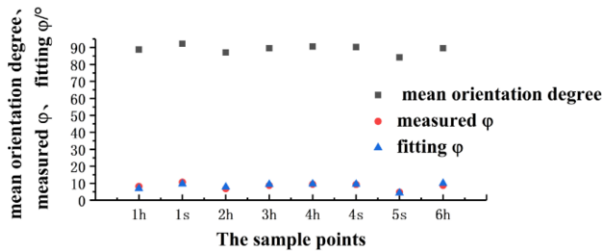


Figure 6. Comparison of measured and fitted angular mean values.

Figure 5 shows that the scatter plot of internal friction angle and particle angle mean has a linear correlation, the correlation coefficient  $R^2$  of fitting regression equation is 0.82, and the correlation coefficient is higher; The  $\varphi$  value of the loess is linear positive correlation with  $\alpha$ , and  $\varphi$  value increases with the increase of  $\alpha$ . Figure 6 is a comparison diagram of the measured value and fitted internal friction angle. The corresponding measured value coincides with the fitted value, and the fitting degree is high. It can be seen that the angle mean of loess particles is one of the factors affecting the value of  $\varphi$ . The larger the angle mean is, the larger the internal friction angle is. The angle mean can be used to predict the internal friction angle of Loess particles.

4.2. Cohesion Fitting Analysis

Coulomb's law shows that the shear strength of soil consists of its internal friction angle and cohesion [19], and the mineral composition between loess particles also influences shear strength. Microcosmic energy spectrum was used to monitor the content of chemical elements of cementing substances around the pores, and to measure the phase relative content of chemical elements, and to calculate the ratios of Fe/Si, Al/Si and K/A. Because of the high content of Fe, more acidic substances, higher cementation strength and higher shear strength. In conclusion, aluminum compounds have high strength, and the ratios of positive correlation are Al/Si and Ca/Fe. The negative correlation is K/Al. In addition, the loess microstructure parameters of original soil at each location, C and phi values, and the relative content statistics of mineral elements are shown in table 1.

Long-axis kurtosis, long-axis skewness, Fe/Si, Al/Si, and K/Al were input into SPSS software as the fitting parameters independent variables. The results were as follows.

Table 2. C value multiple regression analysis table.

The fitting parameters	The equation coefficient	Regression degree of freedom	Degree of error freedom	F	R <sup>2</sup>
Constant term	51.32				
Long axis kurtosis	0.66				
The long axis of skewness	-16.56	5	2	35.43	0.99
Fe/Si	6.89				
Al/Si	60.67				
K/Al	16.74				

Attention: Significance level  $\alpha=0.05$

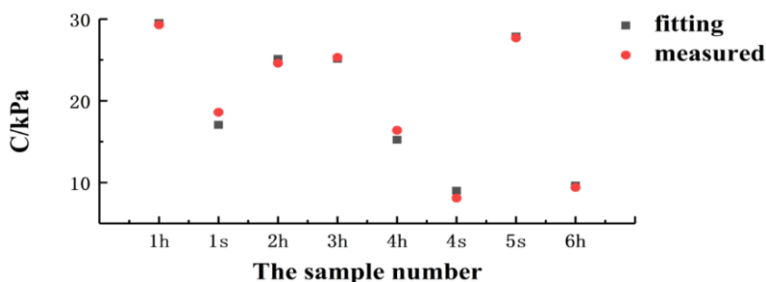


Figure 7. Comparison of measured C-fitting C.

According to table 2, the regression equation (4) can be obtained. In the formula (4) , C,  $k_1$ ,  $k_2$ ,  $k_3$ ,  $k_4$ ,  $k_5$  are cohesion, long axis kurtosis, long axis skewness, ratio of iron to silicon, ratio of aluminium to silicon, ratio of potassium to aluminium, respectively. Regression equation (4), correlation coefficient  $R^2=0.99$ , after checking the F distribution threshold table 2,  $F=35.43 > F_{\alpha}(5,2)=19.30$ .

Therefore, the fitting parameters are linearly related to the C value, and the fitted regression Equation (4) is meaningful and can be used to predict C values. According

to the measured C fit, as shown in figure 7, the fitting C value and the measured C value correspond to each other, and have a high linear correlation. Equation (3) shows that the kurtosis of loess's long axis, Fe/Si, Al/Si and K/Al are positively correlated with the C value, and the C value increases with the increase of these four parameters. However, the skewness of the long axis is negatively correlated with the C value. The larger the kurtosis of the long axis and the smaller the skewness, the better the gradation of the loess, the higher the content of the granules and viscous particles, and the larger the C value.

## 5. Conclusion

(1) The electron microscopic microstructure analysis and direct shear test analysis of the loess in the same area of Lanzhou New District showed that the microscopic parameters and mineral element ratios of the same soil samples with different distances were different with different sampling positions. The long axis of the particles is non-normal distribution, and both have a right-biased trend, and the distribution concentration has obvious peak phenomenon; and the particle orientation degree is roughly bimodal. The larger the long axis kurtosis and the smaller the skewness, the better the grading of the loess, and the higher the content of the powder and clay.

(2) The direct shear strength (C,  $\phi$  value) of the loess in Lanzhou New Area and the multivariate formula fitting of the microstructure parameters indicate that the long axis kurtosis, Fe/Si, Al/Si, K/Al of the loess particles are positively correlated with the C value. The axis skewness is negatively correlated with the C value, and the regression equation satisfies the significance requirement of the variance test. The correlation coefficient of the fitting formula reaches 99%. Has the meaning of predicting C values.

(3) The scatter plot of the internal friction angle and the mean angle of the particle is roughly linearly correlated. The correlation coefficient  $R^2$  of the fitted regression equation is 0.82, and the correlation is high. The corresponding measured value and the fitted value are basically coincident, and the degree of fitting is higher. Thus, the magnitude of the internal friction angle can be predicted by the angle mean.

In short, the microstructural parameters of loess and quantitative analysis of mineral elements can establish a relationship with C and  $\phi$  values, providing new information for engineering applications.

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## References

- [1] Lei XY. Relationship between the microstructure of loess and paleoclimate. *Geological Evaluation*. 1989; 35(4): 333-341.



- [2] Lei XY. Formation age and microstructure characteristics of Qingyuan Caoxian loess. *Acta Geographica Sinica*. 1995; (06): 521-525.
- [3] Deng J, Wang LM, Wu ZJ, et al. Acid modification method and microstructure analysis of seismic collapse deformation of loess. *Rock and Soil Mechanics*. 2012; 33(33): 3625 ~ 3631.
- [4] Song Z, Cheng QG, Zhang W, Meng FC. Analysis of structural strength and deformation characteristics of undisturbed loess. *Journal of Railway Engineering*. 2007; (03):6-11+32.
- [5] Wang HN, Ni WK. Quantitative analysis of loess microstructure based on computed X-ray tomography and scanning electron microscope images. *Rock and Soil Mechanics*. 2012; 33(01):243-247+254.
- [6] Li RK, Wu ZJ, Liang QG, et al. Study on influencing factors of loess dynamic characteristics considering microstructure characteristics. *Chinese Journal of Engineering Geology*. 2008; 26(04):905-914.
- [7] Wang LM. Dynamics of loess. Beijing: Earthquake press. 2003; p. 12-82.
- [8] Wang LM, Deng J, Huang Y. Quantitative analysis of seismic collapsibility of loess. *Journal of Rock Mechanics and Engineering*. 2007; (S1):3025-3031.
- [9] Deng J, Wang LM, Shi YC. Numerical simulation of loess field response waveform based on microstructure particle theory. *Chinese Journal of Geotechnical Engineering*. 2010; 32(02):172-179.
- [10] Deng J, Wang LM, Wu ZJ. Establishment and analysis of elastoplastic loess microstructure and dynamic deformation model. *Journal of Rock Mechanics and Engineering*. 2013; 32(S2):3995-4001.
- [11] Deng J, Wang LM, Wu ZJ. Analysis of microscopic differences and dynamic indexes in lintao loess soil layer. *Seismic Engineering and Engineering Vibration*. 2016; 36(01):164-168.
- [12] An L, Deng J, Wang LM. Experimental study on the microstructure characteristics of loess liquefaction. *Chinese Journal of Seismic Engineering*. 2018; 40(04):752-758.
- [13] Guo QY, Gu TF, Xie WL, et al. Analysis of shear strength and parameter difference of loess under different test methods -- A case study of loess landslide in Heifangtai, Gansu Province. *Journal of Science and Technology of Disaster Prevention*. 2018; 20(01): 25-31.
- [14] Qiu GR, Shi YC, Liu HM. Analysis of the change of microstructure with dynamic stress in loess seismic depression. *Journal of Northwest Seismology*. 2010; 32(01):42-46.
- [15] Wei JY, Gao SC, Lan XL. Research review on soil microstructure. *Forestry Science and Technology Information*. 2010; 42(03):120-123.
- [16] Jia JP, He XQ, Jin YJ. *Statistics*. Beijing: Renmin University of China Press, 2000.
- [17] Xu SF. GB145-90, Classification standard of soil. Beijing: China Planning Press, 1991.
- [18] Wang M, Bai XH, Yang J. Sample preparation for microstructure analysis of collapsible loess. *Journal of Taiyuan University of Technology*. 2010; 41(03):283-286.
- [19] Li ZY, Yang YY. *Introduction to engineering geology*. Wuhan: China University of Geosciences Press, 2011.
- [20] Liu XP, Zhang YL. *Introduction to educational statistics and evaluation*. Beijing: Science Press, 2003.
- [21] Xie J, Chen WW, Cheng J, et al. Relationship between fractal dimension of microstructure and porosity and compressive strength. *Journal of Lanzhou University (Natural Science Edition)*. 2009; 45(02): 26-330.
- [22] Wu WJ, Chen WW, Song BH, et al. Shear test of unaltered Q2 loess in Lanzhou. *Journal of Lanzhou University (Natural Science Edition)*. 2012; 48(06): 21-25.