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# Study on Simplified Calculation Method of Horizontal Displacement of Wharf Pile Foundation Under Horizontal Cyclic Load

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**Abstract.** The upper structure of high piled wharf in coastal soft soil area is loaded by waves or ships, which is transferred to the foundation soil through the pile foundation. The force is small and lasts for a long time, which causes the strength of saturated soft clay in the vibration range to decrease slowly, and the weakening effect of soil gradually accumulates, resulting in the horizontal displacement of pile foundation. In this paper, the maximum horizontal displacement and residual horizontal displacement of pile are deduced and calculated by the analytical method through appropriate simplification, considering the stiffness of pile group, plastic deformation of soil and cyclic weakening. A simplified calculation method of horizontal displacement of pile under horizontal cyclic load is obtained. Compared with the numerical simulation method, the results show that the simplified calculation theory of pile horizontal displacement under horizontal load can better reflect the displacement development law of pile, and then can predict the long-term deformation of pile.

**Keywords.** Horizontal cyclic loading, pile foundation, soft clay, horizontal displacement

## 1. Introduction

The strength of the soil around the pile will weaken with horizontal cyclic load, and the horizontal displacement of the pile foundation will increase with the increase of cyclic loading times. The strength weakening of the soil caused by cyclic loading accounts for a large part of the displacement of the pile. Therefore, the soil weakening can not be ignored in the calculation of pile displacement. The calculation of horizontal displacement of single pile under horizontal load has been studied [1]. For the research of pile group, especially the horizontal displacement of pile group considering the influence of overall stiffness, scholars [2] at home and abroad mostly carry out research by model test or numerical simulation, and reveal the displacement and deformation mechanism of pile group foundation under horizontal cyclic load. Moss and Anderson [3] through 1 of 5 piles × Based on the model test of pile group under horizontal cyclic

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load, the stress mechanism of pile group in cohesive soil under seismic load is analyzed. S. S. Chandrasekaran et al. [4] conducted model tests on pile groups with different pile diameter ratio, pile spacing and length diameter ratio to study the mechanical behavior of pile groups under horizontal cyclic load. Chen Renpenget al. [5] carried out the test of pile-soil interaction under horizontal cyclic load in a large model tank in the laboratory. Jin Weiliang and Song Zhigang [6] analyzed the separation between pile foundation and soil under vibration by using the modified p-y curve and numerical calculation method. And other scholars have done similar research [7].

In this paper, through appropriate simplification, comprehensively considering the factors such as pile group stiffness, soil plastic deformation and cyclic weakening, the maximum horizontal displacement and residual horizontal displacement of piles are calculated by analytical method, and a simplified calculation method is provided for the calculation of pile horizontal displacement under cyclic horizontal load in similar pile foundation projects.

### *1.1. Model Simplification and Analysis*

It is assumed that the pile spacing of high pile cap in the vertical load action direction is equal, there are m + 1 rows of piles along the load action direction, and the pile spacing is L1, L2,..., Li,... LM respectively. Each column of piles acts on the total horizontal load P, as shown in figure 1. Take one span high pile cap along the load action direction for analysis, and its model is shown in figure 2.

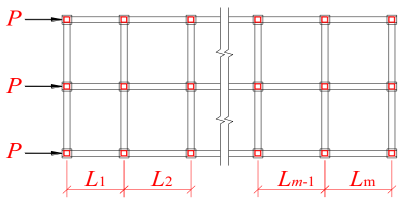



**Figure 1.** Plan of high pile cap under horizontal load.

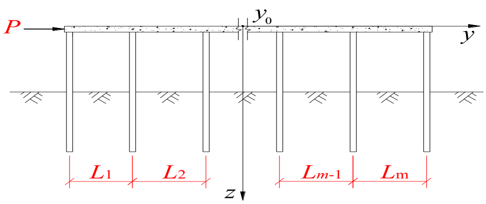



**Figure 2.**Section of high pile cap under horizontal load.

The deformation of single pile under horizontal cyclic load is shown in figure 3. Under the action of cyclic load P, at the peak time, the displacement of pile top is YM and the corresponding pile resistance is PM. After unloading, due to the plastic deformation of soil, the pile cannot be restored to the initial position, the residual displacement of pile top is yr and the corresponding pile resistance is pr. In order to facilitate the analysis, the OA slope KH of the straight line section is used to replace the foundation reaction coefficient during loading, and the AB slope KHR of the straight line section is used to replace the foundation reaction coefficient during unloading.

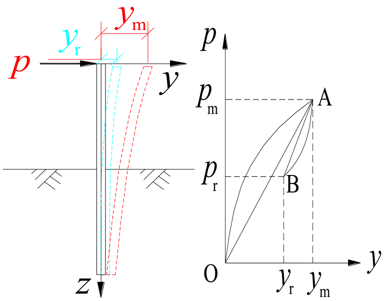



**Figure 3.** Peak value and residual displacement of pile under horizontal cyclic load.

The single pile in the pile foundation can be analyzed by the elastic foundation beam embedded in the foundation soil. According to the stress state of the pile in the soil, the differential equation of beam (pile) deflection can be derived from the theory of structural mechanics.

$$\frac{d^2}{d^2 z_i}(D_i \frac{d^2 y_{mi}}{d^2 z_i}) + p_{mi}(z_i, y_i) = 0 \tag{1}$$

$D_i$—Horizontal lateral stiffness of the pile;

$p_{mi}(z_i, y_i)$—The pile resistance corresponding to the horizontal displacement y at the depth X of the ith pile at the peak loading time.

Similarly, after uninstallation:

$$\frac{d^2}{d^2 z_i}(D_i \frac{d^2 y_{ri}}{d^2 z_i}) + p_{ri}(z_i, y_i) = 0 \tag{2}$$

$p_{ri}(z_i, y_i)$—Pile resistance corresponding to horizontal displacement y at depth X of the ith pile after unloading.

At the peak moment, the pile resistance has the following relationship with the horizontal displacement of the pile.

$$p_{mi}(z_i, y_i) = b_{0i} K_{hmi}(z_i, y_i) y_{mi}^{n_1}(z_i, y_i) \tag{3}$$

$b_{0i}$—Width of the ith pile (equivalent width in case of round pile); $n_1$—index, at the peak time, the soil will undergo plastic deformation, $n_1 < 1$ [8].

The following formula is the approximate analytical solution.

$$p_{mi}(z_i, y_i) = b_{0i} K_{hmi}(z_i, y_i)\left[c y_{mi}(z_i, y_i) + d\right] \tag{4}$$

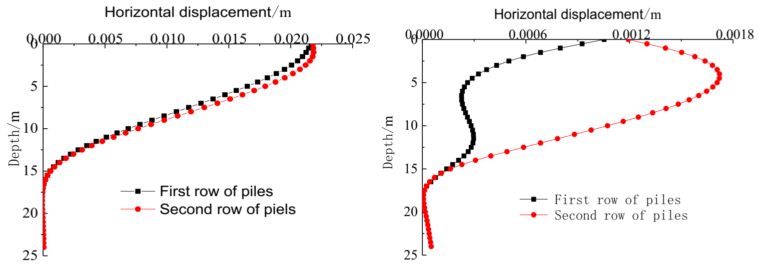
*C*, *d*— $p_1 = y_{mi}^{n_1}(z_i, y_i)$ Slope and intercept of the resulting line.

$y_{mi}(z_i, y_i)=0$, $p_{mi}(z_i, y_i)=0$, $d=0$.

Since the internal force and external resistance of the pile are balanced at the initial time, the internal force and resistance of the pile still need to be balanced in the rebound process.

$$D_i \frac{\mathrm{d}^4 y_{ei}}{\mathrm{d}^4 z_i} + b_{0i} K_{hri} y_{ei} = 0 \tag{5}$$

In addition, along the direction of load action, the horizontal displacement of the i-th pile will squeeze the M-I pile, thus affecting the horizontal displacement of the other M-I piles, as shown in figure 4. Figure 4 shows the peak time and horizontal displacement of the front and rear rows of piles in the direction of load action in the numerical simulation. It can be seen from the figure that the horizontal displacement of the rear row of piles is slightly larger than that of the previous row of piles under the influence of the previous row of piles. Above the top surface of the soil, the displacement difference between the front and rear piles is slightly larger, and the difference is not very obvious under the action of peak load, but after unloading, the displacement difference between the front and rear piles is larger. In the soil, the displacement difference between the front and rear piles is very small. For the convenience of analysis, when the external load acts on the whole structure, it is assumed that the horizontal resistance of the pile body is only related to the stiffness of the pile body.




(a) Loading peak time (b) After unloading

**Figure 4.**Horizontal displacement of pile at different positions.

Since the soil will weaken under cyclic load, in order to reflect the influence of soil weakening on the horizontal displacement of pile, it is assumed that the foundation reaction coefficient after N cycles has the following relationship with the foundation reaction coefficient of undisturbed soil

$$K_{hmi}(N) = K_{hm0i} f(N) \tag{6}$$

$$K_{hi}(N) = K_{hr0i} f(N) \tag{7}$$

The foundation reaction coefficient is similar to the dynamic elastic modulus. The dynamic modulus ecy$E_{cy,N}$ is the cyclic stress amplitude of the nth cycle $\sigma_{cy,N}$ and axial cyclic strain amplitude Ratio of $\varepsilon_{cy,N}$

$$E_{cy,N} = \frac{\sigma_{cy,N}}{\varepsilon_{cy,N}} \tag{8}$$

$$\delta = \frac{E_{cy,N}}{E_{cy,1}} = \frac{\sigma_{cy,N} / \varepsilon_{cy,N}}{\sigma_{cy,1} / \varepsilon_{cy,1}} \tag{9}$$

Then the foundation reaction coefficient can be expressed as:

$$K_{\mathrm{h}i}(N) = K_{\mathrm{h}0i}\delta \tag{10}$$

$$K_{\mathrm{h}i}(N) = K_{\mathrm{hr}0i}\delta \tag{11}$$

Combined formula (1) - formula (5) can be obtained：

$$y_{\mathrm{m}i} = e^{-\beta_{\mathrm{m}i}z}\left(A_{1i}\cos\beta_{\mathrm{m}i}z + A_{2i}\sin\beta_{\mathrm{m}i}z\right) + e^{\beta_i z}\left(A_{3i}\cos\beta_{\mathrm{m}i}z + A_{4i}\sin\beta_{\mathrm{m}i}z\right) \tag{12}$$

$$y_{\mathrm{e}i} = e^{-\beta_{\mathrm{r}i}z}\left(C_{1i}\cos\beta_{\mathrm{r}i}z + C_{2i}\sin\beta_{\mathrm{r}i}z\right) + e^{\beta_i x}\left(C_{3i}\cos\beta_{\mathrm{r}i}z + C_{4i}\sin\beta_{\mathrm{r}i}z\right) \tag{13}$$

Pile rotation angle $\theta$ :

$$\theta_{\mathrm{m}i} = e^{-\beta_{\mathrm{m}i}z}[-A_{1i}(\cos\beta_{\mathrm{m}i}z + \sin\beta_{\mathrm{m}i}z) + A_{2i}(\cos\beta_{\mathrm{m}i}z - \sin\beta_{\mathrm{m}i}z)] \tag{14}$$

$$\theta_{\mathrm{e}i} = e^{-\beta_{\mathrm{r}i}z}[-C_{1i}(\cos\beta_{\mathrm{r}i}z + \sin\beta_{\mathrm{r}i}z) + C_{2i}(\cos\beta_{\mathrm{r}i}z - \sin\beta_{\mathrm{r}i}z)] \tag{15}$$

## 2. Example Verification

Taking a single pile as an example for verification: assuming that the pile is 23m long, the buried depth is 11.5m, the elastic modulus of the pile concrete material is $E$=30Gpa, the stiffness $EI$=156250kN·m$^3$, the external load $P$=100kN, and the kaolin model parameters are selected for the soil (saturation weight 19.2kN/m$^3$, static lateral pressure coefficient 0.7, elastic modulus $E$=30Mpa, undrained shear strength $C$=17 kpa): the foundation reaction coefficient can be obtained through the side pressure test before and after the cyclic load, and it can also be taken as an empirical value, $K_{\mathrm{hm}}=10^4$kN/m$^3$, $K_{\mathrm{hr}}=1.5\times10^4$kN/m$^3$ (empirical value is taken here); By testing the earth pressure in the plastic area on the side of the pile, the straight line is used to approximate the curve $c$=0.8; $\beta_{\mathrm{m}i} = \sqrt[4]{cb_{0i}K_{\mathrm{hm}i}/4D_i}$ , $\beta_{ri} = \sqrt[4]{b_{0i}K_{\mathrm{hr}i}/4D_i}$ , $\beta_{\mathrm{m}}$=0.05, $\beta_{\mathrm{r}}$=0.06.

The displacement of pile top is calculated according to the horizontal displacement and rotation angle of pile body at the top surface of soil mass:

$$y_t = y_0 + L_u\theta \tag{16}$$

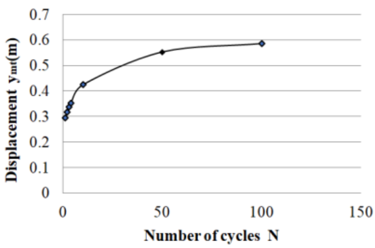
$y_t$, $y_0$, $L_u$ —They are the horizontal displacement of the pile top, the horizontal displacement of the pile body at the top surface of the soil, and the length of the pile body above the top surface of the soil.

Under different cycle times, the horizontal displacement and rotation angle of pile body at the top surface of soil mass are shown in table 1.

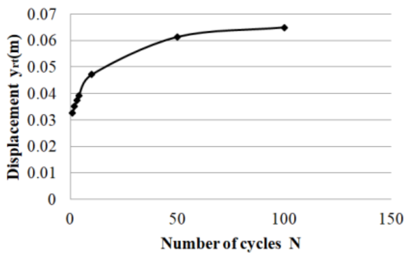
**Table 1.** Displacement and rotation angle of pile body on the top surface of soil under cyclic loading times.

| $N$ | 1 | 2 | 3 | 4 | 10 | 50 | 100 | ∞ |
|---|---|---|---|---|---|---|---|---|
| $y_{0m}$(m) | 0.0763 | 0.0817 | 0.0867 | 0.0911 | 0.1095 | 0.1425 | 0.1509 | 0.1613 |
| $\theta_{0m}$(rad) | 0.0191 | 0.0204 | 0.0217 | 0.0228 | 0.0274 | 0.0356 | 0.0377 | 0.0403 |
| $y_{0r}$(m) | 0.0120 | 0.0129 | 0.0137 | 0.0144 | 0.0173 | 0.0225 | 0.0238 | 0.0255 |
| $\theta_{0m}$(rad) | 0.0018 | 0.0019 | 0.0021 | 0.0022 | 0.0026 | 0.0034 | 0.0036 | 0.0038 |

Substitute the results in table 1 into formula (16) and arrange them into a curve. The results are shown in figure 5. $y_{mt}$in the figure represents the horizontal displacement of pile top at the peak loading time, and $y_{rt}$ represents the residual displacement of pile top after unloading. It can be seen from the figure that when the number of cyclic loading is small, the horizontal displacement of pile top increases rapidly. With the gradual increase of the number of cyclic loading, the limit horizontal displacement of pile top calculated theoretically tends to be stable.




**(a)** Loading peak time




**(b)** After unloading

**Figure 5.** Variation of horizontal displacement of pile top with cyclic loading times.

## 3. Conclusion

In this paper, the peak deformation and residual deformation of pile foundation in horizontal direction under horizontal cyclic load are analyzed, and the following conclusions are drawn:

(1) The deformation of pile group is affected by many factors. For pile group foundation with specific foundation soil layer, when calculating the deformation of pile group, the comprehensive function of main influencing factors such as the structural characteristics of pile group, the resistance of foundation soil around pile and the

number of cyclic loading should be considered. The structural characteristics of pile group include pile body stiffness, bearing platform plate stiffness and the number of spans of pile group along the load direction. The pile body stiffness and bearing platform plate stiffness affect the peak deformation and residual deformation of pile body, and the number of spans of pile group along the load direction affect the deformation of single pile. The resistance of soil around the pile affects the peak deformation and residual deformation of the pile under a certain cyclic load. The number of cyclic loads affects the weakening degree of soil strength, and then affects the cumulative peak deformation and residual deformation of the pile;

(2) The soil around the pile has elastic-plastic deformation, and the resistance of the soil around the pile needs to be calculated by the method that can reflect the elastic-plastic deformation characteristics of the soil;

(3) With the increase of cyclic loading times, the stiffness of foundation soil weakens. In the calculation of soil resistance, the law of stiffness weakening of foundation soil should be considered, and then the cumulative deformation of pile under long-term cyclic load can be carried out in combination with the law of stiffness weakening of foundation soil.

(4) By comprehensively considering the overall stiffness of the structure, the influence of soil elastic-plastic deformation on the resistance of the pile, and the influence of cyclic loading times on the cumulative deformation of the pile, the simplified calculation theory of pile horizontal displacement under horizontal load can better reflect the displacement development law of the pile, and then predict the long-term deformation of the pile.

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