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Measurement Uncertainty Evaluation of a Gauge Block Using Monte Carlo Simulation

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Abstract. The present article describes measurement uncertainty estimation of a Gauge Block (GB) through the Law of propagation of uncertainty (LPU) and Monte Carlo Simulation (MCS) as per JCGM 100 and 101: 2008 recommendations respectively. This paper describes the implementation of the Monte Carlo Simulation Method (MCM) for measurement uncertainty evaluation through octave and R programming, environment for numerical computations. In the present investigation, the measurement uncertainty obtained using the GUM approach is compared with the theoretically simulated results obtained for MCM in octave and R programming. A good agreement has been observed between values obtained for expanded measurement uncertainty using these approaches.

Keywords. Calibration, Metrology, Modeling, Octave, Probability Density Functions, Random Number, R Language, Simulation

1. Introduction

Metrology encompasses all the theoretical and practical concepts that are involved in a measurement, and when applied to a measurement process, can provide results that are accurate and metrological reliable. In this work, uncertainty in measurement for calibration of GB has been evaluated including contributions of various identified error sources. GB, a well-known accurate artefact having flat and parallel opposing surfaces, are important measurement standards in traceable dimensional metrology and industry. GBs are calibrated using two commonly adopted methods, laser interferometry and mechanical comparison.[1-4] However, due to consideration of cost and time, calibration of GB by mechanical comparison method with lowest possible uncertainty is preferred. As per VIM (International Vocabulary of Metrology), the measurement uncertainty is a "non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used".[5] ISO guidelines for uncertainty in measurements (GUM) facilitate harmonization of evaluation and

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expression of uncertainty. We have evaluated uncertainty in measurement for GB using GUM based LPU and MCM.[6]

Two evaluation approaches involve Type A and Type B methods. In Type A uncertainty estimation is performed using statistical analysis of a number of independent observations made under repeatability conditions. Standard uncertainty is given by

$s(\bar{q}) = \frac{s(q)}{\sqrt{n}}$, where $s(\bar{q})$ is square root of experimental variance given by

$s^2(q) = \frac{1}{n-1}\sum_{i=1}^{n}(q_i - \bar{q})^2$ and $\bar{q}$ is average of measured values q_i.

For n number of observation, i ranges from 1 to n.

Type B evaluation is done by methods other than statistical analysis of series of observations e.g. based on previous data, calibration certificates, limit values based on scientific judgments etc. In such cases the limit values are divided by factors based on assumed probability distributions. However, this LPU approach has several limitations such as

- For nonlinear model functions, there are limitations due to first-order approximation of the Taylor series expansion.
- When one or more input quantity is dominant, LPU approach may not be valid.[11]
- It requires calculations of degree of freedom, sensitivity coefficients and input quantities to have symmetric distribution.

MCS approach has been introduced to overcome these limitations of LPU approach for uncertainty estimation.

2. Monte Carlo Approach for Measurement Uncertainty Evaluation

The Monte Carlo method (MCM) is an alternative approach of evaluating measurement uncertainty. It depends on the propagation of input quantities distributions to provide probability distribution of output quantity.[7-11] In this method, random numbers are generated corresponding to each input quantity with specified probability distributions. The steps involved in this approach are

- Define measurand and input quantities and model function as Y=f(x)
- Assign probability distribution functions for each input quantity
- Generate random numbers corresponding to input parameters & run MCS
- Combine all random numbers as per model function to obtain output estimate and standard uncertainty

While generating random numbers, Monte Carlo trials (*M*) are chosen according to the formula given by Eq. 1.[(12)]

$$M > \frac{10^4}{(1-p)} \tag{1}$$

Where, p is the chosen coverage probability. Thus, Monte Carlo trials should be greater than 10^5 for 95% coverage area which corresponds to p equals to 0.95.

3. Measurement Setup

GB Comparator is used for calibration of GB of nominal length 50 mm, and the schematic diagram is shown in Figure 1(a). The setup consists of a mechanical comparator comprising of two probes, Digital Read Out (DRO). The comparator has two templates on its base table where standard GB, and test GB placed. The care is taken that the setup is leveled and anti-vibration paddings are used. For determining the central length deviations (Figure 1(b)), each GB is contacted by two probes[13] and DRO readings are noted.

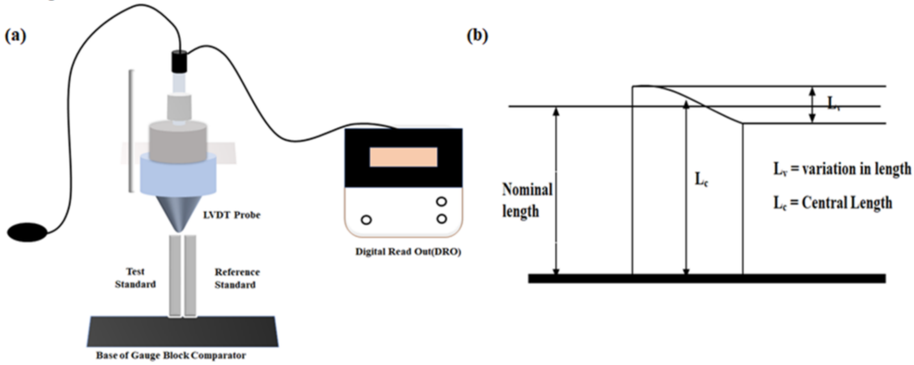

Figure 1. (a) Schematic of Gauge Block Comparator, (b) Measurement of Central Length Deviation of GBs

4. Results and Discussion

Table 1 shows the observations taken for Central Length Deviation (CLD) of GB of nominal length 50 mm using GB comparator.

CLD of the standard as per calibration certificate is, E = 0.01 µm

Initial Temperature = 19.2 °C, Final Temperature = 19.3 °C

Table 1. Observations for Calibration of 50 mm length GB using GB Comparator, E=0.01 µm

S.NO.	Test Standard (t), µm	Reference Standard (s), µm	t -s + E, µm
1.	10.18	10.38	-0.19
2.	10.19	10.39	-0.19
3.	10.18	10.39	-0.20
4.	10.19	10.38	-0.18
5.	10.19	10.38	-0.18
6.	10.19	10.38	-0.18
7.	10.18	10.39	-0.20
8.	10.18	10.39	-0.20
9.	10.19	10.39	-0.19
10.	10.18	10.39	-0.20
		Mean of CLD of Test Gauge	-0.191 µm

Standard Deviation is 8.76 nm and Type A uncertainty is 2.77 nm.

Figure 2 (a) represents the error sources contributing towards the measurement uncertainty for calibration of GBs using a mechanical comparator, and Figure 2(b) shows the measurement traceability pyramid.

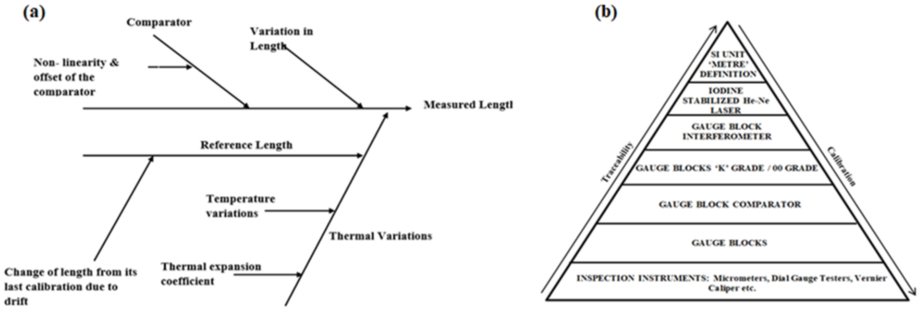

Figure 2. (a) Fishbone diagram showing error sources, (b) Measurement Traceability Pyramid

The model function for this particular case may be represented as[14]:

$$l_x = l_s + \delta l_D + \delta l + \delta l_c - L(\alpha_{avg}\delta t + \delta\alpha\Delta t_{avg}) - \delta l_v \quad (5)$$

l_x **(mm):** Test GB length
l_s **(mm):** Reference GB length at reference temperature and the associated expanded uncertainty as per calibration certificate
δl_D **(mm):** Error due to drift
δl **(mm):** Observations taken for the difference in reference GB length and test GB length
δl_c **(mm):** Error due to nonlinearity and comparator offset correction. The comparator used has the specifications as given in EAL-G21
α_{avg} **($°C^{-1}$):** Error in the average of coefficient of thermal expansion of reference and test GB on the basis of calibration certificate and manufacturer's data.
$\delta\alpha$ **($°C^{-1}$):** Error due to difference in the coefficient of thermal expansion of reference and test GB
δt **(°C):** Error due to difference between reference and test GB temperature
δl_v **(mm):** Corrections due to variation in length of the test GB
Δt_{avg} **(°C):** Deviation from the reference temperature in the mean temperature value of test and reference GB
L **(mm):** Nominal Length of GB (50 mm)
$\delta\alpha \times \Delta t_{avg}$: Second order term considered during evaluation of uncertainty as the estimated value of $\delta\alpha$ and Δt_{avg} is zero. Hence, individually, their contribution to uncertainty becomes zero.
Correlation: It is assumed that input quantities are not correlated.
The detailed uncertainty budget for calibration of GB as per GUM approach is presented in Table 2.

Table 2. Uncertainty Evaluation as per GUM/LPU Approach[14]

Sources	Estimate of the Quantity	Limit	Distribution	Standard Uncertainty	Sensitivity Coefficient	Degree of Freedom (ν)	Uncertainty Contribution (mm)
$l_s(mm)$	50.000040	32×10^{-6}	Normal	16.00×10^{-6}	1.0	∞	16.00×10^{-6}
$\delta l_D(mm)$	0	32×10^{-6}	Triangular	13.10×10^{-6}	1.0	∞	13.10×10^{-6}
$\delta l(mm)$	-191×10^{-6}	-	Normal	2.77×10^{-6}	1.0	9	2.77×10^{-6}
$\delta l_c(mm)$	0	32×10^{-6}	Rectangular	18.50×10^{-6}	1.0	∞	18.50×10^{-6}
δt(°C)	0	0.05	Rectangular	0.029	575×10^{-6}	∞	16.67×10^{-6}
$\delta l_v(mm)$	0	6.7×10^{-6}	Rectangular	3.87×10^{-6}	-1.0	∞	-3.87×10^{-6}
$u_{at}(\delta\alpha\Delta t_{avg})$	0	-	Special	$0.58\times10^{-6}\times0.35$	-L	∞	-10.15×10^{-6}

Combined Standard Uncertainty ± 34.3×10^{-6} mm, Expanded Uncertainty (k=2) ± 68.5×10^{-6} mm
Estimated Length of GB 49.999849 mm ± 68.5×10^{-6} mm

5. Monte Carlo simulation (MCS/MCM) with octave

MCM used for various calculations as required in the evaluation of uncertainty, reduces the analysis effort required for nonlinear model equations. In this investigation, we have used the GNU octave software.[15] In octave, input and output quantities are represented by the dimension of arrays. It has following considerations

- Monte Carlo trials i.e. iterations through which each input quantity is sampled is M, thereby, the model function evaluated is an M-dimension vector. In our case, M is chosen of the order of 10^6.
- Random number generators are used for sampling of probability distributions. For example, unifrnd (a,b,[M,1]) is used for rectangular distribution with limits (a,b). For triangular distribution, we used sum of uniform distributions (unifrnd(a,b,[M,1]) + unifrnd(a,b,[M,1]) with limits half of the interval. In normal distribution normrnd(m,s,[M,1]) is used where m and s are mean and standard deviation of the distribution. Figure 3(a) shows the MCS histogram of output quantity obtained by using octave software.

6. Monte Carlo Simulation (MCS/MCM) with R Language

R programming is used for analyzing statistical information using several functions and graphical representations.[16]. In our case, we generated random numbers for each input quantity through 10^5 iterations for normal or rectangular probability distributions using rnorm() and runif() functions respectively. The average measurand value and standard deviations are computed using mean() and sd() functions. hist() and sprint() functions are used for plotting histogram and output values. The 95% confidence interval is determined through quantile() function and points() function as shown in Figure 3(b).

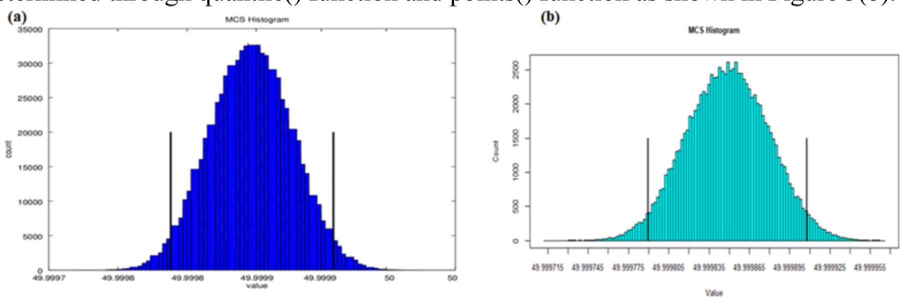

Figure 3 (a) MCS histogram of output quantity using octave, (b) MCS histogram of output quantity using R

Table 3 demonstrates the comparison of results between GUM and MCS.

Table 3. Comparison between GUM and MCS/MCM

Parameters	GUM	MCS with Octave	MCS with R
Measured Length of GB, L (mm)	49.999849	49.999849	49.999849
Expanded uncertainty(× 10-6 mm)	± 68.5 (at k=2)	± 61.1	± 61.1

7. Conclusions

GBs are standard artefacts for dimensional metrology, used to calibrate various instruments. Hence, measurement uncertainty evaluation of GBs is need of the hour. In this paper, estimation of measurement uncertainty of GB using GUM and MCM has been discussed. The uncertainties obtained using GUM and MCM are realized to be close. It has been observed that, the mean measured value of a GB of nominal length 50 mm is (49.999849 ± 0.0000685) mm by LPU method and the mean measured value is (49.999849 ± 0.0000611) mm by MCS approach.

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