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Designing of Fractional Order Inverse Chebyshev Low Pass Filter Using Particle Swarm Optimization

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Abstract. The design of fractional order inverse Chebyshev low pass filter has been presented in this paper. The (1+α) order inverse Chebyshev filters have been designed by comparing the transfer function of fractional order low pass notch filter with the second-order inverse Chebyshev low pass filter. Particle swarm optimization has been utilized to derive the coefficients of the fractional-order filters by varying α from 0.1 to 0.9. The canonical forms of these filters have been realized using multiple input Biquad circuits. The value of circuit elements for each order has been derived. Further, the values for the equivalent RC ladder network of the constant phase element have been derived by fifth-order continuous fractional expansion for the orders 1.2, 1.5 and 1.8. The magnitude plots for the 1.2 and 1.8 order inverse Chebyshev low pass filter have been plotted in SPICE and the magnitude plot for the 1.8 order filter has been compared with PSO approximated plot to confirm the feasibility of the OP-Amp based realized circuit with a mean error of 0.495dB.

Keywords. Inverse Chebyshev filter, Fractional order filter, Particle swarm optimization, Low pass filter, Evolutionary algorithms

1. Introduction

Chebyshev filters find wide application in the fields of signal processing and biomedical instrumentation. They are widely used for filtering signals such as ECG[1,2]. These filters offer higher roll-offs by allowing ripples in the passband or stopband. Depending on the presence of ripples in the passband or stopband, the filters are classified as type 1 and type 2. The proposed work deals with type 2 filters, also known as inverse Chebyshev filters. The ripples exist in the pass-band for inverse Chebyshev filters. Due to their popularity, a lot of research has been done to expand the functionality of the filters in the fractional domain as well. Various works have presented and proposed the methods for designing fractional-order Chebyshev filters of both type 1 and type 2 using various approximation techniques[3–6]. Fractional-order circuits are derived through fractional calculus which is concerned with the derivative and integration of fractional orders. Expansion in the fractional domain offers huge advantages of exploiting the dynamic

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ranges of attenuation and more precise control over the characteristics of the circuits. The dynamic and precise controlling is crucial for applications in fields such as biomedical. In fractional-order filters, while the integer-order counterparts offer the attenuations of orders -20dB/decade, the fractional-order offer the attenuations over the range as -20(n+α) dB/decade. The design of fractional order inverse Chebyshev filter has also been proposed in [7] where the least square approximation method is performed to find the optimum coefficients of fractional order filter. The work presented in this paper proposes the design of the (1+α) fractional-order inverse Chebyshev filter. The proposed work utilizes the metaheuristic, evolutionary, nature-inspired particle swarm optimization algorithm to design the filters. PSO explores the multimodal, multidimensional solution space efficiently to produce the optimum values of coefficients for the design of filters.

The response of an inverse Chebyshev filter can be expressed as a low pass notch filter with the transfer function:

$$C(s) = \frac{s^2 + k^2\omega_0^2}{s^2 + \frac{s\omega_0}{Q} + \omega_0^2} \tag{1}$$

here, k^2 is the DC gain, ω_0 is the notch frequency and the quality factor is given by Q. The transfer function for a general $(1 + \alpha)$ order low pass filter can be given as:

$$H_{LP}^{1+\alpha}(s) = \frac{k_1}{s^{1+\alpha} + k_2 s + k_3} \tag{2}$$

This function is an all poles function, which will not match the notch response. Thus, a different form of general fractional-order low pass filter with both zeros and poles can be given as:

$$H_{LP}^{1+\alpha}(s) = a_4 \frac{a_1 s^{1+\alpha} + 1}{a_2 s^{1+\alpha} + a_3 s^{\alpha} + 1} \tag{3}$$

The purpose of defining a transfer function with both zeros and poles is so that the transfer function for an inverse Chebyshev filter is defined as a notch filter with both poles and zeros present. The transfer function of a second-order low pass inverse Chebyshev filter can be given as:

$$C_2(s) = 0.003162 \frac{s^2 + 1.9999}{s^2 + 0.1123s + 0.0063} \tag{4}$$

This filter has a minimum value of attenuation as 50dB and the value of DC gain is approximately 0dB. The transfer function has both the zeros and the poles unlike the Chebyshev filter [8]. DC gain and high frequency gain for filter given by (3) can be given as a_4 and $a_4 a_1/a_2$. The equations (3) and (4) are compared and the error between these is defined as the cost function for particle swarm optimization. The value of alpha is varied from 0.1 to 0.9.

2. Simulation Results of the Proposed Work

The transfer function of (3) is approximated with respect to (4) such that optimum values of coefficients are derived. These coefficients are derived such that the approximated responses for the ripple and passband characteristics are obtained over a frequency range $[10^{-3}$ to $10^{3}]$. The optimization of the coefficients is done by PSO by setting the parameters of the algorithm as presented in Table 1.

PSO is an evolutionary, nature-inspired, metaheuristic algorithm that is based on swarm behaviour [9–11] The algorithm is presented as a flow diagram in Figure 1. The optimization function to search the coefficients can be given as:

$$Optimization\ Function = \min_{x} \sum_{i=1}^{k} \left(\left| H_{LP}^{1+\alpha}(x, \omega_i) \right| - \left| C_2(\omega_i) \right| \right)^2 \tag{5}$$

α is varied from 0.1 to 0.9 and filter coefficients a_1, a_2, a_3 and a_4 are derived corresponding to different orders. The values of these coefficients have been presented in Table 2. The minimum value of the coefficient is taken as 0.1 and the maximum is considered 200. PSO traverses over a multimodal, non-uniform, multidimensional space over 500 iterations and with an initial population of 50. The algorithm is run multiple times to find the best results.

Table 1. Key parameters used for Particle Swarm Optimization for optimized results.

Parameter	Value
xmin, xmax	0.1,200
Population Size	50
Maximum Iterations	500
Velocity Range	0.2*(x_{min} - x_{max})
κ	1
a_1,a_2	2.05

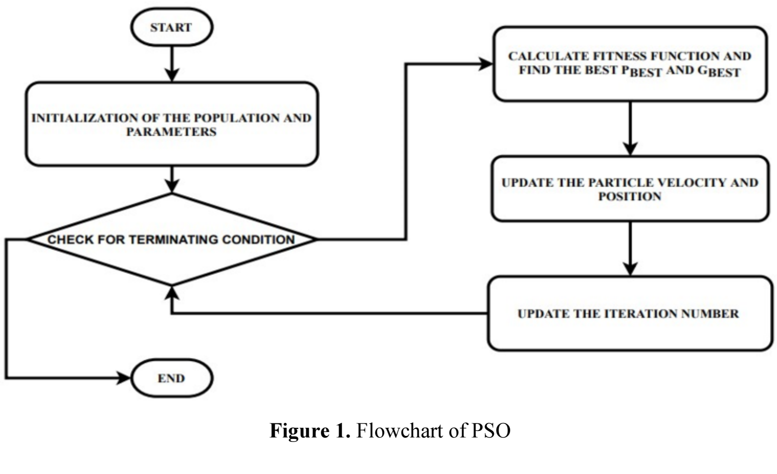


Figure 1. Flowchart of PSO

Table 2. Values of filter coefficients a_1, a_2, a_3 and a_4 obtained through PSO for orders varying from1.1 to 1.9

Filter	α	Values of Coefficients			
		a₁	**a₂**	**a₃**	**a₄**
H_{LP}	0.1	0.726	200.000	0.100	0.870
	0.2	0.681	200.000	0.100	0.929
	0.3	0.644	187.114	0.100	0.918
	0.4	0.615	169.557	0.100	0.871
	0.5	0.593	154.887	0.100	0.826
	0.6	0.574	145.795	0.100	0.804
	0.7	0.557	147.797	0.100	0.839
	0.8	0.541	166.551	0.296	0.974
	0.9	0.518	200.000	13.576	1.220

The magnitude responses of the derived filters are plotted in MATLAB and compared with the second-order low pass filter. The plotted curves are presented in Figure 2. It is observed that the derived filters follow the decreasing trend of slope and thus produce the response similar to ideal fractional-order inverse Chebyshev low pass filters.

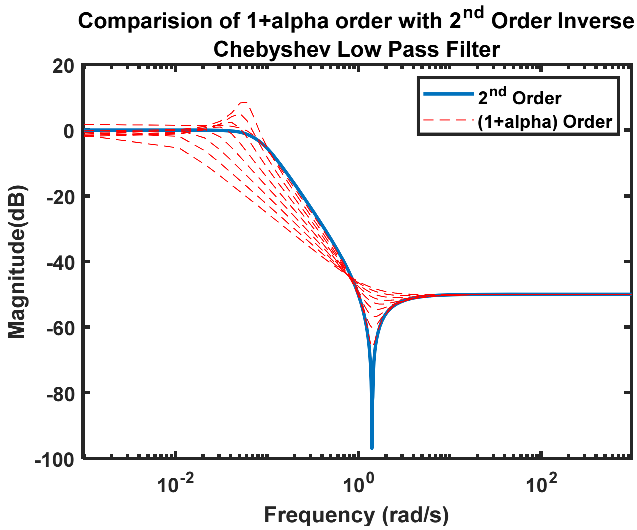


Figure 2. Comparison of designed filters with the second-order Inverse Chebyshev LPF.

From Figure 2 it can be observed that the curves of filters follow the decreasing trend in the attenuation. The value of the ripples varies with α. All the filters follow the behaviour of ideal $(1+\alpha)$ order inverse Chebyshev low pass filters with notch frequency varying with the α. All the filters correspond to a DC gain of 0dB.

2.1. Stability Analysis

The stability of the approximated filters can be determined by conversing the transfer function from the s domain to W-plane [12]. This converts the fractional order to integer-order and further stability analysis can be performed over it. The steps involved are as follows:

1. Convert the s-domain transfer function to W-plane by using $s = W^m$ and $\alpha = k/m$.
2. Select the integers k and m for the corresponding values of the α.
3. Solve the resultant transfer functions for poles in the W plane.
4. Check the minimum value of the absolute pole angles, $|\theta_W|$
5. If $|\theta_W|$ are less than $\frac{\pi}{2m}$ rad/s , the system is unsbale. But, if all values of $|\theta_W|$ are greater than $\frac{\pi}{2m}$ rad/s, then the system is stable.

Using the stated procedure, after conversion to Wplane the value of poles can be calculated by finding the roots of the equation:

$$a_2 W^{m+k} + a_3 W^k + 1 = 0 \tag{6}$$

The poles are calculated by solving equation (6) for α varying from0.1 to 0.9. The value of m is taken as 10 and k is equal to 10α. Table 3 shows the calculated values of minimum absolute pole angles for each order and it can be observed that all values of $|\theta_W|_{min}$ are greater than π/2m=9°. This verifies that the designed filters are stable.

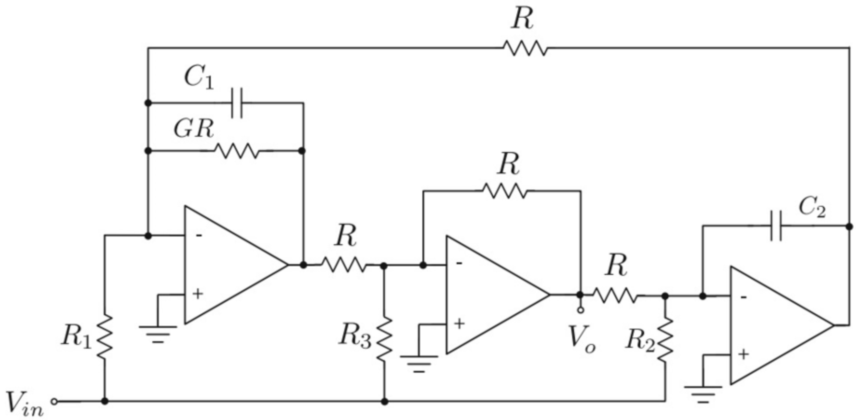
Table 3. Minimum obtained values of absolute pole angles for stability analysis of the designed fractional order filters.

Order	$|\boldsymbol{\theta_W}|_{\boldsymbol{min}}(\boldsymbol{degrees})$
1.1	32.47
1.2	15.01
1.3	27.48
1.4	12.85
1.5	23.86
1.6	11.22
1.7	21.07
1.8	10.01
1.9	18.41

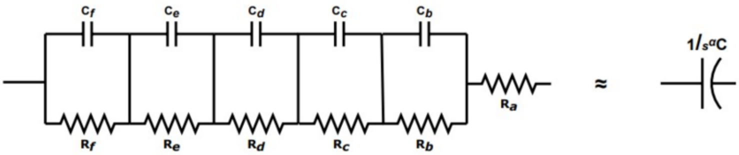
2.2. Circuit Realization

The designed filters are further realized as a circuit by utilizing Op-Amp741 based multiple input Biquad (MIB) circuit shown in Figure 3. C2 is assumed to be a constant phase element and is taken as 1F with an impedance $Z_c = 1/s^\alpha C$. G is taken to be 0.5. The transfer function for the MIB circuit is given as:

$$H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{c_1 s^{1+\alpha} + c_2 s^\alpha + c_3}{s^{1+\alpha} + c_4 s^\alpha + c_5} \tag{7}$$



(a)



(b)

Figure 3. (a)An equivalent MIB notch filter used to realize the fractional-order inverse Chebyshev LPF. (b) Foster 1 Equivalent Ladder Network of C2.

The values of (7) can be compared with (3) and can be represented as:

$$\begin{aligned} c_1 &= \frac{R}{R_3} = \frac{a_4 a_1}{a_2} \\ c_2 &= \left(\frac{1}{R_3 C_1 G} - \frac{1}{R_1 C_1}\right) = 0 \\ c_3 &= \frac{1}{C_1 C_2 R_2 R} = a_4/a_2 \\ c_4 &= \frac{1}{G C_1 R} = \frac{a_3}{a_2} \\ c_5 &= \frac{1}{C_1 C_2 R^2} = \frac{1}{a_2} \end{aligned} \tag{8}$$

The values of circuit elements to realize the filters are derived by solving equation (8) and are presented in Table 4. For practical values circuit parameters, the magnitude values of the elements are scaled by a factor of $1000 (K_m = 1000)$. The central frequency is scaled to be 1kHz ($K_f = 2000\pi$).

Table 4. Values of circuit elements of MIB for the design of filters of different orders varying from 1.1 to 1.9.

Order	R(Ω)	R1(KΩ)	R2(Ω)	R3(KΩ)	ZC1(μF)	ZC2(μF)
1.1	50.00	7.917	57.465	15.834	12738.854	0.159
1.2	50.00	7.913	53.846	15.825	12738.854	0.159
1.3	50.00	7.910	54.475	15.821	11918.059	0.159
1.4	50.00	7.909	82.646	15.818	7471.741	0.159
1.5	50.00	7.908	60.505	15.817	9865.414	0.159
1.6	50.00	7.908	62.225	15.816	9286.324	0.159
1.7	50.00	7.908	59.605	15.815	9413.837	0.159
1.8	147.84	23.381	151.823	46.762	1213.455	0.159
1.9	6788.02	1073.447	5563.468	2146.895	0.691	0.159

Capacitor C2 is represented as an equivalent RC Foster 1 ladder network of fifth-order continuous fraction expansion. The central frequency for the FOE is taken to be 1KHz. This approximation of constant phase element is performed as the fractional-order capacitors are unavailable commercially for use [13,14]. The CFE expands the fractional element in terms of integer orders and is substantial for orders up to 10^4 around the central frequency, depending on the order of approximation [15,16]. Element values for Foster 1 RC Ladder network for orders of 1.2, 1.5 and 1.8 are given in Table 5.

Table 5. Values of R, C elements to realize the equivalent Foster I network using fifth-order continuous fractional expansion.

Element	Order		
	1.2	1.5	1.8
R_a (Ω)	398.99	90.998	13.322
R_b (Ω)	257.19	197.69	62.938
R_c (Ω)	197.19	257.16	130.37
R_d (Ω)	218.49	424.39	320.66
R_e (Ω)	343.55	1.055K	1.345K
R_f (Ω)	1.096K	8.99K	73.37K
C_b (μF)	3.943	0.857	0.296
C_c (μF)	0.039	0.069	0.286
C_d (μF)	0.276	0.256	0.604

The equivalent circuits for 1.2 and 1.8 order inverse Chebyshev LPF using the corresponding parameter values mentioned in Table 4 and Table 5 have been realized in LTSpice. Figure 4 shows the magnitude plots for the filters. The obtained curves display the filter characteristics of equivalent slopes, ripples in the stop-band and passband similar to the ideal filters. The magnitude plot for the 1.8 order filter was further compared with the MATLAB PSO approximated filter in Figure 5. The graph shows the curves closely overlapping. This confirms the feasibility of the OP-Amp based Filter using MIB topology. The values of maximum and mean errors between the magnitude plots is observed to be 9.589dB and 0.495dB respectively.

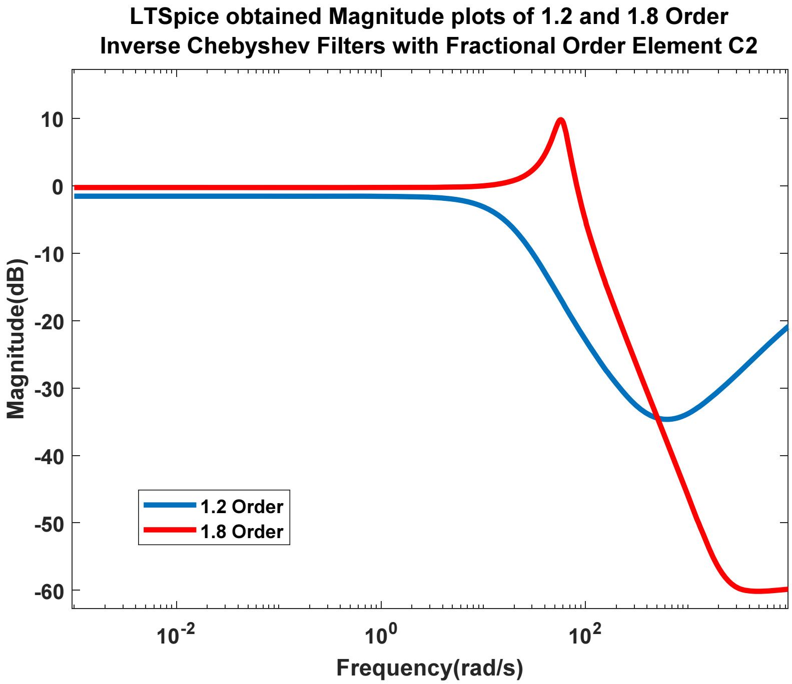


Figure 4. Magnitude plots of 1.8 and 1.2 Order Inverse Chebyshev Fractional Order Low Pass Filter obtained using SPICE implementation of the equivalent MIB circuit.

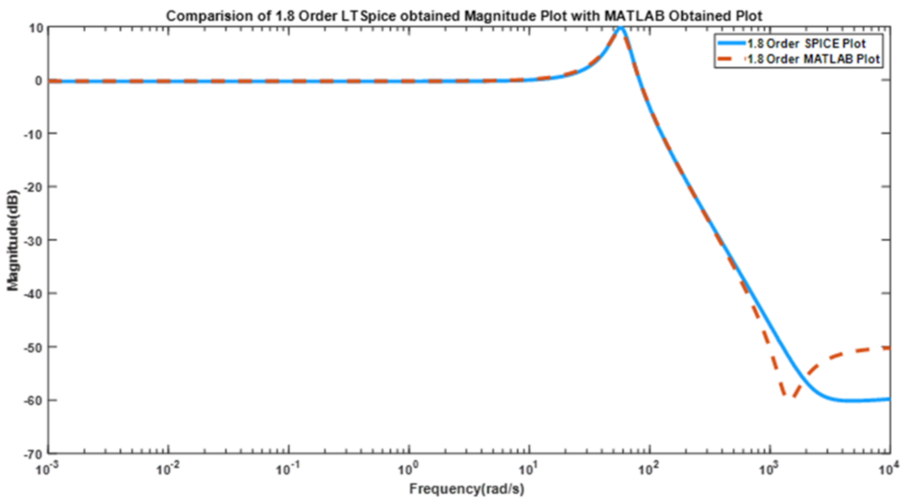


Figure 5. Comparison of MATLAB approximated 1.8 Order filter and SPICE implemented 1.8 Order Filter circuit using MIB.

3. Conclusion

In this paper, a fractional-order low pass inverse Chebyshev filter has been designed using the evolutionary optimization technique, particle swarm optimization. The designed filters showcase high efficiency with low errors and the plots display the designed filters follow the characteristics of the desired fractional order Chebyshev filters. The coefficients of the fractional-order transfer functions have been calculated for orders (1+α). The canonical forms for the derived filters are designed using multiple input Biquad notch filter topology. The value of circuit elements is derived for the orders 1.1 to 1.9. Capacitor C2 is taken as the fractional order element and is expanded as a Foster -1 equivalent RC ladder network using fifth-order continuous fraction expansion. The magnitude plots for orders 1.2 and 1.8 are derived and the realized filters display the approximated behaviour to the ideal filters. The PSO approximated and the Op-Amp realized filter of order 1.8 are compared as magnitude plots. The realized filter closely follows the approximated filter's curve and the errors are calculated between the two curves as well. The maximum and mean errors are calculated to be 9.589dB and 0.495dB respectively. Thus, the proposed work presents a method of designing the fractional order inverse Chebyshev filter which is accurate in terms of magnitude response. The filter is also practical as the stability analysis proves and realization of the filter is feasible as displayed by the error calculations and magnitude plots of the 1.8 order filter's mathematical form and the circuit realized form. The work transcends the design of inverse Chebyshev filters to the fractional domain to provide with more controllability over the attenuation characteristics of the filters thus, providing the dynamic range of working.

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