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Designing of a Tunable Fractional Order Chebyshev High Pass Filter Using Particle Swarm Optimization

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Abstract. Fractional order Chebyshev high pass filters have been designed in this paper for orders (1+α), (2+α) and (3+α), α representing the fractional component. The work in this paper has been done by using the nature-inspired evolutionary metaheuristic technique called particle swarm optimization. MATLAB simulations for these filters have been shown with α varying from 0.1 to 0.9 at a step of 0.1. The attenuation values are compared with the ideal values. The canonical forms of the (1+α) order filters have been realized using OTA based KHN filter. Circuit element values are calculated for orders 1.2, 1.5 and 1.8 using factional order capacitors. These capacitors are approximated as Foster 1 form.

Keywords. Tunable filter, Fractional order Filter, Particle Swarm Optimization, Chebyshev high pass Filter

1. Introduction

Fractional Calculus is a discipline of mathematics concerned with the differentiation and integration of non-integer orders. The branch has grown in popularity in a variety of fields, and it is now employed in a variety of multidisciplinary and applied domains [1]. The definition of the derivative in fractional-order calculus is based on two general approaches; Riemann-Liouville and Caputo definitions.

In the field of electrical and electronics, the same operator is employed as a general fractance device, where the impedance is proportional to the s^{α}, with α as fractional-order [1].

$$Z(s) = \kappa s^{\alpha} \tag{1}$$

Due to the lack of commercially accessible fractance devices, integer-order approximations such as continuous fraction approximation and rational approximation [2] are utilised, and the results are realised using passive RC tree networks. The constant phase element (CPE) or universal fractace device has been utilised to create many electrical and electronic circuits, particularly fractional-order filters (FOFs) and

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associated analogue circuits [3–5]. Operational amplifiers, current conveyors, operational transconductance amplifiers (OTA), and other components are used in the circuits. Tuning circuitry is required in these circuits to compensate for the discrepancies.

In this paper, a fractional order Chebyshev high pass filter (FOHCF) has been designed using particle swarm optimization (PSO), a metaheuristic algorithm. The transfer functions for the traditional Chebyshev HPF and the transfer function for the fractional-order HPF (FOHPF) are compared using MATLAB to design filters corresponding to (1+α), (2+α) and (3+α) orders. The alpha is varied from 0.1 to 0.9 in the steps of 0.1. Moreover, the (1+α) order filter is realised for orders 1.2, 1.5 and 1.8 using OTA based KHN filter.

2. Design of the High Pass filter

The design of fractional order low pass filters has been followed vividly in the research studies. Various fractional-order filters have been designed for Butterworth filters [6] or Chebyshev Filters[4,7–11]. The filters have been designed using various blocks such as field-programmable gate arrays, field-programmable analog arrays, Tow Thomas Biquad filters [4,12]. The proposed work displays a technique to design a fractional-order Chebyshev high pass filter for different orders of $(1+\alpha), (2+\alpha)$ and $(3+\alpha)$. The technique is based on the second order expansion of s^{α}. The transfer function for a (1+α) HPF can be given as:

$$H_{HP}^{1+\alpha}(s) = \frac{a_0\, s^{1+\alpha}}{a_1 s^{1+\alpha} + a_2 s\, + 1} \tag{2}$$

In this paper, the minimum error between the approximated and traditional filter has been calculated by using particle search optimization (PSO). The PSO offers many advantages over traditional approximation techniques as it is more robust, evolutionary and explores the solution space effectively to produce efficient solutions for global minima or maxima. Compared to the other similar evolutionary and metaheuristic techniques, PSO offers a faster convergence along with fewer adjustments to the parameters. The coefficients to design the desired high pass filters are derived using the PSO. The filters are further realized using the OTA based KHN filter topology which offers tuning of the filters.

2.1. Particle Search Optimization

One of the evolutionary computation strategies based on swarm behavior is particle swarm optimization. The flowchart of PSO, defining the steps involved shown in Figure 1.

The different equations for the particle swarm optimization can be given as:

$$\left.\begin{aligned} v_{i,j+1} &= w_j v_{i,j} + a_1 r_1 (P_{pb,i,j} - x_{i,j}) + a_2 r_2 (P_{gb,i,j} - x_{i,j}) \\ x_{i,j+1} &= x_{i,j} + v_{i,j+1} \end{aligned}\right\} \tag{3}$$

here, the various parameters are defined as:

$v_{i,j}$: velocity of particle i at iteration j

r_1, r_2: a random number between 0 and 1
a_1, a_2: acceleration
$x_{i,j}$: position of particle i at iteration j
$P_{pb,i,j}$: best position of particle i at iteration j
$P_{gb,i,j}$: best position of group or swarm at iteration j

The algorithm is applied to optimize the least square error between the fractional-order filter and the integer-order Chebyshev filter.

$$Optimization\ Function = \min_{x} \sum_{i=1}^{k} \left(\left| H(x, \omega_i) \right| - \left| C_n(\omega_i) \right| \right)^2 \tag{4}$$

here, x is the coefficient of filter taken greater than 0.001 to avoid negative coefficients, $|H(x, \omega_i|$ is the magnitude of FOF, $|C_n(\omega_i)|$ is the magnitude of Chebyshev HPF for 3dB ripple of order n at frequency ω. PSO is applied to optimize Eq. (4) using Eqs. (5) to (9) for 0.1< α <0.9. Eqs. (5), (7) and (9) are the transfer functions for 2nd, 3rd and 4th order Chebyshev How pass filters and Eqs. (2), (6) and (8) are transfer functions for the (1+ α) (2+α) and (3 + α) order high pass filters. The algorithm is run for 500 iterations for an initial population size of 50. Minimum and Maximum values of coefficients are defined as 0.001 and 30.

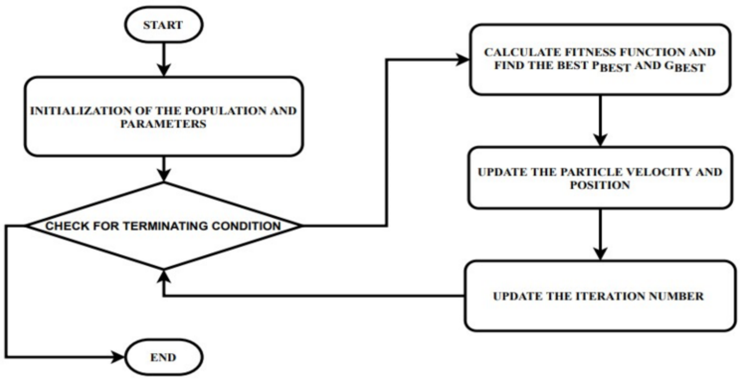

Figure 1. Flowchart of PSO.

$$C_2(s) = \frac{0.708\, s^2}{s^2 + 0.911s + 1.41263} \tag{5}$$

$$H_{HP}^{2+\alpha}(s) = \frac{a_0 s^{2+\alpha}}{a_1 s^{2+\alpha} + a_2 s^2 + a_3 s^{1+\alpha} + a_4 s + 1} \tag{6}$$

$$C_3(s) = \frac{s^3}{s^3 + 3.7043s^2 + 2.383s + 5.6497} \tag{7}$$

$$H_{LP}^{3+\alpha}(s) = \frac{a_0 s^{(3+\alpha)}}{a_1 s^{3+\alpha} + a_2 s^3 + a_3 s^{2+\alpha} + a_4 s^2 + a_5 s^{1+\alpha} + a_6 s + 1} \quad (8)$$

$$C_4(s) = \frac{0.7079 s^4}{s^4 + 2.287 s^3 + 6.605 s^2 + 3.286 s + 5.6497} \quad (9)$$

The transfer function for the $H_{HP}^{1+\alpha}$ is compared with the transfer function for second-order Chebyshev high pass filter C_2. Similarly the transfer functions $H_{HP}^{2+\alpha}$ is compared with C_3 and $H_{HP}^{3+\alpha}$ is compared with C_4. The drawn comparisons reduce the error between the transfer functions to produce the optimum values of coefficients to design the corresponding fractional-order filters which produce the least error when compared with the ideal counterparts effectively. The comparison is done over the frequency range $10^{-2} \leq \omega \leq 10^2$ and the minimum value of coefficients is taken 0.001 to avoid negative coefficients. The coefficients produced through optimization have been specified in Table 1. Figure 2 displays the MATLAB plots of the magnitudes of the obtained FOHCF in comparison to the integer-order Chebyshev high pass filters. The fractional orders curves are represented using dashed lines while integer order curves are presented as solid lines. Figure 2(a) shows the plots for first and second order Chebyshev filters and the plots for $(1+\alpha)$ order are shown. The decrease in the slopes of the plotted curves between 1st and 2nd order curves signifies the changing orders of the filters from α=0.1 to α=0.9. The slopes of the obtained filters are calculated for orders 0.2,0.5 and 0.8 and are shown in Table 1. These are close to the ideal values of $20(n+\alpha)$.

Table 1. Values of filter coefficients obtained through PSO

Filter	α	a_0	a_1	a_2	a_3	a_4	a_5	a_6	Slope (dB/dec)
C2	0.2	1.141	1.609	0.001					24.15
	0.5	0.883	1.244	0.069					30.14
	0.8	0.857	1.203	0.467					36.08
C3	0.2	8.442	25.975	0.001	12.013	14.381			44.12
	0.5	1.190	4.636	0.001	3.151	0.860			50.15
	0.8	1.009	4.362	1.139	3.683	0.001			56.11
C4	0.2	17.325	17.214	0.001	0.001	24.527	30.000	6.596	64.21
	0.5	7.673	7.789	0.001	0.001	29.902	18.420	0.001	70.17
	0.8	6.906	6.935	0.001	8.683	30.000	16.685	5.856	76.13

3. Circuit Realization

The approximated FOHCF of the order $(1+\alpha)$ can be realized using the KHN filter utilizing the OTA. The transfer function for a high pass filter of an OTA based KHN topology as shown in Figure 3 can be given as[13]:

$$H_{LP}(s) = \frac{\frac{g_{m0}}{g_{m5}} \frac{s^2 C_1 C_2}{g_{m1} g_{m2}}}{\frac{s^2 C_1 C_2}{g_{m1} g_{m2}} + \frac{s C_2 g_{m3}}{g_{m2} g_{m5}} + \frac{g_{m4}}{g_{m5}}} \quad (10)$$

The filter comprises six OTAs with transconductance $g_{m0}, g_{m1}, g_{m2}, g_{m3}, g_{m4}$ and g_{m5}. Two capacitors C1 and C2 are also present. The output for the high pass filter is taken across C1 for the KHN filter. The transfer function given in (2) when compared with (10) produces the following relations: $a_0 = \frac{g_{m0}}{g_{m5}} \frac{C_1C_2}{g_{m1}g_{m2}}$, $a_1 = \frac{C_1C_2}{g_{m1}g_{m2}}$ and $a_2 = \frac{C_2g_{m3}}{g_{m2}g_{m5}}$. Assuming, C1=1F and $g_{m1} = g_{m2} = g_{m4} = 1$, we can obtain g_{m0}, g_{m3}, g_{m5} and C_2.

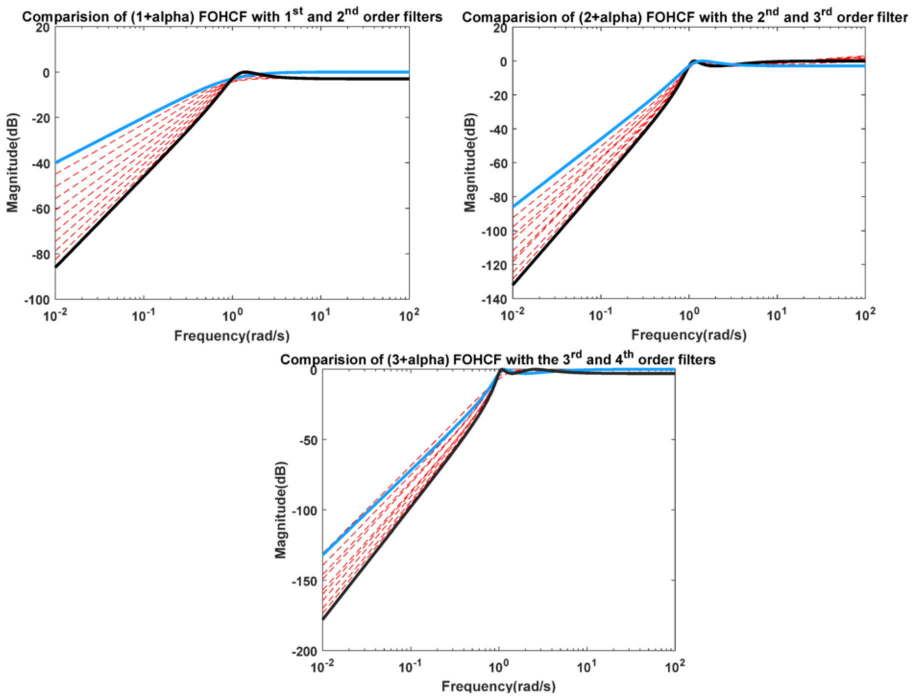

Figure 2. Comparison of approximated fractional-order filters and integer-order filters.

The values of the different transconductance from equation (14) to (16) have been summarized in Table 2. The frequency of operation is scaled to 1KHz and the values of impedance for C1 is given as $Z = 1/s^{\alpha}C_1$ when C1 is taken as the fractional order element. The components are scaled as well by 1000. The capacitor can be implemented using any of the RC ladder networks like Foster I, II or Cauer I, II. [14]. The realization in foster 1 form has been done in this paper by using the fifth-order approximation of the component. The equivalent circuit has been presented in Figure 3 and the values of the elements of the ladder network are presented in Table 3.

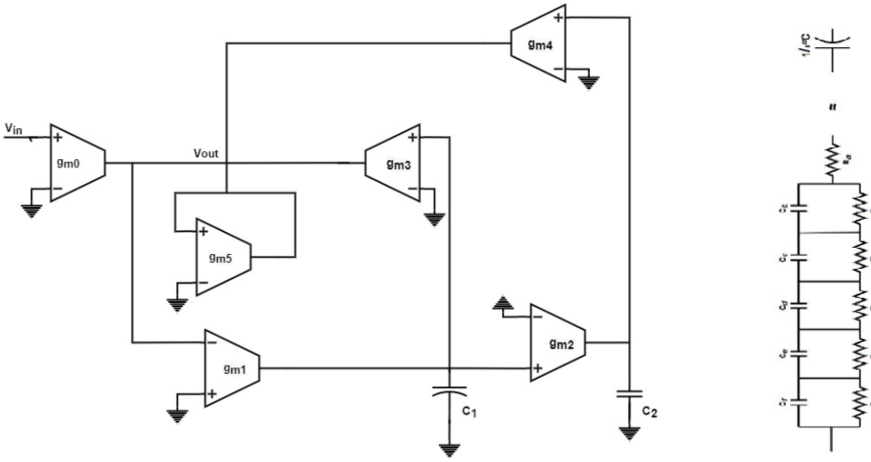

Figure 3. OTA based KHN filter.

Table 2. Values of components to realize Eq. (10) for α=0.2, 0.5, 0.8

Component	Values of FO-LPF of order		
	1.2	**1.5**	**1.8**
Z_{C1} (μF)		0.159	
C_1(μF)	1608	1244	1203
g_{m1}, g_{m4}, g_{m5} (S)		1000	
g_{m0} (S)	708.9	709.8	712.4
g_{m2} (S)	621.6	803.8	831.5
g_{m3} (S)	0.62	55.1	388.2

4. Conclusion

In this paper, fractional-order high pass Chebyshev fractional-order filters have been designed using the evolutionary optimization technique, particle swarm optimization. The coefficients of the fractional-order transfer functions have been calculated for orders $(1+\alpha)$, $(2+\alpha)$ and $(3+\alpha)$. The values for slopes are calculated and are close to the ideal values. The circuit for $(1+\alpha)$ order has been realized using OTA based KHN These filters offer the advantage of simple circuitry and tunability. The values of the circuit elements are calculated for 1.2, 1.5 and 1.8 FOCHF. One of the capacitors is taken to be a CPE and is further approximated with the continuous fractional expansion of fifth-order and represented as the FOSTER I RC ladder network. PSO applied over multiple iterations ensures the searching of solution space for the best solution with high accuracy. The approximated filters display magnitude curves corresponding to ideal behaviour and the values of attenuation confirm the closeness to ideal behaviour. The realization of the filter using OTA based KHN filter offers the advantage of electronic tunability which is necessary in fractional order circuits to reduce any undesirable components in the signal.

Table 3. Values of R, C elements to realize the equivalent Foster I network

Component	**Component values for different orders**		
	1.2	**1.5**	**1.8**
R_a (Ω)	63.4K	14.5K	2.1K
R_b (Ω)	40.9K	31.4K	10K
R_c (Ω)	31.4K	40.9K	20.7K
R_d (Ω)	34.7K	67.5K	50.9K
R_e (Ω)	54.6K	167.7K	213.9K
R_f (Ω)	17.4K	1.4M	11.6M
C_b (nF)	0.24	0.43	1.79
C_c (nF)	1.74	1.61	3.79
C_d (nF)	5.13	3.14	4.96
C_e (nF)	11.15	4.5	4.60
C_f (nF)	24.79	5.4	1.86

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