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Energy Consumption-Based Trajectory Planning for Manipulators

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Abstract. Trajectory planning for robot manipulators is very important in achieving high productivity and excellent accuracy. One of the objectives nowadays is the minimum energy consumption due to the increase in the petroleum prices and the difficulty in the supply lines as well. It is the objective of this paper to design the trajectory of the manipulator based on the minimum energy consumption per cycle of the motors running the manipulator. The selected trajectory will be checked also against the jerk as well to ensure that the robot will not vibrate at the beginning and at the end of any task. A seventh-degree polynomial trajectory is selected to study the effect of the jerk on the trajectory and the torques of the joints as well. The proposed trajectory will be checked through a three degree-of-freedom robotic arm in both horizontal and vertical maneuvering.

Keywords. Optimal, Jerk, Trajectory planning, Energy per cycle

1. Introduction

There are different functions to be applied in designing manipulator trajectories such as trigonometric, Gaussian velocity profile and polynomial of different orders. Polynomial trajectories are preferrable since they can afford continuous velocity, acceleration and jerk based on the degree of the polynomial. If the objective is to control the jerk, seventh-order polynomial is the right choice. There are many applications where robot motion with abrupt changes of jerk is not wanted, such as in transportation of people and goods where dropouts and breakages my easily occur. Also, since jerk control coincides with torque rate control, jerk-bounded trajectories result in much more smoothed actuator loads [Kyriakopoulos and Saridis, 1988].

Eminent researchers have contributed in investigating the effect of jerk on the trajectory and the torque produced by the actuator in performing tasks. Piazzi and Visioli in 2000 solved the global constrained minimax problem to find the minimum jerk cubic spline trajectories based on interval analysis. They validated their algorithm on a six degree-of freedom manipulator and they compared the results with the trigonometric trajectories to prove it.

Macfarlane and Croft in 2003 developed an online optimal trajectory generator for a smooth jerk-bounded trajectory for a single degree of freedom suitable for industrial robot applications. Broquere et al. in 2008 presented a soft motion jerk-limited optimal trajectory generator for multiple DOF. The trajectory consists of seven cubic splines and is suitable for service robotics such as the surgical and nursing robots. Konjevic and Kovacic in 2011 investigated the problem of continuity of position, velocity, acceleration and jerk of electric actuators using two approaches. The first approach separates a planned path and a corresponding velocity profile while the second method combines fifth-order polynomial trajectory.

Zhao and Sidobre in 2015 presented an algorithm to find smooth jerky trajectory for high degree-of-freedom manipulators with soft motion shortcuts that are bounded in velocity, acceleration and jerk. Park et al. in 2017 proposed a smooth speed reference generation algorithm using fifth-order polynomial function for electric actuators. They implemented a simple jerk- bounded time-optimal velocity trajectories with less computational loads. In this paper we are trying to apply the energy per cycle for a DC motor as a criteria for optimal selection of the proper joint trajectory.

The paper is organized as follows: Section 2 represents the trajectory planning as a seventh-order polynomial. Section 3 calculates the energy consumption per cycle for a DC motor and its relation with the angular acceleration and the effect of jerk as well. Section 4 validates the proposed algorithm through a 3 DOF robotic arm in both horizontal and vertical motion. Section 5 contains the discussion and conclusions followed by the references.

2. Trajectory Planning

In general, a spline is a polynomial of a degree k with continuity of derivative of order k-1, at the interpolation point. Low-degree polynomials reduce the effort of computations and the possibility of numerical instability [Fu et al. 1987]. On the other hand, higher-order polynomials enable the control of other variables such as angular acceleration and jerk.

A cubic trajectory gives continuous positions and velocities at the start and finish points but discontinuities in the acceleration and potentially, infinite jerk, at trajectory via points. A discontinuity in acceleration leads to impulsive jerk, which may excite the vibrational modes in the manipulator and reduce tracking accuracy. For this reason, one may wish to specify constraints on the acceleration and jerk as well [Spong et al. 2006].

In order to check the effect of the jerk on the motor torque as well as the energy consumption of the robot, the trajectory is assumed as a 7th order polynomial as follows: $\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 + a_6 t^6 + a_7 t^7$ (1)

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4 + 6a_6 t^5 + 7a_7 t^6 \tag{2}$$

$$\ddot{\theta}(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3 + 30a_6 t^4 + 42a_7 t^5 \tag{3}$$

$$\dddot{\theta}(t) = 6a_3 + 24a_4 t + 60a_5 t^2 + 120a_6 t^3 + 210a_7 t^4 \tag{4}$$

Where $\theta, \dot{\theta}, \ddot{\theta}, \dddot{\theta}$ represent the angular position, velocity, acceleration and jerk respectively. The coefficients $a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7$ are constants that can be determined from the initial conditions.

once the total travelling time of motion has been fixed, minimizing the jerk is desirable because it reduces the actuator and mechanical strain and the joint wear. This implies that trajectory tracking performance by the robot control system is improved and there is also a positive effect on the robot lifespan [Piazzi and Visioli, 2000].

3. Energy Consumption Per Cycle

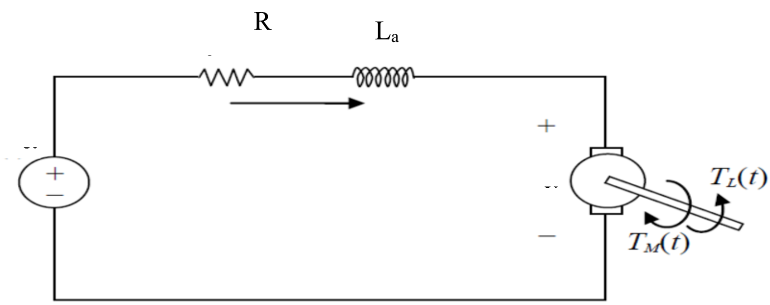

Figure (1) DC motor model.

Consider the DC motor shown in Figure (1) where: $v(t)$, $v_b(t)$, R_a , i_a, L_a, K_T and θ_m, are the input (source) voltage signal, the back emf voltage, the armature resistance, the current of the armature, the inductance of the armature, the back emf constant and the rotor angle position, respectively.The torque of the motor is proportional to the armature current i_a

$$T = K_T i_a \tag{5}$$

The back emf magnitude is proportional to the speed and is given by

$$v_b = K_e \dot{\theta} \tag{6}$$

Thus Kirchhoff's voltage law gives

$$v = i_a R + L\frac{di_a}{dt} + K_e \dot{\theta} \tag{7}$$

Let the torque T_d represent the disturbance torque, from Newton's law one can get

$$I\frac{d\dot{\theta}}{dt} = T - c\dot{\theta} - T_d = K_T i_a - c\dot{\theta} - T_d \tag{8}$$

Where I is the motor inertia and $c\dot{\theta}$ is the damping torque. If we consider the damping coefficient c is small (which is often true in practice), the second equation becomes [Palm, 2000]:

$$I\ddot{\theta} = K_T i - T_d \tag{9}$$

The energy consumption E per cycle is the energy dissipated in the motor resistance R during the time $0 \le t \le t_f$ and can be calculated as:

$$E = R\int_0^{t_f} i^2(t)dt \tag{10}$$

Substitute for i from Equation (9) into Equation (10), yields:

$$E = R\int_0^{t_f} (\frac{I\ddot{\theta} + T_d}{K_T})^2 dt = \frac{R}{K_T^2}\int_0^{t_f} (I\ddot{\theta})^2 dt + \frac{2R}{K_T^2}\int_0^{t_f} I\ddot{\theta}T_d dt + \frac{R}{K_T^2}\int_0^{t_f} T_d^2 dt \tag{11}$$

The first integral on the right-hand side of Equation (12) is independent of T_d and is the energy dissipation when no coulomb friction or load disturbance exists. If T_d is constant, the second integral becomes $\frac{2RIT_d}{K_T^2}\int_0^{t_f}\ddot{\theta}dt = \frac{2RIT_d}{K_T^2}\int_0^{t_f}d\dot{\theta} = 0$. This is simply because for rest-to-rest maneuvering which we assumed here, the angular velocity vanishes at the start and end of the assumed trajectory. Then the Energy dissipated per cycle is given by:

$$E = \frac{RI^2}{K_T^2}\int_0^{t_f}(\ddot{\theta})^2 dt + \frac{RT_d^2 t_f}{K_T^2} \tag{12}$$

Equation (12) shows that if T_d is constant, its effect on the energy dissipation is independent of the velocity profile. Thus, to find the optimal velocity profile we need only to consider the first term in Equation (11) as a measure for the energy dissipated per cycle.

The next step is to study the effect of the jerk in the trajectory by applying the energy per cycle as a criterion to select the optimal trajectory. To do this, the initial conditions will be assumed to calculate the coefficients in the trajectory Equations 1-4. The initial angle, velocity and acceleration as well as the final velocity and acceleration are assumed zero. The final angles are assumed as $\pi/4$, $\pi/3$ and $\pi/2$ and the jerk value in the beginning and end of trajectory are assumed the same for symmetry and their values are changing from zero to 5. Due to this assumption, reference trajectories became symmetric which guarantees the reduction of the computational loads [Park et al., 2017]. This approach is valid also if the velocity and acceleration values at the terminal points are other than zeros. What we need to do is to substitute the optional values of the boundary conditions to find the trajectory coefficients. The results are shown in Table (2). Figure 2 shows the relation between jerk and the Energy Per Cycle for the three joints trajectories.

Table (2) Energy per cycle and maximum torque for the robot joints

Joint Number	θ_f **rad**	J_i **rad/s^3**	J_f **rad/s^3**	**Energy Per cycle (Joule)**
1	$\pi/4$	0	0	0.1256
	$\pi/4$	1	1	0.0730
	$\pi/4$	2	2	0.2008
	$\pi/4$	3	3	0.5089
	$\pi/4$	4	4	0.9974
	$\pi/4$	5	5	1.6663
2	$\pi/3$	0	0	0.2233
	$\pi/3$	1	1	0.1231
	$\pi/3$	2	2	0.2033
	$\pi/3$	3	3	0.4638
	$\pi/3$	4	4	0.9047
	$\pi/3$	5	5	1.5260

3	$\pi/2$	0	0	0.5025
	$\pi/2$	1	1	0.3070
	$\pi/2$	2	2	0.2920
	$\pi/2$	3	3	0.4573
	$\pi/2$	4	4	0.8031
	$\pi/2$	5	5	1.3291

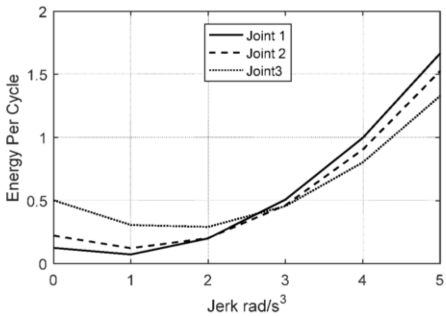

Figure (2) Relation between Jerk and Energy Per Cycle

From Figure (2) it is clear that the energy per cycle decreases for low jerk values (Jerk=0 till 3 rad/s^3) and then increases rapidly after that for the three trajectories. The trend is the same for the three trajectories and this measure can be considered as a basis for selecting the jerk-bounded trajectories for the robot manipulators.

4. Energy Consumption Per Cycle

Consider a 3 DOF planar robot arm with three revolute joints. The required equations of motion for the three links can be determined using Lagrange-Euler technique for the horizontal and vertical motion are given in [Ata et al., 2013]:

The joints torque for the three joints for different values of the jerk as well as the optimum jerk values for the horizontal and vertical motion of the robot arm are shown in Table (2). The highlighted rows represent the optimum values of the joint torque based on the minimum energy per cycle criteria.

Table (2) Comparison of the joints torque for different values of the jerk

Joint Number	J1	J2	J3	Peak torque Nm	
				Horizontal	Vertical
1	0	0	0	3.6151	47.4622
	1	1	1	2.9005	47.0063
	2	2	2	4.2497	48.3489
	3	3	3	5.8533	49.9672
	4	4	4	7.4851	51.5763
	5	5	5	9.1129	53.1752
	1	1	2	3.3297	47.4576
2	0	0	0	1.1287	25.4480

	1	1	1	0.9630	25.4615
	2	2	2	1.4953	26.0063
	3	3	3	2.0970	26.5921
	4	4	4	2.6986	27.1718
	5	5	5	3.3000	27.7453
	1	1	2	1.0057	25.5136
3	0	0	0	0.4219	5.2298
	1	1	1	0.3380	5.2299
	2	2	2	0.5104	5.4101
	3	3	3	0.7130	5.6070
	4	4	4	0.9161	5.8022
	5	5	5	1.1196	5.9957
	1	1	2	0.3660	5.2624

5. Discussion and conclusions

It is clear from Table 2 that there are two sets of jerks that produce the lowest torque for the three joints in both horizontal and vertical motion of the manipulator. These two sets are the optimum case (the last row of each joint) and the second row of each joint. These two sets are similar except for the jerk of the last joint trajectory of the manipulator. This simply because the energy consumption per cycle for joint 3 is the same for the two jerk values of 1 and 2 as can be seen from Table (1). As soon as the jerk increases, the corresponding torque for each joint increases as well. This proves that it is wise to use the energy consumption per cycle as a measure in determining the optimal polynomial trajectory for the robot joints.

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