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One-Dimensional High-Order Dynamic Model of U-Shaped Thin-Wall Arm Segment of Telescopic Boom of Crane

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Abstract. In order to accurately analyse the dynamic performance of the arm segment, a dynamic model of u-shaped thin-walled beam based on one-dimensional high-order beam theory is proposed to predict the three-dimensional displacement of the beam at any point. First, a one-dimensional high-order model is established using Hamilton's principle. The high-order model considers the displacement field by linear superposition of a set of basis functions that vary axially along the beam. A basis function represents a deformation mode, and interpolation polynomials are used to approximate the three-dimensional displacements of nodes on the center line of the section. At the same time, different section discretization methods are analysed, which have different influences on the precision of the model by discretization of the curved surface part of u-shaped section by straight transposition. Finally, the generalized characteristic of the model is worth to obtain the natural frequency, which is compared with the ANASYS plate and shell theory. The error range of the first 16 orders is within 1.5%. The results show that the discrete mode of the model has a certain influence on the frequency error, and the more discrete nodes of the circular arc part, the higher the accuracy.

Keywords. Thin wall structure, one-dimensional high-order model, discrete way

1. Introduction

With the increasing demand of modern professional production development, the crane has become an indispensable mechanical equipment in production, and the telescopic boom of the crane plays an important role. The structure design of telescopic boom has a great influence on the bearing performance of crane. Therefore, it is more important to study the stability of thin-walled structures.

Traditional Timoshenko beam and other theories [1-2] ignore high-order deformation such as section warping and distortion, and the arm segment dynamic modeling has a large error in accurate analysis and calculation. Therefore, thin-wall structure is used to study the performance of the arm segment. In the aspect of thin-walled structure dynamic modeling, there are classical beam theory model, plate and

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shell model, solid model and so on. Yoon et al. [3] established the warpage function for different thin-walled structures and improved the warpage mode of the beam model based on the classical beam theory. Duong et al. [4] analysed the buckling of thin-walled composite beams using Ritz method on the basis of classical beam theory and found that the Angle of thin-walled beams has a certain influence on the buckling load. Carrera et al. [5] carried out nonlinear analysis based on CUF using high-order plate and shell model, and considered the influence on thin-walled members under nonlinear equilibrium state. Dan et al. [6] obtained the free vibration solution of solid beam model based on the refinement theory of displacement variables, which improved the efficiency of numerical analysis. In order to find a theory that can accurately analyse thin-walled beams, Vieira [7] proposed a beam model for thin-walled beam analysis and defined the warpage mode in the high-order thin-walled beam model. Cardenas et al. [8] proposed a unified modeling theory of composite materials with arbitrary plane axial curvature and variable cross-section, which solved the geometric problem of high bending. Choi [9] considered the polygon section with axial variation, adopted the beam frame method to describe the three-dimensional displacement, which achieved good convergence compared with the shell theory, but this method could not be applied to the curved section. Pagani et al. [10] studied the efficiency of the application of radial function method to the theory of high-echelon beams. However, in the application of solid cross section, the curve section cannot be processed accurately and efficiently, so a processing method for curve section is needed.

This paper proposes a method for the dynamic modeling of the curve section. Based on the one-dimensional higher-order theory, the three-dimensional displacement is transformed into a one-dimensional form, and the dynamic modeling of u-shaped thin-walled beams is carried out. Compared with ANASYS plate and shell theory, this method has accurate geometric representation and small error, and improves the dynamic model of curve section.

2. Methods

2.1. Displacement Field

Firstly, the object of our study is a fixed u-shaped section closed thin-walled beam. The space coordinate system (x, y, z) is established with the center of the section as the origin, and the beam length is L, and the wall thickness is t.

The local coordinate system is established on the center line of thin-walled beam section, and the U-shaped section of thin-walled beam is shown in figure 1. For the analysis method of thin-walled beams, N nodes are introduced in the section to capture the deformation of the section [11]. In figure 1, tangential, axial, normal and torsional unit displacements are applied at each node. In addition, the basis function is defined by the form function, and a basis function represents a deformation mode. The model takes into account four degrees of freedom and produces 56 deformation modes, of which 14 are out-of-plane warping and 42 are in-plane distortion. The displacement at the central line of the section is shown in figure 1, where v_i and n_i at the edge j are tangential and outward normal coordinates, P_j is axial coordinates (j=1, 2, 3, 4, 5), u(s, *z*), *v* (*s*, *z*) and *w* (*s*, *z*) are the axial, tangential and normal displacement components of the beam respectively.

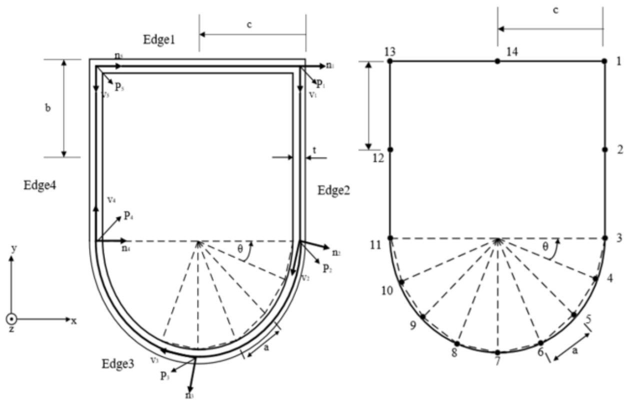

Figure 1. U - shaped thin - wall beam.

The three-dimensional displacement at the center of the section is approximated by the product form of the one-dimensional deformation and the corresponding section shape function:

$$u(z,s)=\sum_{i=1}^{N}\beta_i^T(s)\alpha_i(z)\ ,\ v(s,z)=\sum_{i=1}^{N}\varphi_i^T(s)\alpha_i(z)\ ,\ w(s,z)=\sum_{i=1}^{N}\mu_i^T(s)\alpha_i(z) \tag{1}$$

In the formula: β_{i} -Out-of-plane warping basis function; $\varphi_i(s)$, $\mu_i(s)$ -Distorted basis function in plane; $\alpha_{\mathrm{i}}(z)$ -One-dimensional deformation.

The three-dimensional displacement field X (*u*, *v*, *w*):

$$X = R\phi\alpha \tag{2}$$

In the formula: *R*- Differential operator; ϕ -Coordinate transformation matrix.

ϕ respectively expressed as:

$$\phi=\begin{Bmatrix} \beta_1(s) & \cdots\cdots & \beta_{14}(\mathrm{s}) & 0 & \cdots\cdots & 0 \\ 0 & \cdots\cdots & 0 & \varphi_1(\mathrm{s}) & \cdots\cdots & \varphi_{42}(\mathrm{s}) \\ 0 & \cdots\cdots & 0 & \mu_1(s) & \cdots\cdots & \mu_{42}(\mathrm{s}) \end{Bmatrix} \tag{3}$$

2.2. Dynamic Equation and Finite Element

Under the condition of small displacement, according to Kirchholf hypothesis, the strain component of thin-walled beam is:

$$\chi = CX \tag{4}$$

In the formula: *C*- Differential operator.

According to the elastic constitutive relation of elastic mechanics, the stress component of thin-walled beam is:

$$\sigma = E_q \chi \tag{5}$$

In the formula: E_q - constitutive matrix

In the formula: E -elasticity modulus; *G*-shear modulus; *v*- Poisson's ratio.

According to Hamilton's principle, the dynamic equation of thin-walled beam is obtained:

$$\int_{t_1}^{t_2} (\delta T + \delta U + \delta W) dt = 0 \tag{6}$$

In the formula: *T*- Kinetic energy of boom; *U*- Boom potential energy; *W*- Potential energy force.

Where *T*, *U* and *W* are respectively expressed as:

$$T = \frac{1}{2}\iiint_V \rho \frac{\partial X^T}{\partial t}\frac{\partial X}{\partial t} dV, U = \frac{1}{2}\iiint_V \chi^T \sigma dV, W = -\iiint_V X^T f dV \tag{7}$$

In the formula: ρ —Thin wall beam material density; *f*—The column vectors of distributed forces acting on the cross section.

F can be expressed as:

$$f = \begin{bmatrix} f_x & f_y & f_z \end{bmatrix}^T \tag{8}$$

The dynamic equation of thin-walled beam can be obtained:

$$\iiint_V \rho \delta X^T \frac{\partial^2 X}{\partial t^2} dV - \iiint_V \delta X^T f dV + \iiint_V \delta (CX)^T E_q CX dV = 0 \tag{9}$$

In the study, the finite element method is used to solve the higher-order model, and it is discretized into *N* elements by Lagrange linear interpolation function, mean:

$$\alpha = N d_i, i = 1, 2 \cdots\cdots \text{n} \tag{10}$$

In the formula: i-Cell node Number; *N*- Linear interpolation function; *d*—Nodal displacement vector.

$$d_i = \begin{bmatrix} \psi_1 & \cdots\cdots & \psi_{56}(i) & \psi_1(i+1) & \cdots\cdots & \psi_{56}(i+1) \end{bmatrix} \tag{11}$$

In the formula: ζ_1, ζ_2 - interpolation function; (i),$(i+1)$-Both ends of the unit.

From the above:

$$\rho\iint_{bB}(R\phi\alpha)^{T}R\phi\alpha\frac{\partial^{2}d}{\partial t^{2}}dBd\zeta-\iint_{bB}(R\phi\alpha)^{T}fdBd\zeta+\iint_{bB}(CR\phi\alpha)^{T}E_{q}CR\phi\alpha NdBd\zeta=0 \quad (12)$$

In the formula: *b*-Axial integral region; *B*-Section integral region; *f*-The column vectors of distributed forces acting on the cross section.

The form of the dynamics equation can be arranged as:

$$m\frac{\partial^{2}d}{\partial t^{2}}+k\mathrm{d}=j \quad (13)$$

In the formula: *m*—Element mass matrix; *k*—Element stiffness matrix; *j*-Element load matrix.

The thin-walled beam in this study is in a free vibration state without damping, so the load vector *F* is treated as 0. The mass matrix *M* and stiffness matrix *K* are solved. The overall mass matrix M and stiffness matrix K of integrated thin-walled beams are expressed as:

$$M=\sum_{i=1}^{n}T_{i}^{T}mT_{i}\ ,\quad K=\sum_{i=1}^{n}T_{i}^{T}kT_{i} \quad (14)$$

In the formula: T_i —Transformation matrix of node displacement vector d_i to total node displacement vector *D*.

In this study, the natural frequency of the system could be obtained by solving the generalized eigenvalues of the higher-order model. In the u-shaped section discrete mode, the free vibration analysis of the semi-arc is carried out by replacing the curve with a straight line in different ways, and it is compared with the ANASYS theoretical

3. Results

Different discrete models are established as shown in figure 2, the high-order model was established and the free vibration analysis was carried out to obtain the 7-16 order natural frequencies of the thin-walled beams, and the natural frequencies were compared with those of the ANASYS model, The frequency is expressed by *f*, the frequency of higher-order model is expressed by f_1, and the corresponding error is expressed by δ.

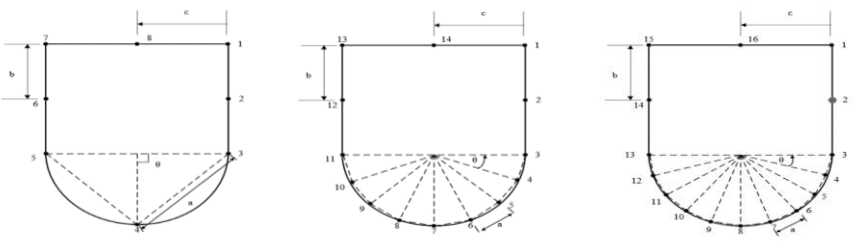

Figure 2. Node processing mode.

Figure 3 shows the comparison of the 7th to 12th order free vibration modes.

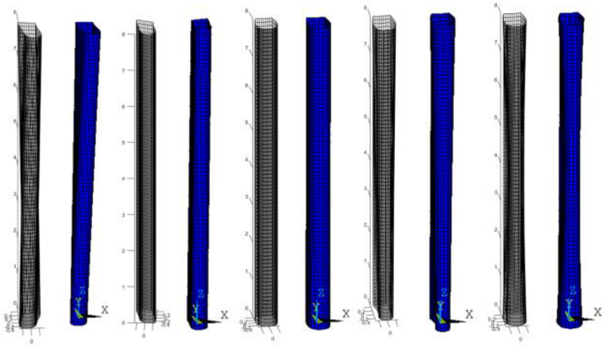

Figure 3. Vibration mode to compare.

Table 1. Natural frequency comparison.

Mode	Present Model	ANSYS Shell	Relative Error
	f_1 (Hz)	f (Hz)	δ_i (%)
7	10.69	10.68	0.09
8	12.22	12.22	0
9	14.00	13.96	0.29
10	14.05	14.00	0.36
11	15.01	14.96	0.33
12	18.10	18.07	0.17
13	20.81	20.69	0.58
14	20.82	20.93	-0.53
15	22.16	22.11	0.23
16	22.30	22.62	-1.41

The errors in the table 1 are shown in the figure 4 below. As can be seen from the above figure 4, when the arc is separated into two segments, the error fluctuates greatly. When the arc is separated into other segments, it can be seen that the error fluctuation trend is the same, and the vibration mode is relatively stable, and the amplitude is small. The error fluctuation is minimum when the arc dispersion is 10 segments.

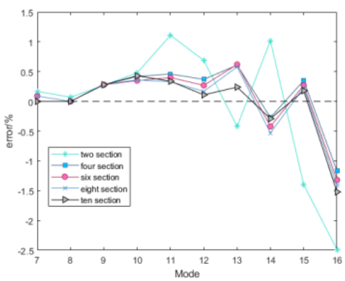

Figure 4. The error analysis.

4. Conclusions and Contributions

A method of finite element analysis to improve the node processing of one-dimensional high-order dynamic model is explored and used to the free vibration analysis. The geometry of thin-walled beams is discretized into a certain number of one-dimensional beam elements, and the results agree well with the reference model. The numerical results are compared with the finite element reference model, and the error range of the first 16 orders is within 1.5%. The results show the discrete mode of the model has a certain influence on the frequency error, and the more discrete nodes of the circular arc part, the higher the accuracy.

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