*Advances in Machinery, Materials Science and Engineering Application*
*M. Chen et al. (Eds.)*


*doi:10.3233/ATDE220452*

# Periodic Acoustic Structure Design Based on the Second-Order Butterworth Acoustic Filter

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**Abstract.** The paper proposes a design method of periodic acoustic structures composed of acoustic filters. Based on acoustic-electric analogy, the second-order Butterworth acoustic filter is first designed, which comprises a main pipe, a Helmholtz resonator, a short tube and an expansion cavity. Second, periodic structures are constructed by taking the acoustic filter as an original unit cell or modifying the acoustic filter to obtain a modified unit cell, and periodically arraying the unit cells along the axial direction of the main pipe. Then, influences of axial and circumferential periodic arrangements of the acoustic structures on the filtering performances are investigated, respectively. Next, multi-period acoustic structure models are established and their transmission losses are simulated and compared. Finally, low-frequency broadband periodic acoustic structures are constructed. The research shows that, the axial period of the acoustic structure composed of the original unit cell has no influence on the resonance frequency of acoustic structure, but contributes to transmission losses and filtering frequency ranges; the increase in the circumferential period can result in a shift in the resonance frequency towards high frequencies and a great increment in the filtering frequency bandwidth; combining unit cells with different resonance frequencies can significantly widen the filtering frequency range; low-frequency broadband performances of the acoustic structure can be implemented by combining unit cells with different resonance frequencies and increasing axial and circumferential periods of the acoustic structures.

**Keywords.** Periodic acoustic structure, acoustic-electric analogy, Helmholtz resonator, shape optimization

## 1. Introduction

The noise generated by vehicles is a complex and irregular multi-band noise [1]. It not only affects performances of the whole vehicle, but also greatly reduces the comfort [2]. Compared with high-frequency noise, low-frequency noise has long wavelength and strong penetrability, and it is more difficult to absorb and attenuate [3]. Therefore, how to absorb and attenuate low-frequency noise has always been a hotspot in acoustic fields.

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With the development of acoustic theory, sound-absorbing structures vary in structure. Among them, the resistant mufflers have excellent comprehensive performances, which are simple in structure and are composed of expansion chambers and Helmholtz resonators [4]. Currently, implementing structural optimization in the design of resistant mufflers is a usual way to increase transmission losses of acoustic waves [5]. In 2011, Wu et al. studied the attenuation of the expansion chamber muffler, and constructed a broadband muffler by determining an appropriate expansion ratio and the angle of the inlet [6]. In 2019, He et al. designed a new type of vortex expansion muffler, and pointed out that transmission losses could be improved through the decreasing the equiaxial angle and increasing the depth of the vortex hole [7]. Also, the periodic structure is adopted to enlarge the frequency bandwidth of acoustic absorption and attenuation. In 2018, Cai et al. studied the influence of layout modes of double Helmholtz resonators with different frequencies on acoustic performances [8], and constructed a periodic double Helmholtz resonator array with a wide frequency band attenuating acoustic waves [9]. In 2019, Wu et al. simulated acoustic performances of the periodic structure composed of multiple Helmholtz resonators [10]. In 2020, Ge et al. built the double-neck Helmholtz resonator models and found that increasing the number of the neck could broaden the resonance frequency band [11].

Acoustoelectric equivalent model is usually used to design acoustic mufflers, providing an alternative approach for designing broadband anechoic structures. Based on acoustic-electric analogy, Jiang et al. built a periodic structure composed of the double-opening Helmholtz to realize low-frequency noise attenuation [12]; Wu et al. designed a parallel micro-perforated plate array and calculated sound absorption coefficients based on a simplified calculation models [13]; Cheng constructed Butterworth band-stop acoustic structures composed of Helmholtz resonators and analyzed the relationship between structure parameters of the acoustic structures and the acoustic performances [4].

In this paper, acoustic-electric analogy is used to obtain the second-order Butterworth acoustic filter in accordance with the corresponding electrical filter, and periodic acoustic structures with low-frequency broadband performances are designed and constructed by modifying the acoustic filter. In order to enlarge the frequency bandwidth effectively attenuating acoustic waves, axial periodicity and circumferential periodicity of the acoustic structure are investigated, and the combination of acoustic filters with different resonance frequencies is adopted.

## 2. Second-order Butterworth Acoustic Filter Design Based on Acoustic-electric Analogy

A Butterworth-type band-stop electrical filter is designed, as shown in figure 1, whose filtering frequency range is set as 100-625Hz with a central frequency of 250Hz. Filter Solutions software is used to design the filter and simulate the transmission losses. Electrical parameters are calculated and given in table 1, including inductances $L_1$ and $L_2$, capacitances $C_1$ and $C_2$. The characteristic impedance is 1Ω. In figure 2, a transmission loss curve is given, where the transmission losses reach a maximum of 108.79dB at 250Hz and are greater than 3dB in the range of 100-625Hz, meeting the design requirements.

**Table 1.** Electrical parameters.

| $L_1$ (mH) | $C_1$ (μF) | $L_2$ (μH) | $C_2$ (mF) |
|---|---|---|---|
| 1.891 | 214.4 | 214.4 | 1.891 |

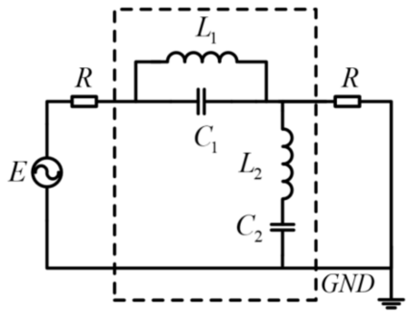


**Figure 1.** Electrical filter circuit.

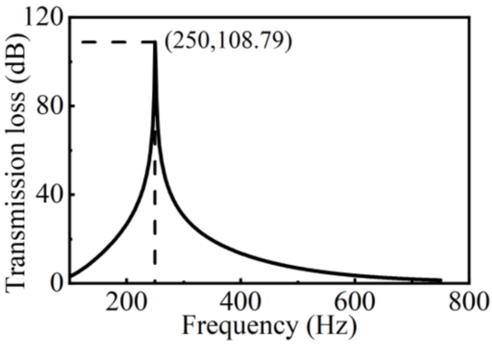


**Figure 2.** Transmission losses.

Based on acoustic-electric analogy, acoustic inductance $M_t$ and acoustic capacitance $C_A$ can be constructed by a short tube and an expansion cavity, respectively, and can be estimated by

$$M_t = \frac{\rho_0 \cdot l_e}{S_t} \qquad C_A = \frac{V_A}{\rho_0 \cdot c_0^2} \tag{1}$$

where, $\rho_0$ is the density of medium; $l_e$ stands for the effective length of the short tube, and can be calculated by its radius $r_t$ and its length $l_t$, i.e. $l_e = l_t + 1.7r_t$; $S_t$ the area of the short tube; $V_A$ the volume of the expansion cavity; $c_0$ the sound velocity in the medium.

A short tube and an expansion cavity can form a Helmholtz resonator whose resonant frequency can be estimated by using equation (1), i.e.

$$f = \frac{c_0}{2\pi}\sqrt{\frac{S_t}{l_e \cdot V_A}} \tag{2}$$

According to acoustic-electric analogy, a parallel connection of an inductance and a capacitance in an electric circuit is analogous to a parallel connection of a short tube and an expansion cavity in an acoustic structure, and a series connection of an inductance and a capacitance in an electric circuit is analogous to a Helmholtz resonator in an acoustic structure. In this way, based on the electrical filter circuit in figure 1, an equivalent second-order Butterworth acoustic filter is designed, as shown in figure 3.

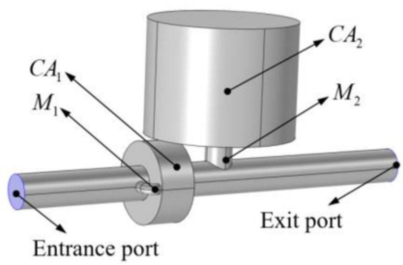


Figure 3. Acoustic filter.

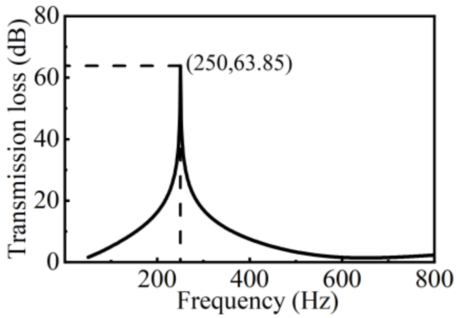


Figure 4. Transmission losses.

Assuming that the temperature is 20°C and the sound velocity is 343m/s. Based on table 1 and equation (1), structural parameters of the acoustic filter can be figured out. Here, the characteristic impedance of the acoustic filter is

$$R_A = \frac{\rho_0 \cdot c_0}{S_m} \tag{3}$$

where $S_m$ is the area of the main pipe. The characteristic impedance of the acoustic filter is different from the characteristic impedance in figure 1, so the parameters $L$ and $C$ in table 1 should be transformed by

$$M_t = \frac{R_A}{R} L \qquad C_A = \frac{R}{R_A} C \tag{4}$$

Table 2 gives structural parameters of the acoustic filter in figure 3, where $r_{M_1}$, $l_{M_1}$, $r_{M_2}$ and $l_{M_2}$ are radiuses and lengths of the short tubes $M_1$ and $M_2$, and $r_{CA_1}$, $l_{CA_1}$, $r_{CA_2}$ and $l_{CA_2}$ are radiuses and lengths of the expansion cavities $CA_1$ and $CA_2$.

**Table 2**. Structural parameters of the acoustic filter with a resonance frequency of 250Hz.

| $r_{M_1}$ (mm) | $l_{M_1}$ (mm) | $r_{M_2}$ (mm) | $l_{M_2}$ (mm) | $r_{CA_2}$ (mm) | $l_{CA_2}$ (mm) | $r_{CA_1}$ (mm) | $l_{CA_1}$ (mm) |
|---|---|---|---|---|---|---|---|
| 3.00 | 30.37 | 8.00 | 15.00 | 43.70 | 66.69 | 24.00 | 21.00 |

The acoustic filter model is built by using COMSOL, and the transmission losses are analyzed. Set the radius and the length of the main pipe to be 10mm and 250mm, respectively. A transmission loss curve is shown in figure 4, under the plane wave incident of 1Pa assumption. Through comparing figure 2 and figure 4, it can be found that, the two transmission loss curves have a consistent variation trend.

## 3. Periodicity of the Acoustic Structures Composed of Acoustic Filters

Based on the acoustic filter in figure 3, several types of periodic acoustic structures are built, and the influence of structural periodicity on acoustic performances is investigated. In this section, axial periodicity and circumferential periodicity are considered.

The acoustic filter in figure 3 is regarded as an original unit cell, and a type of periodic acoustic structure is constructed by periodically arraying the original unit cell along the axial direction of the main pipe. The axial periodicity is described by the number of unit cells arrayed axially inside the periodic structure. In order to investigate the influence of the axial periodicity of the acoustic structure on filtering performances, two-, three- and four-period models are built and analyzed, respectively. Figure 5 is a three-period acoustic structure. Under the plane wave incident of 1Pa assumption, the transmission losses of the three models are shown in figure 6, where the transmission loss curve of the original unit cell model in figure 3 is also given to facilitate the comparison. In figure 6, it can be found that, with an increase in the number of the axial period, maximum of the transmission losses rises from 63.85dB to 131.04dB, 169.44dB and 279.98dB, and the filtering frequency range with the transmission losses greater than 20dB widens from 68Hz to 207Hz, 292Hz and 327Hz. The results verify that, axial periodicity of the acoustic structure has no influence on the resonance frequency of the acoustic structure, but contributes to transmission loss values and the filtering frequency range.

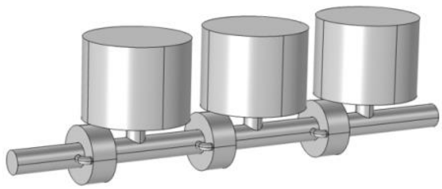

**Figure 5**. A three-period acoustic structure.

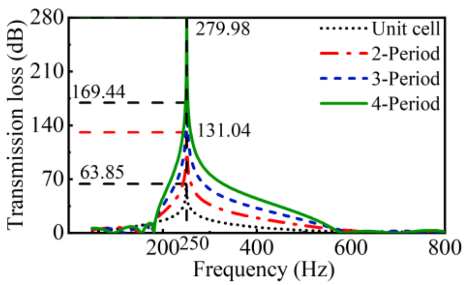


**Figure 6.** Transmission losses.

In order to research the influence of circumferential periodicity of the acoustic structure, the original unit cell in figure 3 is modified by symmetrically arraying several Helmholtz resonators along the circumferential direction of the main pipe. The circumferential periodicity is described by the number of Helmholtz resonators arrayed circumferentially inside a unit cell. Based on the modified unit cells, a new type of periodic acoustic structure is obtained by periodically arraying the modified unit cells along the axial direction of the main pipe. Here, three acoustic structure models with two periods along the axial direction of the main pipe are built and analyzed, and three transmission loss curves are shown in figure 7, where 1-HR stands for the two-period model composed of the original unit cell in figure 3, 2-HR the two-period model composed of the modified unit cell comprising 2 symmetrical Helmholtz resonators, and 3-HR the two-period model in figure 8 composed of the modified unit cell comprising 3 symmetrical Helmholtz resonators.

In figure 7, it can be found that, different from the axial periodicity of the acoustic structure composed of the original unit cell, the circumferential periodicity or the number of Helmholtz resonators arrayed circumferentially inside a unit cell, can contribute to the resonance frequency, transmission loss values and filtering frequency range. The increase in the number of Helmholtz resonators results in a little shift in the resonance frequency towards the high frequency, a significant rise in the maximum of the transmission losses from 131.04dB to 158.48dB and 175.91dB, and an increment in filtering frequency range with transmission losses greater than 20dB from 207Hz to 383Hz and 514Hz. Relatively speaking, the periodic acoustic structure in figure 8 shows the best acoustic performances.

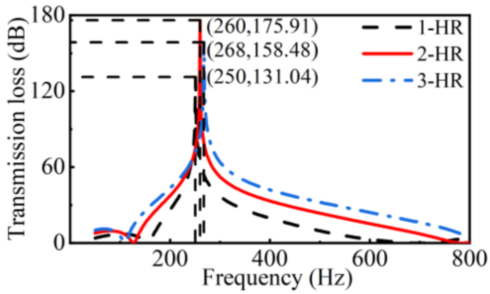


**Figure 7**. Transmission losses (2-period models).

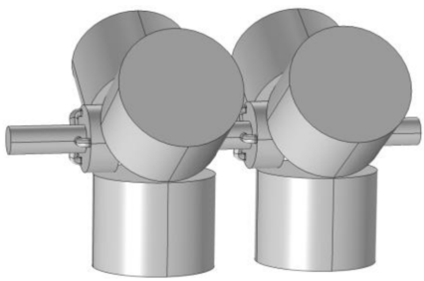

**Figure 8.** Two-period structure model.

Due to good performances of the periodic acoustic structure in figure 8, the modified unit cell comprising 3 Helmholtz resonators (3-HR) is adopted to build one-, two-, three- and four-period structure models by periodically arraying the modified unit cell along the axial direction of the main pipe. Under the plane wave incident of 1Pa assumption, the transmission losses of the four models are shown in figure 9.

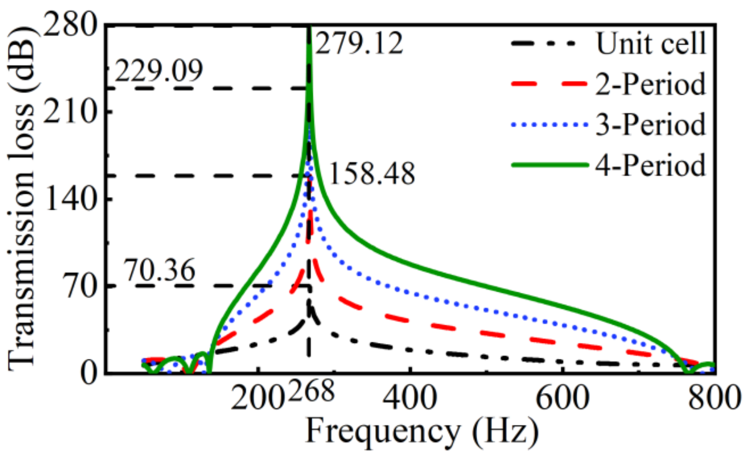


**Figure 9.** Transmission losses (3-HR).

For the acoustic structures composed of the modified unit cell comprising 3 symmetrical Helmholtz resonators, the axial periodicity can affect the resonance frequency of the periodic structure, different from the acoustic structures composed of the original unit cell in figure 3. With an increase in the number of the axial period, the resonance frequency moves to high frequencies and is finally fixed at 268Hz. The increase in the number of the axial period also results in a significant rise in the maximum of the transmission losses from 70.36dB to 158.48dB, 229.09 dB and 279.12dB, and a great increment in the filtering frequency range with transmission losses greater than 20dB from 213Hz to 514Hz, 584Hz and 645Hz.

## 4. Periodic Acoustic Structures with Low-frequency Broadband Performances

In order to further widen filtering frequency range of the acoustic structure, two acoustic filters with different resonance frequencies are combined to form a combination unit cell, as shown in figure 10. The left acoustic filter has a resonance frequency of 250Hz and its parameters are same as those in table 2; and the right acoustic filter has a resonance frequency of 350Hz and its parameters are listed in table 3.

**Table 3**. Structural parameters of the acoustic filter with a resonance frequency of 350Hz.

| $r_{M_1}$ (mm) | $l_{M_1}$ (mm) | $r_{M_2}$ (mm) | $l_{M_2}$ (mm) | $r_{CA_2}$ (mm) | $l_{CA_2}$ (mm) | $r_{CA_1}$ (mm) | $l_{CA_1}$ (mm) |
|---|---|---|---|---|---|---|---|
| 3.00 | 29.09 | 8.00 | 15.00 | 38.39 | 44.58 | 21.00 | 17.00 |

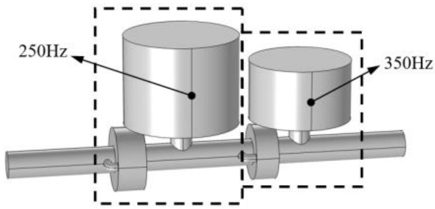


**Figure 10.** A combination unit cell.

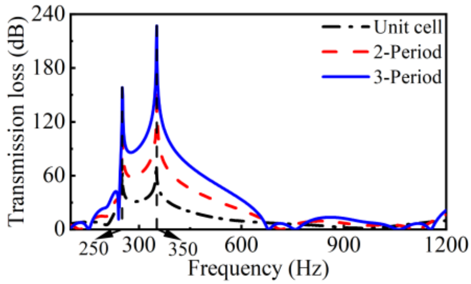


**Figure 11.** Transmission losses.

The periodic acoustic structure is constructed by periodically arraying the combination unit cell along the axial direction of the main pipe. The simulation results are given in figure 11, where three transmission loss curves stand for the combination unit cell model, two- and three-period models, respectively. Axial periodicity of the acoustic structure composed of the combination unit cell has consistent influence with that of the acoustic structure composed of the original unit cell, including unchanged resonant frequencies, increased transmission losses and enlarged filtering frequency range. By comparing figure 6 and figure 11, it can be observed that, the filtering frequency bandwidth with transmission losses greater than 20dB is enlarged significantly, for one-period models rising from 68Hz to 217Hz, for the two-period models rising from 207Hz to 394Hz, for the three-period models rising from 292Hz to 461Hz, verifying the effectiveness of the combination unit cell.

Based on the research on circumferential periodicity of the acoustic structures, the two acoustic filters in figure 10 are modified, respectively, by symmetrically arraying several Helmholtz resonators along the circumferential direction of the main pipe, to construct modified combination unit cells. Here, two modified combination unit cell models are built, where each acoustic filter comprises 2 Helmholtz resonators (2-HR) or 3 Helmholtz resonators (3-HR). Simulation results are shown in figure 12, where the transmission loss curve 1-HR stands for the original combination unit cell in figure 10. The two modified combination unit cell (2-HR and 3-HR) have frequency bandwidths of 415Hz and 663Hz, respectively, bigger than that of the combination unit cell in figure 10. The modified combination unit cell, corresponding to the curve 3-HR, has the best filtering performances.

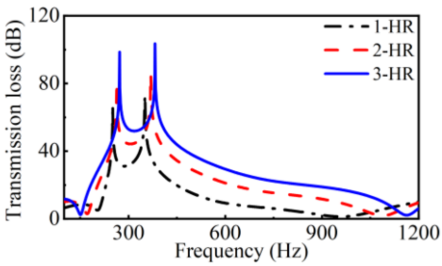


**Figure 12**. Transmission losses (1-period models).

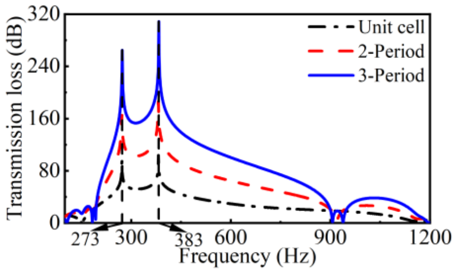


**Figure 13.** Transmission losses (3-HR).

And then, based on the modified combination unit cell having the best filtering performances, the periodic acoustic structure is obtained by periodically arraying the unit cell (3-HR) along the axial direction of the main pipe. Figure 13 shows the simulation results of one-, two- and three-period structure models. Compared with figures 11 and 12, the acoustic structures composed of the modified combination unit cell have better filtering performances, whose frequency bandwidths with transmission losses greater than 20dB are 663Hz, 891Hz and 937Hz, respectively.

It can be observed that, the periodic acoustic structure corresponding to the blue solid line in figure 13 shows the best filtering performances, which has 2 acoustic filters with resonance frequencies of 250Hz and 350Hz inside a unit cell, 3 circumferential periods and 3 axial periods. The simulation results verify that, combining unit cells with different resonance frequencies and properly increasing axial and circumferential periods is an effective design method of periodic acoustic structures composed of acoustic filters.

## 5. Conclusions

A design method of the periodic acoustic structure composed of acoustic filters is proposed, and based on the method, periodic acoustic structures with low-frequency broadband filtering performances are constructed by modifying the unit cell of the periodic structure and properly increasing axial and circumferential periods. Main conclusions are as follows:

- While taking the second-order Butterworth acoustic filter as an original unit cell, the axial period of the acoustic structure composed of the original unit cell has no influence on the resonance frequency of acoustic structure;

- While modifying the original unit cell by increasing the number of Helmholtz resonators arrayed circumferentially inside a unit cell, the axial period of the acoustic structure composed of the modified unit cell contributes a little to the resonance frequency of acoustic structure, and the increase in the axial period results in a little shift of the resonance frequency towards the high frequency;
- For the periodic structure composed of acoustic filters, the filtering frequency bandwidth can be enlarged effectively by increasing the number of unit cells arraying axially in the periodic structure, raising the number of Helmholtz resonators arrayed circumferentially inside a unit cell, or combining unit cells with different resonance frequencies;
- Combining unit cells with different resonance frequencies and properly increasing axial and circumferential periods is an effective design method of periodic acoustic structures with low-frequency broadband performances.

## Acknowledgments

This research was financially supported by the National Science Foundation No.51975083, No.51775080, No.11372059 and No.11272073.

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