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Performance Comparison of Analytical and Numerical Methods of Elastic Properties of Composite Micromechanics

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Abstract. Because of the anisotropic nature of composite materials, they can show different material properties in different directions. Therefore, it is decisive to determine the mechanical properties of composites. In this study, performance comparison of analytical methods and numerical methods are investigated. The most well-known analytical method, the strength of the material method, and the relatively more complex and widely used Chamis' equations and Tsai-Halpin equations were selected. Numerically, the finite element-based representative volume element (RVE) homogenization method without periodic boundary conditions is chosen. As a result, the modulus of elasticity in the fiber direction and the major Poisson's ratio values are similar for all methods. Tsai-Halpin method and RVE Homogenization method can predict the transverse elasticity value close to each other, Chamis' equations also provide results relatively close to these results, but the results obtained from the strength of material approaches are not acceptable. From the study, it was revealed every method can be used to identify modulus of elasticity in fiber direction and major Poisson's ratio but modulus of elasticity in the transverse direction and shear modulus can be calculated in other methods.

Keywords. Composites, elastic properties, chamis, Tsai-Halpin, RVE

1. Introduction

Composites embody more than one discrete material. There are two major structural elements in composite materials called fiber and matrix. While the fiber is mostly responsible for the strength, the matrix is responsible for the bonding and integrity of the structure. The mechanical properties of composites not only appertain fiber and matrix properties but also depends on manufacturing defects. However, in this study manufacturing defects such as the volume of voids are not included. Our goal is to compare some analytic and numerical methods for predicting elastic properties. Elastic properties of composite materials emerge from usage materials with different compositions, where the discrete ingredients detain their unique characters and function with each other to provide appropriate mechanical properties [1]. Although, much study has been done to overcome these design issues [2]. The modulus of elasticity of composite material is linked to the understanding of its microstructural behavior [3,4]. Several analytical and numerical methods are usable, like Chamis' equations [5] and the Tsai-Halpin approach [6]. Nevertheless, these methods are not sufficient to appraise the

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elastic properties of composites. Hence, employing a FEM based method like representative volume element (RVE) is better to evaluate elastic properties [7]. RVE term was applied for the first time by Hill [8] and identified as the minimum volume of composite material which represents a sufficiently accurate model to the effective elastic response. Sadik L. Omairey [9] has developed an ABAQUS plugin that predicts the elastic properties of composite materials. Methods need to be compared to determine the approach that comes as close as possible to the true results. Pal [10] compared results that are found from the strength of materials approach, Tsai-Halpin equations, and FEA-based RVE analysis. There are very few studies in the literature that compare the methods of calculating the transverse modulus of elasticity and finding the correct value. This study sheds light on the above problem by comparing some methods. As a result of this study, it was found that Tsai-Halpin and the RVE Method gave the closest values in the result, on the other hand, Chamis' equations provided convergence, but the strength of materials method did not give sufficient values.

2. Materials and Method

2.1. Material Properties

The model of matrix and fiber in table 1 has the dimension of 7.42x7.42x7.42 which represents the width, height, and length of the RVE with a fiber fraction measurement of 0.7. The model is designed in ABAQUS software and the C3D8R element is chosen for meshing.

Table 1. Material Properties.

Properties (GPA)	Fiber (T300)	Epoxy Resin
E	230	2.8
n_{12}	0.2	0.3

2.2. Analytical Methods

The anisotropic nature of the composite materials allows these values to vary according to the fiber direction. Since the laminas are in plane stress, four different engineering constants have to be determined. These constants are E_1, E_2, v_{12}, and G_{12}. There are various analytical, semi-empirical, and more sophisticated models for determining these elastic properties of composite materials. These are the Strength of Materials approach, the Tsai-Halpin model, and the Chamis' equations. While all the analytical method satisfies in longitudinal modulus of elasticity (E_1) and major Poisson's ratio (v_{12}), they differ in transverse elastic modulus (E_2) and shear modulus (G_{12}) formulas.

2.3. Numerical Methods

The fundamental thought of the computational homogenization techniques is to achieve mechanical properties of composites by solving the boundary value problem with proper boundary conditions. To estimate the mechanical properties of composite numerically, a RVE that corresponds to a regular fiber packing arrangement is often used. The RVE term was used first by [8] and it can be described as minimum material volume which can provide an accurate macroscopic material response. Hence, the RVE should be modeled so that the RVE can show accurate material response at the material response [11]. Typical RVE with single fiber is shown in figure 1(a).

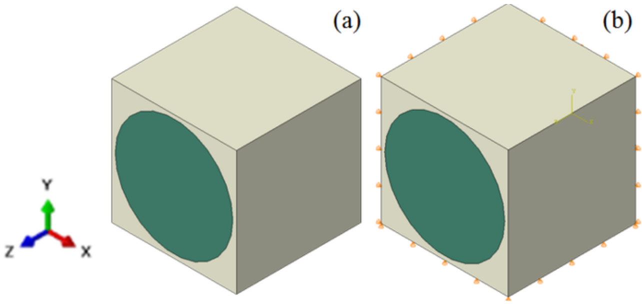

Figure 1. (a) RVE (Micro-scale), (b) Boundary Conditions of RVE.

The composite lamina is described in classical lamination theory as a homogenous orthotropic material with specific effective moduli that characterize the composite's typical material properties. Macro-stress and strain are calculated by averaging the stress and strain tensor over the volume of the RVE to describe this macroscopically homogenous material.

$$\bar{\sigma}_{ij} = \frac{1}{V}\int_V \sigma_{ij}(x,y,z)dV \tag{1}$$

$$\bar{\varepsilon}_{ij} = \frac{1}{V}\int_V \varepsilon_{ij}(x,y,z)dV \tag{2}$$

2.4. Finite Element Analysis

The displacement of the RVE is constrained by displacement in the x, y, and z directions by back faces. Boundary conditions of RVE can be seen in figure 1(b). It is fixed in the direction of the normal back surfaces and the other two degrees of freedom are not constrained. To determine the elastic properties, a load must be applied and this load is set to 10 depending on the directions. These loads and the elastic constants E_1, E_2, E_3, G_{12}, G_{23}, and G_{13} that can be found are given below in table 2.

Table 2. Load directions.

Direction	Load	Displacement	To determine
x	σ_x	ε_x	$E_1 = \frac{\sigma_x}{\varepsilon_x}$
y	σ_y	ε_y	$E_2 = \frac{\sigma_y}{\varepsilon_y}$
z	σ_z	ε_z	$E_3 = \frac{\sigma_z}{\varepsilon_z}$
yz	τ_{yz}	γ_{yz}	$G_{23} = \frac{\tau_{yz}}{\gamma_{yz}}$
xz	τ_{xz}	γ_{xz}	$G_{13} = \frac{\tau_{xz}}{\gamma_{xz}}$
xy	τ_{xy}	γ_{xy}	$G_{12} = \frac{\tau_{xy}}{\gamma_{xy}}$

3. Result and Discussion

Figures 2(a), (b), and (c) show the stress distribution of RVEs according to elastic constants. For all the analytical methods, longitudinal modulus of elasticity (E_1) and major Poisson's ratio;

$$E_1 = Ef \times V_f + E_m \times V_m \tag{3}$$

$$\nu_{12} = \upsilon_f \times V_f + \nu_m \times V_m \tag{4}$$

where E_f, E_m is the elastic modulus of composite components, and V_f, V_m is the volume of fiber and matrix fraction respectively.

$$\frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m} \tag{5}$$

$$\frac{1}{G_{12}} = \frac{V_f}{G_f} + \frac{V_m}{G_m} \tag{6}$$

where G_f, G_m is the elastic modulus of composite components. Because the strength of the material approach doesn't sufficient to calculate E_2 and G_{12}. This has led to the development of more complex models just as the Tsai-Halpin equation. The obtained results from the test don't agree well with the test results. To solution for this Tsai-Halpin [6] developed their model by curve fitting. E_2 and G_{12};

$$\frac{E_2}{E_m} = \frac{1+\xi\eta V_f}{1-\eta V_f} \tag{7}$$

$$\eta = \frac{\frac{E_f}{Em}-1}{\frac{E_f}{Em}+\xi} \tag{8}$$

where x is the reinforcing factor term. It rely on loading conditions, fiber and packing geometry. If the lamina has circular fibers with a square arrangement ξ equal to 2, if it has hexagonal fibers, ξ is equal to 2(a/b). To find shear modulus;

$$\frac{G_{12}}{G_m} = \frac{1+\xi\eta V_f}{1-\eta V_f} \tag{9}$$

$$\eta = \frac{\frac{G_f}{G_m}-1}{\frac{G_f}{G_m}+\xi} \tag{10}$$

According to [6], $\xi = 1$ to find the shear modulus of fiber consisting of circular fiber with a square array and $\xi = \sqrt{3\log_e(a/b)}$ find the shear modulus of a fiber with a hexagonal array. On the other hand, $\xi = 1$ give satisfactory results only up to $V_f = 0.55$. To solve this [12] proposed the following equation;

$$\xi = 1 + 40V_f^{10} \tag{11}$$

Chamis proposed another model [5]. The most complex theory to evaluate E_2 of the unidirectional laminates was derived by Hashin. Hashin's model is very complex and hard to evaluate material constants. When compared with Chamis' formula, it is found that the results are similar and Chamis' formula is far simple. To find E_2 and G_{12} according to Chamis' formula;

$$E_2 = \frac{E_m}{1-\left(1-\frac{E_m}{E_f}\right)V_f} \tag{12}$$

$$G_{12} = \frac{G_m}{1-\left(1-\frac{G_m}{G_f}\right)V_f} \tag{13}$$

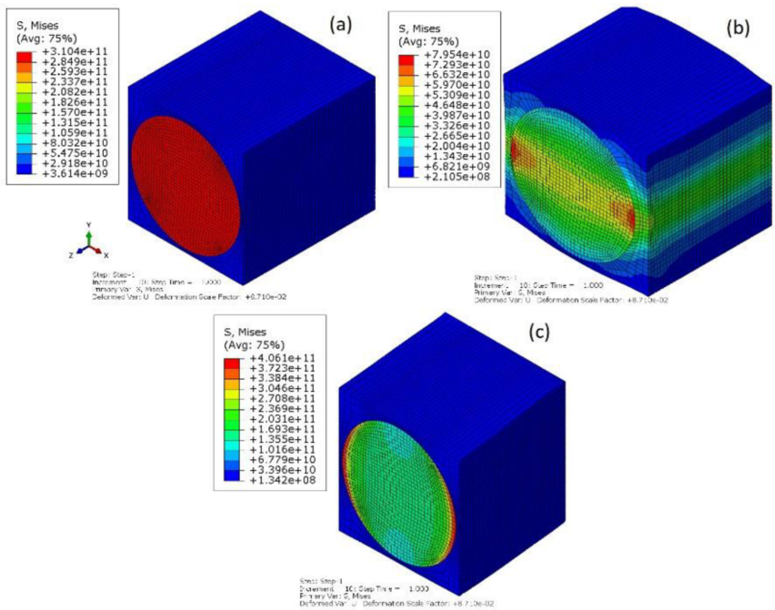

Figure 2. (a) Tensile stress in the fiber direction caption, (b) Tensile stress in the transverse direction, and (c) Shear stress in the XZ plane.

Table 3. Comparison of elastic constants.

	E_1 (GPa)	E_2 (GPa)	ν_{12}	G_{12} (GPa)
Strength of Materials Approach	161.84	9.08	0.23	3.4980
Chamis' Equations	161.84	16.14	0.23	6.2343
Tsai-Halpin's Equations	161.84	20.25	0.23	8.0458
RVE Homogenization Method	161.54	21.35	0.23	3.0159

Using analytical methods and numerical methods, elastic constants are determined without periodic boundary conditions. Table 3 shows the comparison of E_1 in the strength of the material, Tsai-Halpin equations, Chamis' equations, and the RVE Homogenization method. The RVE homogenization method provides the identical result to the strength of materials approach, Chamis' equations, and Tsai-Halpin equations. For E_2 values Tsai-Halpin equations and the RVE Homogenization method gave a good agreement. The reason why the strength of materials approach gave different reasons probably is that they do not take into account the fiber alignments, loading directions, and packing arrangements. Tsai-Halpin equations have a reinforcing factor (ξ) that includes the effect of these terms. Because of this term, the Tsai-Halpin equations give more appropriate values than the others. For major Poisson's ratio (υ_{12}) every method calculates the same values.

4. Conclusion

The focus of this study is to compare predicted elastic properties. According to table 3, elastic modulus in fiber direction (E_1) and major Poisson's ratio (υ_{12}) are consistent in all methods. So, methods are sufficient to determine fiber direction properties. On the other hand, the transverse tensile modulus (E_2) values are consistent for the two methods. In Chamis' equations value of E_2 seems quite sufficient to predict but it is not efficient as Tsai-Halpin equations. Additionally, the shear modulus (G_{12}) values are inconsistent. Since the values from the methods differ from each other, the calculated values should be correlated with the experimental data.

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