Abstract. The Colorado River basin is facing a growing water shortage. Under a fixed set of water supply and demand conditions, we establish a linear programming model based on time of water supply to solve the problem of water supply allocation. Take the supply time as the objective function, get the distribution coefficient $\alpha$, $\beta$ to obtain the water supply from the Hoover dam and Glen Canyon dam. Finally it is concluded that the optimal objective function is obtained when $\alpha$ is 0.8 and $\beta$ is 1.0. The water supply of Hoover Dam is 6947519530 m$^3$ and Glen Canyon dam is 9309140232 m$^3$.

Keywords. Objective programming, allocation, water, electricity

1. Introduction

A reservoir can function for multiple purposes through impounding water and controlling streamflow [1]. Hydropower, large and small, remains by far the most important of the “renewables” for electrical power production worldwide, providing 19% of the planet’s electricity [2]. Water from Glen Canyon dam is supplied to Arizona, Colorado, Wyoming, New Mexico, and water from Hoover dam is supplied to Arizona, California. According to actual situation, water outflows from the Glen Canyon dam supply part of the water input to the Hoover dam. Then, it inputs to Arizona. Arizona receives the water from Glen Canyon dam and Hoover dam. Figure 1 shows the dam supply.

Figure 1. Supply plan of dams.

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2. Problem Analysis

In order to better solve the problems, it is expected to collect a large amount of data on water and electricity demands in Arizona, Colorado, Wyoming, New Mexico California and Mexico. We need to convert electricity supply to water supply, because power stations use a certain amount of water to produce electricity. Then set up a time objective function to obtain the distribution coefficient.

3. Modeling Analysis

This section establishes the objective programming model, we need to solve the total water supply of dams according to the water supply and power supply distribution of each state. Since Arizona receives water supply and power supply from two dams, we set the corresponding coefficient $\alpha$ to reflect the supply situation of the two dams to Arizona. At the same time, the water supply is constrained by the current water level of the two dams, the lowest water level and the water flowing into the downstream.

3.1. Water Supply Calculation

Given in the question, the water level of Lake Powell is $P$, and Lake Mead is $M$. We set $X_i$ as the water supply of each state. According to the supply relationship between each dam and each state, the water supply of the two dams should be

\[(1-\alpha)X_2 + X_3 + X_4 + X_5 = V_{\text{Powell}} \tag{1}\]
\[X_1 + \alpha X_2 = V_{\text{Mead}} \tag{2}\]

where $V_{\text{Powell}}$ denotes the water supply in Lake Powell, $V_{\text{Mead}}$ denotes the water supply in Lake Mead, $\alpha$ denotes the proportion of water supply from Hoover dam to Arizona, and $1-\alpha$ is the proportion of water supply from Glen Canyon dam to Arizona.

The conditions to be met are

\[\max_{\text{raw}} (P - \text{inflow}) - S_P - V_{\text{draw}} + V_{\text{inflow}} \tag{3}\]
\[V_{\text{max, Powell}} \geq (1-\alpha)X_2 + X_3 + X_4 + X_5 \tag{4}\]
\[V_{\text{max, Mead}} = (M - M_{\text{min}})S_M + V_{\text{draw}} - V_{\text{Mead}} \tag{5}\]
\[V_{\text{max, Mead}} \geq X_1 + \alpha X_2 \tag{6}\]

where $S_P$ is the water surface area of Lake Powell and $S_M$ is the water surface area of Lake Mead. $V_{\text{draw}}$ is the amount of water flowing from Lake Powell to Lake Mead. $V_{\text{inflow}}$ is the amount of water flowing from Lake Powell upper reaches to Lake Powell. $V_{\text{Mead}}$ is the water supply of Mexico. Lake Powell’s water supply should be less than the maximum water supply minus the amount flowing downstream, while Lake Mead’s...
water supply should be less than the maximum water supply plus the amount flowing upstream and ensuring that the amount flowing into Mexico meets the requirements.

There is a certain relationship between water consumption and power generation of hydropower station. For hydroelectric power plants, the larger the head, the more power generated by the same amount of water [4]. The relationship between power generation and water consumption is as follows

\[ E_M = C_M t, E_P = C_P t \]  

\[ W_M = \frac{N_M}{\eta H_M} \cdot 3600t, W_p = \frac{N_p}{\eta H_p} \cdot 3600t, (\eta = 8.5) \]  

\[ K_M = \frac{W_M}{E_M}, K_p = \frac{W_P}{E_P} \]  

where, \( E_M \) is annual energy output of Lake Mead, \( C_M \) is total capacity of generator set of Lake Mead, \( t \) is number of power plant operating hours per year, \( W_M \) is annual water consumption of Lake Mead, \( K_M \) is power output of hydropower station of Lake Mead, \( \eta \) is efficiency coefficient, \( H_M \) is rated head of hydropower station of Lake Mead. \( \frac{W_M}{E_M} \) is a coefficient that converts electricity supply into water supply. Subscript \( P \) indicates the parameter of Lake Powell.

By analyzing state data from 1950 to 2015, we can figure out the average water supply and the average electricity supply for each state each year, which includes hydro and other power generation. We include hydro in renewable energy generation. Water supply is used for agriculture, industry, and residence. Electricity is divided into renewable and non-renewable electricity. At the same time, we should also pay attention to the relevant data of water supply in Mexico, and obtain the average water supply in Mexico every year to ensure Mexico’s rights and benefits.

The electricity generated by hydroelectric power generation in the electricity consumption is obtained by a certain proportion, and then the electricity is converted into water supply according to the relationship between electricity and water quantity.

\[ X_i = Xin_i + Xir_i + Xps_i \]  

\[ Y_i = \frac{Gelec_i}{Gre_i + Gunre_i} d_i \]  

where \( Xin_i \) is average industrial water consumption, \( Xir_i \) represents agricultural, \( Xps_i \) represents residential. \( Y_i \) is average water power consumption, \( Gelec_i \) represents hydropower generation, \( Gre_i \) represents renewable energy generation, and \( Gunre_i \) represents non renewable energy generation. \( d_i \) is state power consumption.
3.2. The Time Programming Model

The competing interests of the Colorado River between areas and fields have been controversial problems since the last century, especially during continuous drought conditions [4]. We analyzed the water storage maintenance time of the two dams by constructing the objective function of time. On the basis of analyzing the present situation and existing problems of water resources allocation, the practical allocation problem is transformed into a mathematical model, in order to obtain the water allocation scheme efficiently and accurately [5]. Finally, the shortest time is taken from the supply time of the two dams to optimize the total supply time and the distribution of water supply and electricity supply in each state.

Based on the above analysis, we get the following formula:

\[ V_{\text{Mead}} = X_i + \alpha X_i + (Y_i + \beta Y_i) \cdot K_{st} \]  
(12)

\[ V_{\text{Pawell}} = (1 - \alpha)X_i + \sum_{i=3}^{5} X_i + [(1 - \beta)Y_i + \sum_{i=3}^{5} Y_i] \cdot K_{p} \]  
(13)

\[ T_x = \frac{X_i}{x_i}, T_y = \frac{Y_i}{y_i}, T_{\text{MEX}} = \frac{V_{\text{max,MEX}}}{V_{\text{min,MEX}}} \]  
(14)

where \( \beta \) is proportion of power supplied from dam Hoover dam to Arizona, \( T_x \) is the water supply time of state \( i \), \( T_y \) power supply time of state \( i \), \( T_{\text{MEX}} \) is the water supply time of Mexico. \( x_i \) is the water consumption of state \( i \), \( y_i \) is the power consumption of state \( i \), \( V_{\text{max,MEX}} \) is water supply of Mexico, \( V_{\text{min,MEX}} \) is water consumption of Mexico (surface water).

At present, 40% of the agricultural irrigation water sources in Arizona come from groundwater, 20% from Colorado River water, and the rest 40% depend on accumulated spring and autumn rain, which has played a significant effect on the protection of groundwater resources [6]. Based on the principle of sustainable development and aiming at the characteristics of sustainable utilization of water resources, objective optimal allocation model of water resources was established [7]. Through this model, the economic benefits of water resources utilization can be optimized by coordinating the structure of national economy with the distribution of water resources [8]. If the time is optimized, then water supply and power supply are allocated to each state in this way to optimize the distribution mode, and time is selected as the objective function

\[ \max \{ \min \{ T_x, T_y, T_{\text{MEX}} \} \} \]  
(15)

The constraints that need to be met are

\[ \begin{align*}
V_{\text{Pawell}} &\leq V_{\text{max,Pawell}} \\
V_{\text{Mead}} &\leq V_{\text{max,Mead}} \\
V_{\text{MEX}} &\geq V_{\text{min,MEX}} \\
X_i &\geq x_i \\
Y_i &\geq y_i \\
0 &\leq \alpha \leq 1 \\
0 &\leq \beta \leq 1
\end{align*} \]  
(16)
where water supply $X_i$ should be greater than average water consumption $x$, water power supply $Y_i$ should be greater than average water power consumption $y$, water supply of Mexico $W_M$ should be greater than average water consumption of Mexico $V_{Mex}$. We define $V_{min_{MEX}}$ as the product of the average per capita water supply in Mexico and the number of people without access to safe drinking water.

3.3. Model Solution

For our model, it needs to be given in advance $\alpha$ and $\beta$. We select the data of water and electricity consumption in 2015 and bring them into our model. After repeated verification, we selected the most appropriate one $\alpha$ and $\beta$ to solve the model, the selection process is shown in Figure 2.

![Figure 2. $\alpha$, $\beta$ value distribution when the objective function is optimal.](image)

According to the Figures 3 and 4, we choose 36 combinations of $\alpha$ and $\beta$ to solve the model. The optimal objective function is obtained when $\alpha$ is 0.8 and $\beta$ is 1.0. When the water consumption in 2015 is substituted into equations (1) and (2), the water supply $V_{Mead}$ of Hoover Dam is $6947519530 \text{ m}^3$ and $V_{Powell}$ is $9309140232 \text{ m}^3$. The optimal solution of the objective function is 0.6 years. In order to meet the fixed demand, the amount of additional recharge is similar to water supply of the two dams.

![Figure 3. $\alpha$ value distribution.](image)

![Figure 4. $\beta$ value distribution.](image)
According to the Figure 5, it is concluded that the best distribution mode in 2015 is CA accounting for 34.64%, AZ accounting for 10.12%, CO accounting for 27.23%, NM accounting for 4.53% and WY accounting for 23.48%. For electricity, CA accounting for 83.38%, AZ accounting for 11.39%, CO accounting for 4.23%, NM accounting for 0.23% and WY accounting for 0.76%.

4. Conclusion

We came to the conclusion that the optimal objective function is obtained when $\alpha$ is 0.8 and $\beta$ is 1.0. At this time, the objective function is 0.6 years. The water supply of Hoover Dam is 6947519530 m$^3$ and Glen Canyon dam is 9309140232 m$^3$.

References