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A Simple Method for Position Analysis of Stephenson-III Spherical Six Bar Mechanism

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Abstract. Aiming at the position analysis of Stephenson III spherical six bar mechanism, a simple method to solve its input-output equation is given. The Stephenson III spherical six bar mechanism is regarded as composed of basic spherical four-bar chain and spherical two-bar group. The basic coordinate system and branch coordinate system are established respectively. The coordinates of each hinge point are solved with the help of geometric principle and displacement rotation theory. Based on the motion constraints of the basic spherical four-bar chain and the coupling constraints with the spherical two-bar group, the constraint equations of the spherical six bar mechanism are established by using spherical trigonometry. The constraint equations are simplified and eliminated by Sylvester's resultant elimination method and triangular transformation formula, and then the constraint equations of the mechanism are obtained.

Keywords. Location analysis, Displacement rotation theory, Sylvester resultant elimination method, Spherical trigonometry

1. Introduction

The research content of mechanism kinematics can be divided into motion analysis and motion synthesis. Kinematics analysis includes position analysis, velocity analysis and acceleration analysis. Position analysis is the most basic task of motion analysis [1,2]. Based on the constraint relationship between mechanism components, the establishment of constraint equations and the forward and inverse solution of mechanism position can not only lay a solid foundation for the analysis of mechanism speed and acceleration, but also lay a solid foundation for the solution of mechanism workspace. Then we can more systematically and deeply study all kinds of existing mechanisms, master the law and scope of their motion transformation, and lay a good foundation for designing new and better mechanisms or machines.

The spherical surface has the characteristics of both planar mechanism and spatial mechanism, it has always been highly concerned by scholars at home and abroad. Based on spherical trigonometry, On the basis of a new 4-DOF spherical parallel mechanism proposed by Liu et al.[3], the positive and negative kinematics of the spherical parallel mechanism was solved, and its good kinematic characteristics were

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verified. Hernandez s et al. [4] comprehensively designed the spherical mechanical wrist based on the analysis of Stephenson spherical six bar mechanism based on displacement rotation theory. This paper based on the geometric principle, displacement rotation theory and spherical trigonometry, the input-output equations of Stephenson-III spherical six bar mechanism are solved by analytical method, The solution process is simple, easy to understand and program.

2. Establishment of constraint equations of spherical six bar mechanism.

In the analysis and synthesis of spherical mechanism, in order to facilitate description and calculation, it is necessary to use spherical coordinates to study spherical mechanism. The position of any point on the sphere can be expressed by coordinate parameters $P(r,\alpha,\beta)$ [5],as

$$\begin{cases} P_x = \cos\beta\cos\alpha \\ P_y = \cos\beta\sin\alpha \\ P_z = \sin\beta \end{cases} \tag{1}$$

The variation ranges of r , α and β are $-\infty \le r \le \infty$, $-180° \le \alpha \le 180°$ and $-90° \le \beta \le 90°$ respectively. As shown in Figure 1.

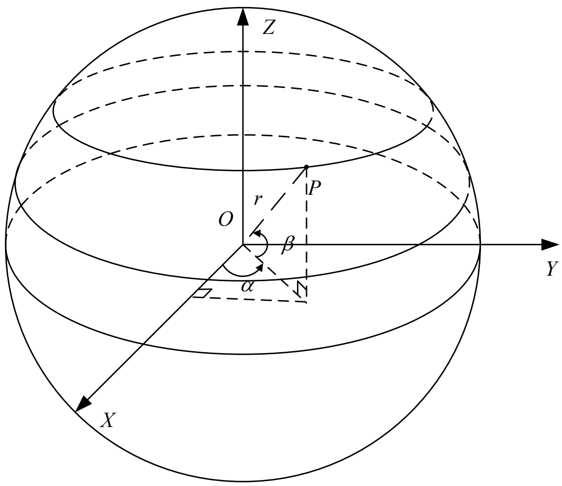

Figure 1. Spherical coordinate representation of spatial points

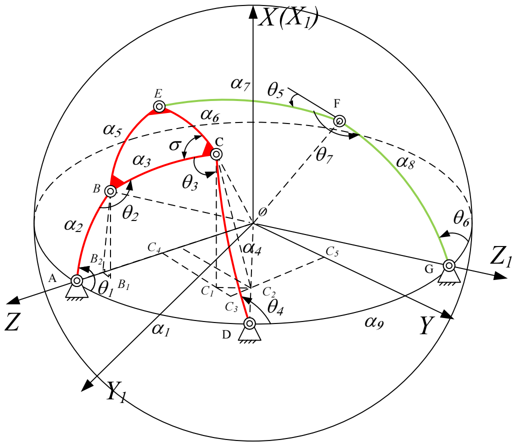

Figure 2. Schematic diagram of Stephenson-III spherical six bar mechanism

Figure 2 is the schematic diagram of Stephenson-III spherical six bar mechanism. In order to keep the generality of the research, it is assumed that the spherical six bar mechanism is located on the unit sphere. O is the center of the ball, A、D、G are the fixed hinge point, B、C、E、F are the movable hinge point. In the schematic diagram of Stephenson-III spherical six bar mechanism, AB is the input lever, CD and FG are output rods, BCE and EF are the connecting rod, AD and DG are the rack. The length of each rod is expressed by its ball center angle $\alpha_k\ (k=1,2,\cdots,9)$, Enter dihedral angle $\angle BAD=\theta_1$,Output dihedral angle 1 is $\angle CDG=\theta_4$, The output dihedral angle 2 is the complement of $\angle FGD$ and is recorded as θ_6 . The spherical six bar mechanism is regarded as a basic spherical four-bar chain $ABCD$ and spherical two-bar group EFG , and the basic coordinate system $OXYZ$ and branch coordinate system $OX_1Y_1Z_1$ are established respectively, In the basic coordinate system $OXYZ$, the X axis is perpendicular to the plane composed of OA and OD ,The Z axis coincides with the OA axis, and the Y axis is in the plane composed of OA and OD . In the branch coordinate system, the X_1 axis coincides with the X axis of the basic coordinate system, the Z_1 axis coincides with the OG axis, and the Y_1 axis is determined according to the right-hand rule [6].

Note that $\sin(\theta_i)=s\theta_i$, $\cos(\theta_i)=c\theta_i(i=1,2,\cdots 6)$, $\sin(\alpha_i)=s\alpha_i$, $\cos(\alpha_i)=c\alpha_i(i=1,2,\cdots 9)$ in the basic spherical four-bar chain, according to the motion relationship of spherical members and the condition of constant rod length, the constraint equation is

$$\overrightarrow{OB}\bullet\overrightarrow{OC}=c\alpha_3 \tag{2}$$

According to the constraint coupling between spherical four-bar mechanism and spherical two-bar group, the constraint equation between them can be obtained as

$$\begin{cases} \overrightarrow{OC} \bullet \overrightarrow{OE} = c\alpha_6 \\ \overrightarrow{OB} \bullet \overrightarrow{OE} = c\alpha_3 c\alpha_6 - s\alpha_3 s\alpha_6 c\sigma \end{cases} \tag{3}$$

The constraint equations of Stephenson-III spherical six bar mechanism can be obtained by combining equation (2) and equation group (3), as follows:

$$\begin{cases} \overrightarrow{OB} \bullet \overrightarrow{OC} = c\alpha_3 \\ \overrightarrow{OC} \bullet \overrightarrow{OE} = c\alpha_6 \\ \overrightarrow{OB} \bullet \overrightarrow{OE} = c\alpha_3 c\alpha_6 - s\alpha_3 s\alpha_6 c\sigma \end{cases} \tag{4}$$

3. Solution of constraint equations of spherical six bar mechanism

In the basic spherical four-bar chain, it is easy to know that the coordinates of fixed hinge point A and point D are $(0,0,1)^T$ and $[0, s\alpha_1, c\alpha_1]^T$ respectively. According to the geometric principle $BB_2 = OB \bullet s\alpha_2$, $\angle BB_2B_1 = \angle BAD = \theta_1$, $BB_1 = BB_2 \bullet s\theta_1$, $B_2B_1 = BB_2 \bullet c\theta_1$, At the same time, due to $\angle CC_2C_1 = \angle CDA = \pi - \theta_4$, $OB = OC = 1$, we can know that $C_1C_4 = OC_5 - C_1C_3 = c\alpha_4 s\alpha_1 + s\alpha_4 c\theta_4 c\alpha_1$, $CC_2 = s\alpha_4$, $CC_1 = s\alpha_4 s\theta_4$, $C_3C_5 = C_2C_5 + C_2C_3 = c\alpha_4 c\alpha_1 - s\alpha_4 c\theta_4 s\alpha_1$, To sum up, the coordinates of each hinge point in the basic spherical four-bar chain are

$$\begin{cases} \mathbf{A} = [0,0,1]^T \\ \mathbf{B} = [s\alpha_2 s\theta_1, s\alpha_2 c\theta_1, c\alpha_2]^T \\ \mathbf{C} = [s\alpha_4 s\theta_4, s\alpha_1 c\alpha_4 + c\alpha_1 s\alpha_4 c\theta_4, c\alpha_1 c\alpha_4 - s\alpha_1 s\alpha_4 c\theta_4]^T \\ \mathbf{D} = [0, s\alpha_1, c\alpha_1]^T \end{cases} \tag{5}$$

Referring to the literature[7], the fixed-point transformation of the spherical coordinate system can be obtained by rotating around three (fixed) coordinate axes, and the coordinate transformation matrices around the X axis and Z axis are

$$\mathbf{R}(X,\alpha_i) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\alpha_i & -s\alpha_i \\ 0 & s\alpha_i & c\alpha_i \end{pmatrix}, \mathbf{R}(Z,\theta_i) = \begin{pmatrix} c\theta_i & -s\theta_i & 0 \\ s\theta_i & c\theta_i & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{6}$$

Therefore, the coordinate transformation from the base coordinate system $OXYZ$ to the branch coordinate system $OX_1Y_1Z_1$ is the counterclockwise rotation angle $(\alpha_1 + \alpha_9)$ around the X axis, and its coordinate transformation matrix is

$$\mathbf{R}(X,(\alpha_1+\alpha_9))=\begin{pmatrix}1 & 0 & 0\\ 0 & c(\alpha_1+\alpha_9) & -s(\alpha_1+\alpha_9)\\ 0 & s(\alpha_1+\alpha_9) & c(\alpha_1+\alpha_9)\end{pmatrix} \tag{7}$$

Further, it can be obtained that the coordinate of the fixed hinge point G in the basic coordinate system $OXYZ$ is

$$\mathbf{G}=\mathbf{R}(X,(\alpha_1+\alpha_9))\bullet\mathbf{G}_1=\begin{pmatrix}1 & 0 & 0\\ 0 & c(\alpha_1+\alpha_9) & -s(\alpha_1+\alpha_9)\\ 0 & s(\alpha_1+\alpha_9) & c(\alpha_1+\alpha_9)\end{pmatrix}\bullet\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{bmatrix}0\\ -s\alpha_1 c\alpha_9-s\alpha_9 c\alpha_1\\ c\alpha_1 c\alpha_9-s\alpha_1 s\alpha_9\end{bmatrix} \tag{8}$$

Therefore, based on the displacement rotation theory [8], it can be seen that the coordinates of movable hinge points F and E in the basic coordinate system $OXYZ$ can be expressed as

$$\begin{aligned}\mathbf{F}&=\mathbf{R}(Z,\theta_6)\bullet\mathbf{R}(X,\alpha_8)\bullet\mathbf{G}\\ \mathbf{E}&=\mathbf{R}(Z,\theta_5)\bullet\mathbf{R}(X,\alpha_7)\bullet\mathbf{F}\end{aligned} \tag{9}$$

It can be obtained by calculating in the mathematical software maple. The coordinates of hinge points E , F and G in the basic coordinate system $OXYZ$ are

$$\begin{cases}\mathbf{E}=\left[a_1c\theta_5s\theta_6+a_2s\theta_5c\theta_6-a_3s\theta_5,\ a_4s\theta_5s\theta_6\text{-}a_5c\theta_5c\theta_6+a_6c\theta_5,\ a_7c\theta_6\text{-}a_8\ \right]\\ \mathbf{F}=\left[a_9\bullet s\theta_6,\ a_{10}\bullet c\theta_6,\ a_{11}\right]^T\\ \mathbf{G}=\left[\ 0,\text{-}s\alpha_1c\alpha_9-s\alpha_9c\alpha_1,c\alpha_1c\alpha_9-s\alpha_1s\alpha_9\right]^T\end{cases} \tag{10}$$

In equation (10)

$$\begin{aligned}
a_1&=c\alpha_8c\alpha_9s\alpha_1+c\alpha_8s\alpha_9c\alpha_1-s\alpha_8s\alpha_9s\alpha_1+s\alpha_8c\alpha_9s\alpha_1\\
a_2&=c\alpha_7c\alpha_8c\alpha_9s\alpha_1+c\alpha_7c\alpha_8s\alpha_9c\alpha_1-c\alpha_7s\alpha_8s\alpha_9s\alpha_1+c\alpha_7s\alpha_8c\alpha_9c\alpha_1\\
a_3&=s\alpha_7s\alpha_8c\alpha_9s\alpha_1-s\alpha_7s\alpha_8s\alpha_9c\alpha_1-s\alpha_7c\alpha_8s\alpha_9s\alpha_1+s\alpha_7c\alpha_8c\alpha_9c\alpha_1\\
a_4&=c\alpha_8c\alpha_9s\alpha_1+c\alpha_8s\alpha_9c\alpha_1-s\alpha_8s\alpha_9s\alpha_1+s\alpha_8c\alpha_9c\alpha_1\\
a_5&=c\alpha_7c\alpha_8c\alpha_9s\alpha_1-c\alpha_7c\alpha_8s\alpha_9c\alpha_1+c\alpha_7s\alpha_8s\alpha_9s\alpha_1-c\alpha_7s\alpha_8s\alpha_9c\alpha_1\\
a_6&=\mathrm{s}\alpha_7s\alpha_8c\alpha_9s\alpha_1+s\alpha_7s\alpha_8s\alpha_9c\alpha_1+s\alpha_7c\alpha_8s\alpha_9s\alpha_1
\end{aligned} \tag{11}$$

$$\begin{aligned}
a_7&=s\alpha_7s\alpha_8s\alpha_9s\alpha_1-s\alpha_7c\alpha_8c\alpha_9s\alpha_1-s\alpha_7c\alpha_8s\alpha_9c\alpha_1-s\alpha_7s\alpha_8c\alpha_9c\alpha_1\\
a_8&=c\alpha_7s\alpha_8c\alpha_9s\alpha_1-c\alpha_7s\alpha_8s\alpha_9c\alpha_1-c\alpha_7c\alpha_8s\alpha_9s\alpha_1+c\alpha_7c\alpha_8c\alpha_9c\alpha_1\\
a_9&=c\alpha_8c\alpha_9s\alpha_1+c\alpha_8s\alpha_9s\alpha_1-s\alpha_8s\alpha_9s\alpha_1+s\alpha_8c\alpha_9s\alpha_1\\
a_{10}&=s\alpha_8s\alpha_9s\alpha_1-s\alpha_8c\alpha_9s\alpha_1-c\alpha_8s\alpha_9s\alpha_1-c\alpha_8c\alpha_9s\alpha_1\\
a_{11}&=c\alpha_8c\alpha_9c\alpha_1-c\alpha_8s\alpha_9s\alpha_1-s\alpha_8s\alpha_9c\alpha_1-s\alpha_8c\alpha_9s\alpha_1
\end{aligned} \tag{12}$$

By substituting equation (5) and equation (10) into equation (4), the constraint equations of spherical six bar mechanism can be obtained as

$$\begin{cases} f_1c_4 + f_2s_4 + f_3 = 0 \\ g_1s_5s_6 + g_2c_5c_6 + g_3s_5c_6 + g_4c_5s_6 + g_5c_6 + g_6c_5 + g_7s_5 + g_8 = 0 \\ k_1s_5s_6 + k_2c_5c_6 + k_3s_5c_6 + k_4c_5s_6 - k_5c_5 + k_6s_5 + k_7c_6 + k_8 = 0 \end{cases} \tag{13}$$

In equation (13) $f_i\left(i=1,2,3\right), g_i, k_i\left(i=1,2,\cdots,8\right)$ are constant containing mechanism structure parameters and input parameters.

Make $\tan\left(\theta_4/2\right)=x_4, \tan\left(\theta_5/2\right)=x_5, \tan\left(\theta_6/2\right)=x_6$, we can get $s_4 = 2x_4/\left(1+x_4^2\right)$, $c_4=\left(1-x_4^2\right)/\left(1+x_4^2\right)$, $s_5 = 2x_5/\left(1+x_5^2\right)$, $c_5=\left(1-x_5^2\right)/\left(1+x_5^2\right)$, $s_6 = 2x_6/\left(1+x_6^2\right), c_6=\left(1-x_6^2\right)/\left(1+x_6^2\right)$. Substituting s_4, c_4, s_5, c_5, s_6, c_6 into equation (13) and simplify it, the input-output equations (14) of spherical six bar mechanism can be obtained as

$$\begin{cases} a_{11}x_4^2 + a_{12}x_4 + a_{13} = 0 \\ a_{21}x_6^2 + a_{22}x_6 + a_{23} = 0 \\ a_{31}x_6^2 + a_{32}x_6 + a_{33} = 0 \end{cases} \tag{14}$$

In equation (14)

$$\begin{aligned} &a_{11}=\mathrm{g}_3-\mathrm{g}_1 \quad a_{21}=x_5^2(k_2+k_5-k_7+k_8)+x_5(2k_6-2k_3)+(k_8-k_2-k_5-k_7) \\ &a_{12}=2\mathrm{g}_2 \quad ,a_{22}=-2k_4x_5^2+4k_1x_5+2k_4 \\ &a_{13}=\mathrm{g}_1+\mathrm{g}_3 \quad a_{23}=x_5^2(k_5+k_7+k_8-k_2)+x_5(2k_3+2k_6)+(k_2-k_5+k_7-k_8) \end{aligned} \tag{15}$$

$$\begin{aligned} &a_{31}=x_5^2(f_2-f_5-f_6+f_8)+x_5(2f_7-2f_3)+(f_8+f_6-f_5-f_2) \\ &a_{32}=-2f_4x_5^2+4f_1x_5+2f_4 \\ &a_{33}=x_5^2(f_5+f_8-f_2-f_6)+x_5(2f_3+2f_7)+(f_2+f_5+f_7+f_8) \end{aligned} \tag{16}$$

Stephenson-III spherical six bar mechanism has two output angles θ_4 and θ_6. x_4 can be obtained from the first equation of equation group (14) as

$$x_4=\frac{-a_{12}\pm\sqrt{a_{12}^2-4a_{11}a_{13}}}{2a_{11}} \tag{17}$$

And since $\tan\left(\theta_4/2\right)=x_4$,we can get

$$\theta_4 = \arctan\left(\frac{-a_{12} \pm \sqrt{a_{12}^2 - 4a_{11}a_{13}}}{2a_{11}}\right) \tag{18}$$

The sign in equation (18) corresponds to two circuits (assembly configuration) of the basic spherical four-bar chain. The second and third equations of equation group (14) can be solved by Sylvester's resultant elimination method as

$$\begin{aligned} & a_{21}^2 a_{33}^2 - a_{21}a_{22}a_{32}a_{33} - 2a_{21}a_{23}a_{31}a_{33} \\ & + a_{21}a_{23}a_{32}^2 + a_{22}^2 a_{31}a_{33} - a_{22}a_{23}a_{31}a_{32} \\ & + a_{23}^2 a_{31}^2 = 0 \end{aligned} \tag{19}$$

By substituting $a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}$ into equation (19) and simplifying it, we can get

$$d_1 x_5^8 + d_2 x_5^7 + d_3 x_5^6 + d_4 x_5^5 + d_5 x_5^4 + d_6 x_5^3 + d_7 x_5^2 + d_8 x_5 + d_9 = 0 \tag{20}$$

In equation (19), $d_i\left(i = 1, 2, \cdots, 9\right)$ is only a function of x_4 . therefore, by substituting the structural parameters of the mechanism into equations (17) and (20), 8 solutions of the positive position solution x_5 of the mechanism can be obtained. Because of $\tan\left(\theta_5 / 2\right) = x_5$, 8 solutions of the joint angle θ_5 can be obtained accordingly. It can be seen that there are at most 8 circuits (assembly configuration) of Stephenson-III spherical six bar mechanism. At the same time, it can also be obtained that the output angle θ_6 is

$$\theta_6 = 2\arctan\left(\frac{a_{21}a_{33} - a_{31}a_{23}}{a_{31}a_{22} - a_{21}a_{32}}\right) \tag{21}$$

4. Conclusion

Referring to the division method of Stephenson-III planar six bar mechanism, Stephenson-III spherical six bar mechanism is regarded as composed of basic spherical four-bar chain and spherical two-bar group. On this basis, the position analysis of Stephenson III spherical six bar mechanism is carried out, its input-output equations are obtained, and the correctness of the equations is verified by loop analysis, It not only lays a solid foundation for the kinematic analysis of this type of mechanism, but also contributes to the research of its dynamic analysis, and provides a theoretical basis for the better application of Stephenson III spherical six bar mechanism.

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