Applied Mathematics, Modeling and Computer Simulation
C. Chen (Ed.)

doi:10.3233/ATDE220014

Edge Detection Algorithm Based on Threshold Function De-Noising and Wavelet Neural Network

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Abstract. In this paper, threshold function denoising algorithm and wavelet neural network edge detection algorithm are combined to apply to image edge detection. Firstly, an improved threshold function is constructed in this paper, compared with the traditional soft and hard threshold functions and some existing improved threshold functions, the improved threshold function is adjustable and differentiable everywhere. It approximates the soft threshold function and the image at the threshold point is smoother. It can retain more true information and have an obvious effect in image de-noising. Finally, this paper presents wavelet neural network edge detection algorithm. Selecting the modulation Gaussian function wavelet as its excitation function, which is applied to extract the edge of the image after threshold de-noising. Thus, a new edge detection algorithm is proposed, which combines threshold function de-noising algorithm and wavelet neural network edge detection algorithm. The simulation results show that the edges detected by the new algorithm are clearer, contain less noise, and the continuity and accuracy are also improved.

Keywords. Threshold function, wavelet neural network, image de-noising, edge detection algorithm.

1. Introduction

The edge is the most fundamental characteristic of images. It contains some important information about the image, so it is a major basis for image analysis and segmentation. The consideration of image de-noising effect and edge detection effect is an important part of current research. Traditional Canny operator [1], Sobel operator [2] and other operators can get clear images without noise, but the ability to suppress noise is weak under the noise. Wavelet neural network has the nature of general approximation function of wavelet analysis and classification features, inherits the characteristics of multi-resolution analysis of wavelet transform. And it is gradually applied to image edge detection [3].

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In order to elevate the precision of image edge detection more usefully and reduce the impact of noise on image edge detection, pre-de-noising of images with high concentration of noise before edge extraction has become a new hotspot in image processing. There are many ways of image de-noising, such as Wavelet transform [4], Bilateral filtering [5], Curvelet transform and Contourlet transform [6], among which the wavelet threshold de-noising is easier to implement, easy to operate and has wider adaptability [7]. Therefore, this paper combine the wavelet threshold de-noising with wavelet neural network edge detection algorithm to apply to image edge detection, so as to extract image edges more accurately and clearly.

Wavelet transform is conducive to the energy compression. Therefore, wavelet transform is used to decompose the image and noise to get the wavelet coefficient, the wavelet coefficients whose absolute value of the high frequency part is less than the threshold value are taken as noise removal, and the other part is retained [8]. Finally, low frequency and high frequency coefficients are fused to obtain images [9]. However, the hard threshold function is discontinuous [10], the wavelet coefficient of the soft threshold function is biased from the wavelet coefficient of the real image to some extent. For these problems, some improvement methods have been proposed. Huang [11] proposed an improved threshold function that could adjust the denoising effect by adjusting the value of parameters. The function is continuous, but the denoising effect is not ideal. Lu [12] proposed an improved threshold function with good continuity, whose wavelet coefficient contracted less than that of the soft threshold function, but its image at the threshold point is not smooth, so the denoising effect is not ideal. A new threshold function which is derivable everywhere and adjustable is given in this paper, which gradually approximates the soft threshold function, and the image at the threshold point is smoother.

In addition, the modulation Gaussian function wavelet is selected as the excitation function of wavelet neural network to carry out edge extraction of the image denoised by the improved threshold function, not only make the edge detection algorithm have stronger ability in noise abatement, but also enhance the accuracy of edge detection. Therefore, the effective combination of the two kinds of image processing methods will also provide a reference for image research in the future.

2. The Traditional Threshold Functions

Traditional threshold functions include hard and soft threshold functions.

$$\hat{\omega}_{i,l} = \begin{cases} \omega_{i,l}, & |\omega_{i,l}| \geq S \\ 0, & |\omega_{i,l}| < S \end{cases} \tag{1}$$

And

$$\hat{\omega}_{i,l} = \begin{cases} \mathrm{sign}(\omega_{i,l})(|\omega_{i,l}| - S), & |\omega_{i,l}| \geq S \\ 0, & |\omega_{i,l}| < S \end{cases} \tag{2}$$

Where, S is the threshold, $\omega_{i,l}$ is wavelet coefficients, $\hat{\omega}_{i,l}$ is the processed wavelet coefficient after de-noising.

The threshold functions in literature [11]-[12] are all odd functions on $(-\infty,+\infty)$, differentiable on $(-\infty,-S)\cup(-S,S)\cup(S,+\infty)$ and continuous at $\omega_{i,l}=\pm S$. The expressions are in order

$$\hat{\omega}_{i,l}=\begin{cases}\operatorname{sign}(\omega_{i,l})[|\omega_{i,l}|-S(\lg(|\omega_{i,l}|^t-S^t+10))^{-1}], & |\omega_{i,l}|>S\\ 0, & |\omega_{i,l}|\le S\end{cases}$$

$$\hat{\omega}_{i,l}=\begin{cases}\operatorname{sign}(\omega_{i,l})\{|\omega_{i,l}|-T(e^{3[\alpha(|\omega_{i,l}|-S)/S]})^{-1}\}, & |\omega_{i,l}|\ge S\\ 0, & |\omega_{i,l}|<S\end{cases}$$

Where, $\operatorname{sign}(\cdot)$ is the symbolic function, S is the threshold and the thresholds are $S=\sigma\sqrt{2\ln N}\cdot i^{-1}, S=\sigma\sqrt{2\ln N}\cdot\left(\log_2(i+1)\right)^{-1}$, i is the corresponding number of decomposition layers, σ is the standard deviation of noise, N is the total number of wavelet coefficients of the image, t, $\alpha(\alpha>0)$ is the adjustment parameter, $\omega_{i,j}$ is wavelet coefficients and $\hat{\omega}_{i,l}$ is the processed wavelet coefficient after de-noising.

It can be seen that the threshold functions in literature [11]-[12] are continuous at the threshold point, and the images of the threshold functions given in these three literatures are all between soft and hard threshold functions.

3. Improved Threshold Function

Theorem 1 Let function

$$T(z)=\begin{cases}z-\operatorname{sign}(z)\alpha, & |z|>\beta\alpha\\ \operatorname{sign}(z)P(|z|)\cdot V(|z|), & \gamma\alpha<|z|\le\beta\alpha\\ 0, & |z|\le\gamma\alpha\end{cases}\tag{3}$$

Where, $\gamma+\beta=2$, $\alpha,\gamma,\beta>0$, $\beta>\gamma$, $V(\gamma\alpha)=V'(\gamma\alpha)=V'(\beta\alpha)=0$, $V(\beta\alpha)=1$, $P(\gamma\alpha)=P'(\gamma\alpha)=0$, $P(\beta\alpha)=\beta\alpha-\alpha$, $P(z)$ and $V(z)$ are sufficiently smooth on $(\gamma\alpha,\beta\alpha)\cup(-\beta\alpha,-\gamma\alpha)$, then $T(z)$ is an odd function on $(-\infty,+\infty)$ and it differentiates on $(-\infty,+\infty)$.

Prove If $|z|\le\gamma\alpha$, there is

$$T(-z)=-T(z)$$

If $\gamma\alpha<|z|\le\beta\alpha$, there is

$$T(-z)=\operatorname{sign}(-z)\cdot P(|-z|)\cdot V(|-z|)=-T(z)$$

If $|z|>\beta\alpha$, there is

$$T(-z)=-z-\operatorname{sign}(-z)\cdot\alpha=-T(z)$$

Thus $T(z)$ is an odd function on $(-\infty,+\infty)$.

And for any $P(\gamma\alpha)=P'(\gamma\alpha)=0$, $P(\beta\alpha)=\beta\alpha-\alpha$, $P'(\beta\alpha)=1$, $V(\beta\alpha)=1$, $V(\gamma\alpha)=V'(\gamma\alpha)=V'(\beta\alpha)=0$,there is

$$\lim_{z\to(\beta\alpha)^+} T(z)=\beta\alpha-\alpha$$

$$\lim_{z\to(\beta\alpha)^-} T(z)=\lim_{z\to(\beta\alpha)^-} P(z)\cdot V(z)=\beta\alpha-\alpha$$

Thus $\lim_{z\to(\beta\alpha)^+} T(z)=\lim_{z\to(\beta\alpha)^-} T(z)$. Similarly, we can get

$$\lim_{z\to(\gamma\alpha)^+} T(z)=\lim_{z\to(\gamma\alpha)^-} T(z)=0$$

$$\lim_{z\to(-\gamma\alpha)^+} T(z)=\lim_{z\to(-\gamma\alpha)^-} T(z)=0$$

Thus $T(z)$ is continuous at $z=\pm\beta\alpha$ and $z=\pm\gamma\alpha$. Because

$$T_+'(\beta\alpha)=\lim_{z\to(\beta\alpha)^+}(\mathrm{z}-\alpha-T(\beta\alpha))(\mathrm{z}-\beta\alpha)^{-1}=1,$$

$$T_-'(\beta\alpha)=\lim_{z\to(\beta\alpha)^-}(P(z)V(z)-T(\beta\alpha))(\mathrm{z}-\beta\alpha)^{-1}=1$$

Thus $T'(\beta\alpha)=T_+'(\beta\alpha)=T_-'(\beta\alpha)=1$. Similarly, we can get $T'(\gamma\alpha)=0$, $T'(-\beta\alpha)=1$, $T'(-\gamma\alpha)=0$. Thus $T(z)$ is differentiable at $z=\pm\beta\alpha$ and $z=\pm\gamma\alpha$. Pay attention to the $T(z)$ is differentiable on $(-\beta\alpha,-\gamma\alpha)\cup(\gamma\alpha,\beta\alpha)$. In summary, odd function $T(z)$ is differentiable on $(-\infty,+\infty)$.

A class of improved threshold functions is constructed according to theorem 1.

$$\hat{\omega}_{i,l}=\begin{cases}\omega_{i,l}-\mathrm{sign}(\omega_{i,l})\cdot S, & |\omega_{i,l}|>\beta S\\ \mathrm{sign}(\omega_{i,l})P(|\omega_{i,l}|)\cdot V(|\omega_{i,l}|), & \gamma S<|\omega_{i,l}|\le\beta S\\ 0, & |\omega_{i,l}|\le\gamma S\end{cases} \tag{4}$$

Where, γ and β are adjustment parameters and $\gamma+\beta=2$, $\gamma>0$, $\beta>0$, $\gamma<\beta$, $\omega_{i,l}$ is wavelet coefficients and $\hat{\omega}_{i,l}$ is the processed wavelet coefficient after de-noising. S is the threshold and the thresholds is $S=\sigma\sqrt{2\ln N}(i+1)^{-1}$, i is the corresponding number of decomposition layers, σ is the standard deviation of noise, N is the total number of wavelet coefficients of the image.

To facilitate the simulation experiment, we select

$$g(z)=\begin{cases}e^{-z^{-2}}, & z\neq 0\\ 0 & z=0\end{cases}$$

$$V(z)=g(z-\gamma\alpha)\cdot[g(z-\gamma\alpha)+g(\beta\alpha-z)]^{-1}$$

$$P(z)=\left(2\alpha(\beta-\gamma)\right)^{-1}\cdot z^2-\gamma\left(\beta-\gamma\right)^{-1}\cdot z+\gamma^2\alpha(2(\beta-\gamma))^{-1}$$

The image of the improved threshold functions is shown in figure 1.

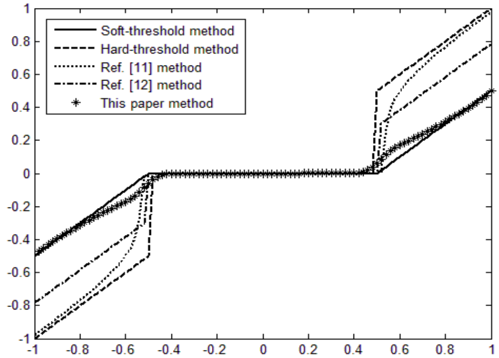
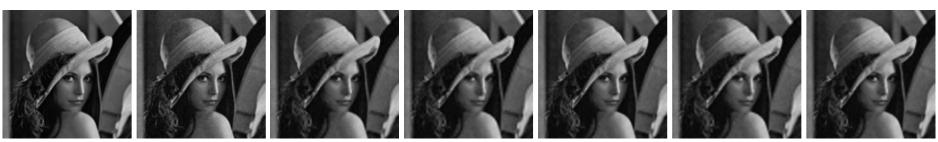


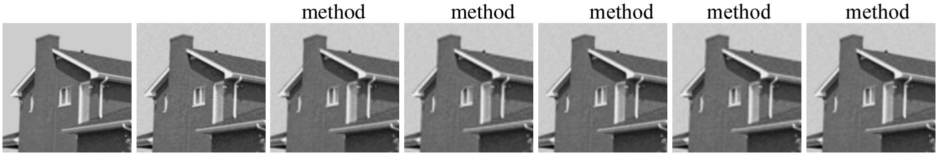
Figure 1. The improved threshold functions in this paper

4. Image De-Noising Method of Improved Threshold Function

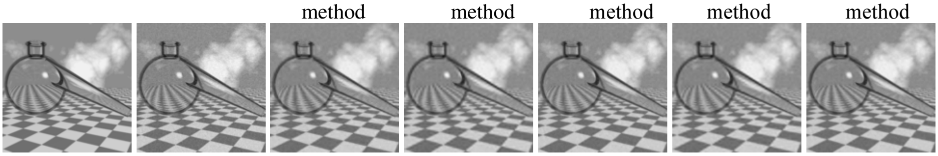
In this paper, Gaussian noise with variance of 20 is added to Lena, House and Bottle respectively to carry out simulation experiments with Matlab. This paper have established objective evaluation indexes PSNR, MSE, SSIM, correlation coefficient and degree of distortion to verify the superiority of the method, thus to avoid some subjective errors caused by the visual characteristics of human eyes. The simulation results are shown in figure 2 and table 1. According to the results in table 1, the improved threshold function is applied to denoise the image, and the values of PSNR, SSIM and correlation coefficient are larger, while the values of MSE and distortion degree are smaller. Therefore, the improved new threshold function has better denoising effect, and the image distortion after denoising is smaller and more close to the original image.



(a)Lena (b)Noisy image (c)Hard-threshold method (d)Soft-threshold method (e)Ref. [11] method (f)Ref. [12] method (g).This paper method



(a)House (b)Noisy image (c)Hard-threshold method (d)Soft-threshold method (e)Ref. [11] method (f)Ref. [12] method (g).This paper method



(a)Bottle (b)Noisy image (c)Hard-threshold method (d)Soft-threshold method (e)Ref. [11] method (f)Ref. [12] method (g).This paper method

Figure 2. Lena, House and Bottle images threshold de-noising results

Table 1. Objective evaluation results of Lena, House and Bottle

Image	De-noising method	Objective evaluation results				
		MSE (db)	PSNR (db)	SSIM	Correlation coefficient	Degree of distortion
Lena	Hard-threshold	10.3536	27.8290	0.9792	0.9794	7.5809
	Soft-threshold	10.3768	27.8095	0.9787	0.9796	7.1277
	Ref. [11] method	8.7874	29.2536	0.9850	0.9853	6.7621
	Ref. [12] method	10.3474	27.8342	0.9788	0.9797	7.0627
	This paper method	8.6355	29.4051	0.9855	0.9858	6.6270
House	Hard-threshold	8.6923	29.3481	0.9888	0.9887	6.4331
	Soft-threshold	7.9118	30.1653	0.9906	0.9906	5.7642
	Ref. [11] method	7.8599	30.2224	0.9908	0.9908	5.7898
	Ref. [12] method	7.5515	30.5701	0.9914	0.9914	5.6684
	This paper method	7.2382	30.9382	0.9922	0.9921	5.5527
Bottle	Hard-threshold	11.8756	26.6377	0.9617	0.9612	8.3864
	Soft-threshold	11.9815	26.5606	0.9590	0.9604	7.7381
	Ref. [11] method	11.2060	27.1418	0.9658	0.9654	8.0965
	Ref. [12] method	11.9726	26.5670	0.9591	0.9605	7.5405
	This paper method	10.8358	27.4336	0.9677	0.9675	7.4410

5. Fusion Algorithm

The edge detection algorithm flow of wavelet neural network is shown in figure 3.

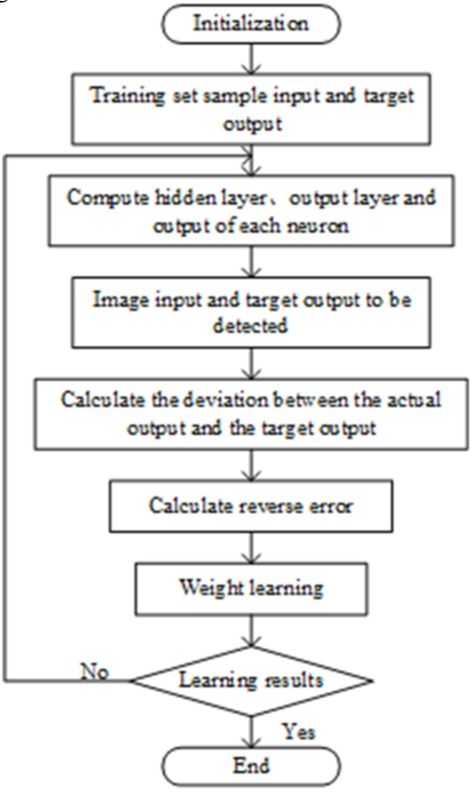


Figure 3. Wavelet neural network training process

The specific flow chart of the image edge detection algorithm combining the improved threshold de-noising algorithm with wavelet neural network edge detection algorithm is as follows in figure 4.

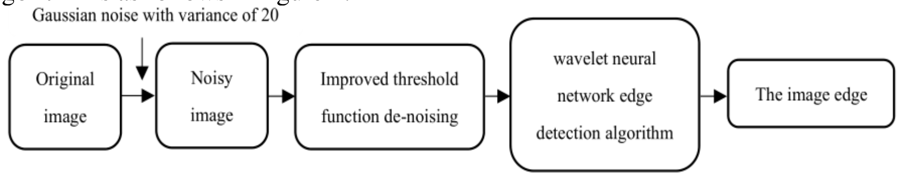
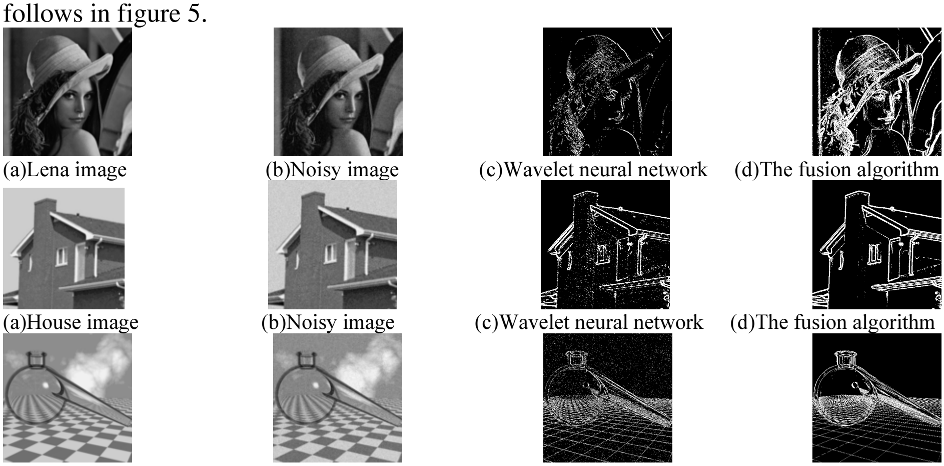


Figure 4. The fusion image edge dedection algorithm

6. The Simulation Results

For checking the effectiveness of the de-noising method in noise image edge detection, this paper selects Lena, House and Bottle images to test. The simulation results are as follows in figure 5.



(a)Lena image (b)Noisy image (c)Wavelet neural network (d)The fusion algorithm

(a)House image (b)Noisy image (c)Wavelet neural network (d)The fusion algorithm

(a)Bottle image (b)Noisy image (c)Wavelet neural network (d)The fusion algorithm

Figure 5. Lena, House and Bottle images edge detection results

Table 2. Objective evaluation results

Algorithm	**Meangradient**			**Informationentropy**		
	Lena	House	Bottle	Lena	House	Bottle
Wavelet neural network	33.0510	10.3265	33.9613	0.6124	0.4171	0.5371
The fusion algorithm	35.2438	11.8785	37.7758	0.7922	0.4879	0.7260

The experimental results in table 2 show that by using the modulation Gaussian wavelet as the excitation function of the wavelet neural network, the image edges which are denoised by the improved threshold function are extracted, and the image edges which are smoother, less noisy and have better continuity are obtained.

7. Conclusion

In this paper, a threshold function is proposed, which is adjustable and differentiable everywhere, to make the image at the threshold point smoother. Then, the edge detection algorithm of wavelet neural network is used to extract the edge of the image denoised by the threshold function, and the modulated Gaussian wavelet is used as the excitation function of wavelet neural network.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (Grant NO.51875142) and (Grant NO. 11871181).

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