

Research on the Inversion Method and Application of Random Mechanical Parameters for Arch Dam and Its Nearby Mountain Structure

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Abstract. First, prior distribution of the mechanical parameters of arch dam body and its nearby mountain rock mass was determined according to the meso-modelling of dam concrete and test data of its nearby mountain rock mass, and a finite element model of the arch dam and its nearby mountain was established. Based on Bayesian theory, random inversion objective function was established. Afterwards, inversion analysis of mechanical parameters was conducted with Differential Evolution Adaptive Metropolis (DREAM_(sz)) on the basis of Markov chain random theory, for the inversion of random distribution parameters of the elasticity modulus of dam concrete and overall deformation modulus. This method was applied to some high-arch dam project, and it was verified to be effective by the results.

Keywords. Arch dam and its nearby mountain structure, random parameters, inversion method, bayesian theory

1. Introduction

Monitoring effect quantity as well as mechanical parameters of arch dam and its nearby mountain structure are uncertain. By taking various uncertain variables as random ones, determining their distributed parameters and establishing a random inversion model, it can effectively improve the inversion precision of the mechanical parameters of arch dam and its nearby mountain structure. Bayesian method is one of the important approaches of the inversion analysis of random mechanical parameters, and it has

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already been widely applied in the inversion of mechanical parameters of dam concrete and rock-soil. Marjoram et al. [1] first proposed the Markov Chain Monte Carlo method without using the likelihood function since it is difficult to be determined; Daniel et al. [2] increased the computing efficiency of this method by regulating the tolerance of Markov Chain; Zhang et al. [3] inverted random parameters of the landslide mass of some expressway, such as anchor force of sliding surface, shearing strength, etc. with MCMC (Markov Chain Monte Carlo) Bayesian inversion method; Zhang et al. [4] inverted permeability coefficient of unsaturated soil and other random parameters of a slope with Bayesian method, on the basis of pore water pressure monitoring data; Li et al. [5] inverted the deviation factor and rock mass elasticity modulus of the analysis model with multi-step Bayesian update method based on the incremental deformation monitoring data of a high slope.

In actual projects, elasticity modulus of concrete and overall deformation modulus of rock mass have a huge impact on the change of arch dam and nearby mountain structure. Therefore, this paper mainly took the elasticity modulus of dam concrete and overall deformation modulus of nearby mountain rock mass as random variables, and conducted the inversion to determine the probability distribution of above-stated random mechanical parameters, since probability distribution could be described with the mean value, variance and other characteristic value. This paper mainly studied the inversion of random mechanical parameters of arch dam and nearby mountain structure with Bayesian principle and random finite element method on the basis of the in-situ monitoring data.

2. Inversion Mode of Random Mechanical Parameters of Arch Dam and Nearby Mountain Structure

According to the dam safety monitoring theory, deformation component sequence for the inversion of random mechanical parameters of the structure could be obtained by deducting the temperature-dependent component $\delta_T(x, y, z)$ and time-dependent component $\delta_\theta(x, y, z)$ from the measured deformation of any point (x, y, z) in the arch dam and its nearby mountain structure, as shown below:

$$u(x, y, z) = \delta(x, y, z) - \delta_T(x, y, z) - \delta_\theta(x, y, z) = f_{st}(s, H, x, y, z) \quad (1)$$

Where u is a random water pressure component of deformation, which is a random variable; s is a random parameter to be inverted; H is hydraulic load; f_{st} is a random finite element derived function.

The random finite element governing equation is:

$$\{K\}\{u^*\} = \{R_H\} \quad (2)$$

Where, $\{K\}$ is the overall stiffness matrix; $\{u^*\}$ is the structural node deformation matrix under the impact of water pressure; $\{R_H\}$ is hydraulic load matrix, and here $\{K\}$, $\{u^*\}$ and $\{R_H\}$ are all random variables.

In random finite element analysis, the overall stiffness matrix $\{K\}$ was decomposed into two parts, namely the overall stiffness matrix $\{K_0\}$ at the mean value of elasticity modulus of all units and its fluctuation $\{\Delta K\}$, namely $\{K\} = \{K_0\} + \{\Delta K\}$. Neumann expansion [6, 7] was conducted for the inverse matrix $\{K\}^{-1}$ of overall stiffness matrix, $\{\Delta K\}$ satisfying:

$$\{\Delta K\} = F(\varepsilon_x, \theta, f(\mu), L, S) \tag{3}$$

Where, ε_x is the error term of elasticity parameters; θ is a parameter reflecting the spatial correlation; $f(\mu)$, L and S reflect the unit size of calculation area and restricted variables respectively.

For the inversion of random parameters, this paper introduced the Bayesian inversion model, which deemed the mechanical parameter s to be inverted as a random variable with some prior probability density $p(s)$, and then, other measured variable u which was associated with s was analyzed to extract the information of parameter s from the input and output data. The information could be inferred by calculating the posterior probability density function $p(s|u)$ of parameter s , and the mutual relationship was related by the Bayesian model, in the following expression:

$$p(s|u) = \frac{p(u|s)p(s)}{p(u)} \propto p(u|s)p(s) \tag{4}$$

Where, $p(u)$ is a constant, and $p(u|s)$ is likelihood function, denoted as $L(s)$.

The relationship between u and finite element simulation value $u^*(s)$ was:

$$u = u^*(s) + \varepsilon_H \tag{5}$$

Where, $u^*(s)$ is the calculated value matrix obtained on the basis of random finite element calculation; ε_H is the error matrix of monitoring effect quantity, which is supposed to be in normal distribution with the mean value of 0, namely $\varepsilon_H \sim (0, \text{cov}(\varepsilon_H))$, where $\text{cov}(\cdot)$ is the covariance matrix. According to the above assumptions, the likelihood function could be obtained:

$$L(s) = p(u|s) = \frac{1}{(2\pi)^{n/2} |\text{cov}(\varepsilon_H)|^{1/2}} \exp\left[-\frac{1}{2}(u - u^*)^T \text{cov}^{-1}(\varepsilon_H)(u - u^*)\right] \tag{6}$$

Suppose the probability density distribution function of parameter matrix s to be inverted is $f(s)$ and $\text{cov}(s, \varepsilon_H) = 0$, the expression of posterior distribution could be obtained:

$$P_D(s) = L(s)f(s) = \frac{f(s)}{(2\pi)^{N/2} |\text{cov}(\varepsilon_H)|^{1/2}} \exp\left[-\frac{1}{2}(u-u^*)^T \text{cov}^{-1}(\varepsilon_H)(u-u^*)\right] \quad (7)$$

If the prior distribution $f(s)$ also conforms to normal distribution, namely $s \sim (\mu_s, \text{cov}(s))$, the prior distribution $f(s)$ and posterior distribution $P_D(s)$ are conjugate function. The posterior distribution of mechanical parameters can be worked out directly through analysis, but in actual project, there are numerous parameters to be inverted, and prior distribution does not usually conform to the normal distribution or satisfy the conjugate condition, so posterior distribution shall be worked out through random sampling and numerical simulation. According to equation (7), prior distribution of random mechanical parameters and spatial correlation have significant impacts on the parameter estimation results. Next, methods of determining the prior information will be studied.

3. Determination of the Prior Information of Random Mechanical Parameters

It can be learnt from expression (7) inferred according to the posterior probability density distribution of random mechanical parameters that prior information of parameter x shall be determined first in order to decide the posterior distribution of the mechanical parameters of arch dam and its nearby mountain structure, namely probability density function of mechanical parameters of dam concrete and rock mass, as well as related distance. Next, the determination methods would be studied respectively.

(1) Prior distribution of elasticity modulus of dam

Dam concrete is a multi-phase composite material composed of aggregate and mortar layer, but different components differ greatly in mechanical properties, which results in the randomness of mechanical parameters of concrete. In this paper, random mechanical parameters of the dam were taken as the elasticity modulus, and meso-modelling of dam concrete and macro-mechanical property equivalent method were studied from the perspective of meso-composition of dam concrete, to explore the statistical characteristics of elasticity modulus.

Size distribution and geometry of aggregates play a significant role in the meso-modelling of concrete. A large number of studies have shown that Fuller-grading may contribute to the outstanding density and macroscopic strength of dam concrete. In this paper, it was supposed that aggregate distribution satisfied Fuller-grading curve, and its expression was:

$$p(d) = 100 \left(\frac{d}{d_{\max}} \right)^n \quad (8)$$

Where, $p(d)$ is the cumulative volume fraction of aggregates whose particle size is smaller than d ; d_{\max} is the maximum aggregate size; n is a parameters, ranging between 0.45 and 0.70.

The above-stated Fuller grading curve [8] is a 3D grading curve of aggregate in concrete. To simplify the calculation, meso simulation calculation of concrete material was conducted on a 2D plane, which required the 2D grading curve converted from 3D grading curve. According to statistical rules, Walraven J. C gave the conversion formula for Fuller grading curve [9] as below:

$$P_2(d > D) = 100 \left(1 + \frac{1}{2} \varepsilon^2 \right) (1 - \varepsilon^2)^{\frac{1}{2}} - \int_{\varepsilon}^1 f(\varepsilon^*) W(\varepsilon, \varepsilon^*) d\varepsilon^* \quad (9)$$

Where, $P_2(d > D)$ is the probability that aggregate size d is over D ; $\varepsilon = \frac{D}{D_{\max}}$ is the

dimensionless aggregate size; ε^* is integration variable of ε ; $f(\varepsilon^*)$ is 3D grading curve; $W(\varepsilon, \varepsilon^*) = \frac{3}{2} \left(\frac{\varepsilon^4}{\varepsilon^{*4}} \right) \frac{1}{(\varepsilon^{*2} - \varepsilon^2)^{\frac{1}{2}}}$.

According to test code for hydraulic concrete [10], the size of standard 2D grading concrete specimen is 150×150×150 (mm), while that of 3D specimen is 300×300×300 (mm), and 4D 450×450×450 (mm). Aggregate size scope and particles of different-grading standard concrete specimen are shown in table 1.

Table 1. Aggregate size range and particles of standard concrete specimen of distinct grading.

Equivalent size (mm)	80~120	40~80	20~40	10~20	5~10
Aggregate particles of 2D grading specimen	0	0	6	0	56
Aggregate particles of 3D grading specimen	0	6	21	102	0
Aggregate particles of 4D grading specimen	3	2	4	160	0

On the basis of Python language, 2D random aggregate modelling program was written, and 2D random aggregate model generated was shown in figure 1. Element equivalence was conducted for the macro mechanical parameters using Voigt parallel model, and the macro-equivalent mechanical parameter of the element E_{eq} was expressed as:

$$E_{eq} = R_{ag} E_{ag} + R_{mo} E_{mo} \quad (10)$$

Where, E_{ag} and E_{mo} are the respective mechanical parameters of aggregate and mortar; R_{ag} and R_{mo} are the respective volume fractions of aggregate and mortar;

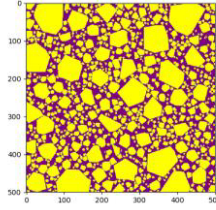


Figure 1. Random aggregate model of concrete.

Macro element equivalent chart is shown in figure 2. Different colors represent different assignments of macro-equivalent elasticity modulus.

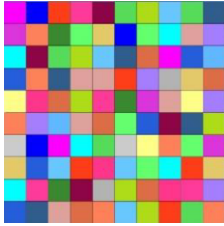


Figure 2. Macro-mechanical property equivalence (100 elements)

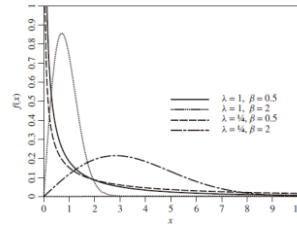


Figure 3. Weibull distribution.

Random distribution of macro elasticity modulus of concrete was described with Weibull distribution. Figure 3 gave the probability density function curve in Weibull distribution, and the expression of probability density function of two-parameter Weibull distribution function was:

$$f(x) = \frac{\beta}{x} (\lambda x)^\beta \exp[-(\lambda x)^\beta] \tag{11}$$

The expression of cumulative probability distribution function was:

$$1 - F(x) = \exp[-(\lambda x)^\beta] \tag{12}$$

Taking the double logarithm on both sides of equation (12), and:

$$\ln[-\ln(1 - F(x))] = \beta \ln x - \beta \ln \lambda \tag{13}$$

A scatter diagram was drawn with $\ln x$ as the horizontal ordinate X and $\ln[-\ln(1 - F(x))]$ as the longitudinal ordinate Y , and linear function $Y = bX + a$ was adopted for fitting, and here $a = -\beta \ln \lambda$, $b = \beta$. Weibull distribution parameters β and λ could be worked out with the least squares fitting. Weibull distribution parameters could be calculated with graphic method. In figure 4, it is the histogram of the equivalent elasticity modulus of three-graded concrete element.

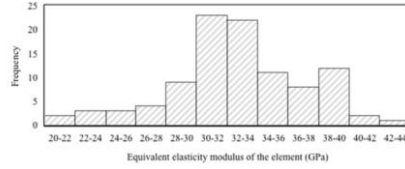


Figure 4. Histogram of the macro equivalent elasticity modulus of three-graded concrete unit (100 elements).

Numerical characteristic calculation formula of Weibull distribution is as follows:

$$\text{Mean value: } E(x) = \lambda \Gamma\left(1 + \frac{1}{\beta}\right) \tag{14}$$

$$\text{Variance } D(x) = \lambda^2 \left[\Gamma\left(1 + \frac{2}{\beta}\right) - \left(\Gamma\left(1 + \frac{1}{\beta}\right) \right)^2 \right] \tag{15}$$

In conclusion, random parameters of macro elasticity modulus of dam concrete were achieved as follows: first meso modelling and macro element equivalence were conducted according to the grading level of concrete, and then mechanical parameters of the macro equivalent element were fitted with Weibull distribution. On this basis, the prior distribution of the elasticity modulus of dam concrete could be obtained.

(2) Prior distribution of the overall deformation modulus of rock mass

The overall deformation modulus of rock mass was random. According to the Unified Standard for Reliability Design of Hydraulic Engineering Structures (GB50199-2013) [11], the distribution model of overall deformation modulus of rock mass shall be described with log-normal distribution, and the probability distribution density function is:

$$f(x) = \frac{1}{x\zeta\sqrt{2\pi}} \exp\left[-\frac{(\ln x - \lambda)^2}{2\zeta^2} \right] \tag{16}$$

The numerical characteristic calculation formula is as follows:

$$\text{mean value: } E(x) = \exp\left(\lambda + \frac{\zeta^2}{2}\right) \tag{17}$$

$$\text{variance: } D(x) = \exp(2\lambda + \zeta^2) [\exp(\zeta^2) - 1] \tag{18}$$

Where, λ is the mean value of $\ln x$, and ζ is the standard deviation of $\ln x$.

If a large number of test data are available, statistical analysis can be conducted to get the prior distribution of the overall deformation modulus of rock mass. If the test data are limited, the overall deformation modulus obtained by nob-random inversion can be taken as the prior mean value. On this basis, parameters of the prior probability distribution density function of overall deformation modulus could be settled, to get the prior distribution of the overall deformation modulus of rock mass.

(3) Spatial correlation of random mechanical parameters of dam and its nearby mountain structure

Random mechanical parameters of dam and nearby mountain structure have spatial correlation. During the random simulation of finite element, if the correlation is ignored, the random simulation may be inaccurate. Therefore, parameters must be introduced to describe the spatial correlation among random mechanical parameters and invert the parameter value of spatial correlation.

According to the random field theory, the random field is claimed to have strict stationarity. In actual projects, it is usually difficult to determine the joint probability density function of mechanical parameters of materials, so it is necessary to introduce another definition of stationary random field. Suppose random field $Z(\mathbf{X})$ is a two-order moment field. If it satisfies: (1) for any $X_i \in \mathbf{X}$, $E[Z(X_i)] = m$ (constant); (2) for any $X_i \in \mathbf{X}$, correlation function of $Z(X_i)$, namely $\rho(X_i, X_i + h)$, has nothing with X_i , and it can be merely expressed as the function of h and $\rho(h)$, and the random field is claimed to have wide stability. Accordingly, the random field is called wide or weak stationary field.

In this paper, based on the assumption of wide stationary random field, the spatial correlation between random mechanical parameters of arch dam and its nearby mountain structure could be expressed by a correlation function $\rho(h)$:

$$\rho(h) = \rho(\Delta x, \Delta y, \Delta z) = \exp(-\pi[(\frac{\Delta x}{\theta_x})^2 + (\frac{\Delta y}{\theta_y})^2 + (\frac{\Delta z}{\theta_z})^2]) \tag{19}$$

Where, θ_x , θ_y and θ_z are the correlation distance in x , y and z axis directly, for representing the relevance of spatial correlation between mechanical parameters of rock mass; $h = (\Delta x, \Delta y, \Delta z)$ is the distance between two points, namely (x_1, y_1, z_1) and (x_2, y_2, z_2) , satisfying $\Delta x = x_1 - x_2$, $\Delta y = y_1 - y_2$, $\Delta z = z_1 - z_2$.

So far, prior distribution of the random mechanical parameters of arch dam and its nearby mountain structure $f(\mathbf{x})$ can be represented by several parameters, such as distribution parameter, correlation distance, etc.

4. Inversion Method Flow of Random Mechanical Parameters

According to the previous analysis, prior distribution and posterior distribution of random mechanical parameters for inversion fail to satisfy the conjugate conditions, and it is difficult to be analyzed for solving the posterior distribution, so numerical calculation is required for the max estimation of posterior parameters. Aiming at this problem, this paper adopted the DREAM (sz) [12, 13] method for posterior distribution estimation on the basis of Markov chain random theory, and the basic principle was as follows:

Markov chain is a random discrete process and suppose a group of random sequence satisfies the following definition:

$$P(x_t | \dots, x_{t-2}, x_{t-1}) = P(x_t | x_{t-1}) \tag{20}$$

The sequence is in Markov state, namely the conditional probability at x_t moment only depends on the state at x_{t-1} moment. Markov sequence [14-16] has convergence, that is, Markov chain that satisfies certain conditions would converge to stationary distribution. Spatial sampling can be conducted by virtue of this nature according to a certain acceptance – rejection probability. When Markov chain reached stationary distribution, Monte-Carlo simulation can be conducted with steadily distributed samples, to solve the posterior distribution [17] of parameters.

First, it generates a $N \times n$ dimension sample matrix \mathbf{X} , containing N different Markov chains $\{x^i, i=1, 2, \dots, N\}$, n is the number of parameters for inversion. It establishes Matrix \mathbf{Z} according to the present and past state of the chain, and the expression of the t^{th} candidate sample z^i of the i^{th} Markov chain is:

$$z^i = x_{t-1}^i + (1 + e_n)\gamma(\delta, n') \left[\sum_{j=1}^{\delta} \mathbf{Z}^{r_1(j)} - \sum_{k=1}^{\delta} \mathbf{Z}^{r_2(k)} \right] + \varepsilon_n \tag{21}$$

Where, x_{t-1}^i is the $t-1^{\text{th}}$ sample of the i^{th} Markov chain; e_n is a uniformly distributed random number, $e_n \sim U(-b, b)$; $\gamma(\delta, n')$ is a scale factor related to parameter δ and n' , δ is the number of parallel Markov chains that generate the candidate sample z^i , n' is the number of updated parameters during the generation of candidate sample z^i ; $\mathbf{Z}^{r_1(j)}$ and $\mathbf{Z}^{r_2(k)}$ represent the $r_1(j)^{\text{th}}$ and $r_2(k)^{\text{th}}$ sample randomly extracted from matrix \mathbf{Z} , $r_1(j) \neq r_2(k)$, $j, k = 1, 2, \dots, \delta$; ε_n is a random number, conforming to the normal distribution, $\varepsilon_n \sim (0, b^*)$. The suggested value of equation (21) is as follows: $N = 3 \sim 5$, $\delta = 1$, $b = 0.05$, $b^* = 1 \times 10^{-12}$, $\gamma = 2.38 / \sqrt{2\delta n'}$, the scale factor $\gamma = 1.0$ is reset every five times of random sampling; after generating the candidate sample z^i according to the current location of the chain x_{t-1}^i , it will decide whether z^i would be taken as the location of the next sample x_t^i according to the acceptance rate, and the expression of $\alpha(x_{t-1}^i, z^i)$ is:

$$\alpha(x_{t-1}^i, z^i) = \begin{cases} \min\{1.0, \frac{P_D(z^i)}{P_D(x_{t-1}^i)}\} & P_D(x_{t-1}^i) > 0 \\ 1.0 & P_D(x_{t-1}^i) = 0 \end{cases} \tag{22}$$

Where, $L(\cdot)$ is likelihood function, and the criterion is: if $\alpha(x_{t-1}^i, z^i) \geq u$, z^i is accepted as the next sample point $x_t^i = z^i$, or $x_t^i = x_{t-1}^i$, in which u is a random number evenly distributed in interval $[0, 1]$, namely $u \sim U(0, 1)$.

To improve the random sampling efficiency, adaptive random sampling strategy is introduced, and crossover probability CR can be used to revise the parameter n' , CR can adjust adaptively with the moving of Markov chain, the initial value of n' is n , and then n' decreases with the increase of sample x_{t-1}^i accepted by candidate sample z^i , the expression is:

$$z_j^i = \begin{cases} x_{j,t-1}^i & u \leq 1 - CR \\ z_j^i & u > 1 - CR \end{cases} \quad (23)$$

Gelman-Rubin convergence diagnostic value [18] is adopted as the criterion of convergence of Markov chain. When diagnostic value R satisfies $R < 1.2$, it can be considered that Markov chain reaches the state of convergence.

The specific steps for the inversion of random mechanical parameters of arch dam and nearby mountain structure are as follows:

Step 1: based on the measured data of the deformation of arch dam and its nearby mountain structure, random water pressure component sequence matrix \mathbf{u} was established according to equation (1), for the uncertainty analysis, to get the distribution of error $\boldsymbol{\varepsilon}_H \sim (0, \text{cov}(\boldsymbol{\varepsilon}_H))$;

Step 2: prior distribution of random mechanical parameters of arch dam and nearby mountain structure $f(x)$ was obtained, and the number of parameters for inversion n , the number of Markov chain N and calculation steps T were determined. According to the prior distribution $f(x)$, FFT random field simulation method was applied for N random sampling to generate the initial Markov chain matrix \mathbf{X} and M random sampling to generate matrix $\mathbf{Z} = \{x_j^k\} (k = 1, 2, \dots, M; j = 1, 2, \dots, n)$, $M = 10n$;

Step 3: aiming at the t^{th} sample on the i^{th} Markov chain in sample matrix \mathbf{X} , random sample z^i was generated according to equation (21);

Step 4: stiffness matrix $\{\Delta K\}$ was generated accord to random samples, and matrix \mathbf{u}^* for calculating the deformation of arch dam and its nearby mountain structure is obtained with the random finite element calculation, which can be substituted in equation (7) for the calculation of posterior probability density function value $P_D(z^i)$ and $P_D(x_{t-1}^i)$. And then, the acceptance probability $\alpha(x_{t-1}^i, z^i)$ of candidate sample z^i was calculated according to equation (22), to generate a random number u in interval $[0, 1]$. If $\alpha(x_{t-1}^i, z^i) \geq u$, sample point $x_t^i = z^i$ was accepted, or $x_t^i = x_{t-1}^i$;

Step 5: for N Markov chains, N groups of samples were added into matrix \mathbf{Z} whenever ten samples were generated, and the number of samples of matrix \mathbf{Z} was updated to $M + N$;

Step 6: when N parallel Markov chains satisfy Gelman-Rubin convergence diagnosis condition $R < 1.2$ or when Markov chain moves to length T , $10n$ groups of samples in \mathbf{Z} were deleted, and the probability distribution of random mechanical samples of arch dam and its nearby mountain structure can be inferred according to the remaining samples in \mathbf{Z} .

5. Engineering Example

The main hydraulic structure of some hydropower station is composed of concrete arch dam, plunge pool behind the dam and subsidiary dam, spillway tunnel on the right bank and underground powerhouse in the middle of right bank, etc. The dam crest height is

1885.0m, the maximum dam height is 305.0m. Figure 5 is the 3D finite element model of arch dam and its nearby mountain structure.

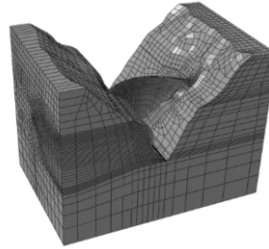


Figure 5. 3D FEM of arch dam and its nearby mountain structure.

First, prior information of the elasticity parameters of dam concrete and rock mass was determined according to the engineering design and test data. Prior distribution probability density function of the elasticity modulus of dam was described with Weibull distribution. Prior distribution probability density function of the overall deformation modulus of nearby mountain was described with log-normal distribution, and spatial correlation was described with correlation distance. Since the correlation distance of the elasticity modulus of dam concrete was relatively small, relative to the dam structural size, the spatial correlation had a small impact on the posterior distribution, so it was not taken into account in this example. The prior information of all structural area was shown in table 2.

Table 2. Prior information of overall deformation modulus parameters of near-dam mountain.

Area	λ	ζ	Expected value (GPa)	Standard deviation (GPa)	Correlation distance (m)
Dam	34.5	7.4	32.4	3.16	/
Nearby mountain	16.9	4.7	15.5	3.7	50

3D finite element model of arch dam and its nearby mountain structure was established. For the random sampling of mechanical parameters, a structural area for inversion was divided into several subareas. The elasticity parameters of different subareas within the same structural area share the same distribution function, and there was also spatial correlation. The spatial distance was represented by the distance to geometric center of the sub-area. The Markov chain process line of the elasticity modulus of dam is shown in figure 6. According to the prior distribution of elasticity parameters, FFT method was applied for the sampling of Markov chain.

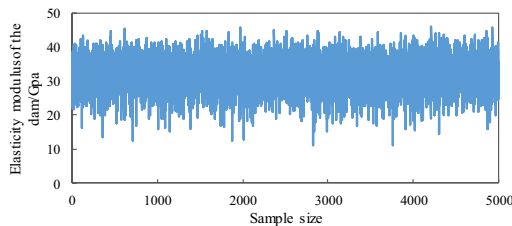


Figure 6. Markov chain process line of the electricity modulus of dam.

The number of parallel Markov chain $N = 5$, length $T = 1000$. Posterior distribution inference method proposed in this paper was applied for parameter inference. Figure 7 was the changing curve of convergence diagnostic value with five samples, and the posterior distribution statistics of the elasticity modulus of dam and overall deformation modulus of nearby mountain were shown in table 3.

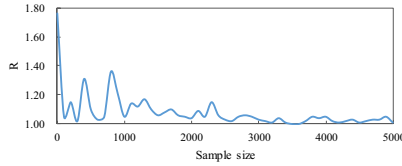


Figure 7. Changing curve of diagnostic value of convenience with the total number of samples.

Table 3. Statistical results of the posterior distribution of random mechanical parameters.

Subarea	Mean value (GPa)	Standard deviation (GPa)	Correlation distance (m)
Dam	31.8	2.31	/
Nearby mountain	13.2	2.38	51

Figure 8 was the comparison diagram of calculated value and measured value of the displacement incremental of survey points, from which it was clear that the calculated value matched the measured value perfectly, and verified the effectiveness of the inversion method proposed in this paper.

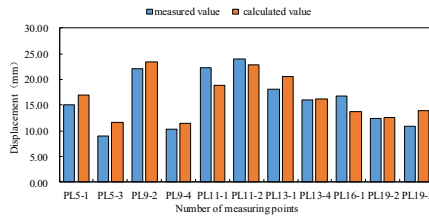


Figure 8. Comparison diagram of the calculated value and measured value of radial displacement increment of the dam.

6. Conclusion

Based on Bayesian theory and Markov chain random theory, the inversion of elasticity modulus of dam concrete and overall deformation modulus of rock mass of the arch dam and nearby mountain is put forward in this paper. The inversion method takes the randomness of physical and chemical parameters into account, which is closer to engineering practice. Through the inversion of engineering project, the mean value and standard deviation of the elasticity modulus of dam, as well as the mean value, standard deviation and correlation distance of the overall deformation modulus of nearby mountain are obtained, which verify the effectiveness of the inversion method.

Acknowledgements

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