Advances in Manufacturing Technology XXXIV M. Shafik and K. Case (Eds.) © 2021 The authors and IOS Press. This article is published online with Open Access by IOS Press and distributed under the terms of the Creative Commons Attribution Non-Commercial License 4.0 (CC BY-NC 4.0). doi:10.3233/ATDE210045

Multimodal Biometric Recognition Using Iris and Face Features

Sulaiman ALSHEBLI^{*a*,1}, Fatih KURUGOLLU^a and Mahmoud SHAFIK^{a,2} ^a University of Derby, Derby, United Kingdom

Abstract. Multimodal biometrics has recently gained interest over single biometric modalities. This interest stems from the fact that this technique offers improvements in recognition and more security. In this ongoing research programme, we propose a new feature extraction technique for a biometric system based on face and iris recognition. The extraction of iris and facial features is performed using the Discrete Wavelet Transform combined with the Singular Value Decomposition. Merging the relevant characteristics of the two modalities is used to create a pattern for each individual in the dataset. The evaluation process is performed using two datasets (i.e., Faces94 Faces dataset and IIT Delhi Iris dataset). The experimental results carried out in this programme showed the robustness of the proposed technique.

Keywords. Face, Iris, Multimodal Biometrics, recognition, DWT.

1. Introduction

Biometric systems are tools used to automatically and accurately recognizing people. This recognition is usually based on unique physiological characteristics such as iris, face, and fingerprints, or behavioural characteristics such as signature and gait [1]. Nowadays, the growing demand for reliable and secure recognition systems, used in many fields, proves that more attention needs to be paid to biometrics. These systems are used in many areas such as military, banking, passport verification, airports, buildings, mobile phones, and identity cards [2]. Many biometric systems are based on a single characteristic of the human body. These systems have many limitations related to this single characteristic such as noise, intra-class variation, and identity fraud [2][3]. To overcome these problems, multimodal biometrics was developed. This technique consists of a combination of information from several biometric sources. Multiple biometric images obtained from different sensors can be combined at a different level: (i) sensor level: a data obtains from different sensors is combined and formed as a new dataset and further we create a feature extraction vector. (ii) feature level: at this level, we first extract features from different obtained biometric images and combine them to create a new feature vector. (iii) score level: at this level, different characteristics are extracted, compared individually and a combined match score is obtained. In this work, we are interested in feature-level fusion. The recognition is based on a comparison between the test sample and the template stored in the database as an indicator of similarity for the modalities. A biometric system has two phases, training, and

¹ Sulaiman ALSHEBLI. sulaimanalshebli@gmail.com

²Mahmoud Shafik. mshafik@derby.ac.uk

recognition. For the training phase, the biometric modality is captured and processed using specific algorithms, to obtain a reference biometric template for each user which is stored in a database. For the recognition phase, a biometric sample is captured and processed as in the training phase and compared with the biometric templates stored in the database. The result is either a match found (if the user is found in the database) or not recognized [4]. The main contribution of this research programme is to pre-process the images before exposing them to the recognition model. This model exploits the performance of the Discrete Wavelet Transform (DWT) method for feature extraction and then uses the Singular Value Decomposition (SVD) method to reduce the size of the feature space. The Euclidean distance method was used to find the match in the database.

The rest of the paper is organized as follows: section two covers an investigation into the state-of-the-art of multimodal recognition. Section three introduces the new proposed model for pre-processing, feature extraction, and matching. This is followed with results analysis, and discussion section. The paper concluded with some of the initial finding and potential ongoing work about the research framework programme.

2. State of the Art of Multimodal Recognition

The recognition rate of multimodal systems depends on multiple factors, such as the fusion technique, the feature extraction technique, and the classification method. In recent years, several researchers have focused on the proposal of reliable, multimodal biometric systems. The proposed models have been based on the combination of at least two characteristics. Among the works are those based on face and voice [5][6], face and fingerprint [7], face and palm print [8], fingerprint and iris [9], face and iris [10] [11], or fingerprint and hand geometry [12] [13].

In this work, we have chosen to propose a biometric system based on the face and iris. We made this choice because the face is the natural means of identifying people and the iris is currently considered to be one of the most accurate biometric systems [10] [11].

Eskandri and Toygar proposed to use left and right iris patterns with optimized features of local and global based facial feature extraction methods using Particle Swarm Optimization (PSO) and Bird Swarm Algorithm (BSA) to remove redundant data for the fusion of face-iris multimodal system with Tanh Score Normalization and Weighted Sum Rule fusion method where the weights are also optimized using PSO and BSA [14].

In their article [15], the authors have proposed a Face–iris multimodal biometric scheme based on feature level fusion. They used a 2D Gabor filter with different scales and orientations for feature extraction on the face and iris, then transform them by histogram statistics into an energy-orientation. After that, Principal Component Analysis method (PCA) was used for dimensionality reduction. Finally, they used Support Vector Machine (SVM) for the matching.

In the work [16], the authors proposed a multimodal face–iris biometric system that combines the advantages of score level, feature level and decision level fusion by considering the optimized information of face and iris biometrics at each level of fusion. The optimized output of one fusion level provides appropriate input for the next fusion level to construct a new and efficient scheme. The optimized scores are computed based on the extracted and fused optimized features of the face and iris modalities and finally the optimized decisions are made according to the optimized ROC's obtained from score level fusion. The authors have used the Log-Gabor transform to extract the facial and Iris features and the BSA feature selection algorithm to select the relevant features.

In the study [17], the authors proposed a weighted score level fusion technique to combine face and iris. They have used the Daugman method for iris recognition where an automatic segmentation is performed using circular Hough transform to localize iris and pupil area and 1D Log-Gabor filters were used to encode the unique features of the iris into a binary template. For face recognition, a PCA based method was used. For the matching, they have used the Hamming distance for iris and the Euclidean distance for faces. The min-max normalization technique has been used for normalizing the matching score of the iris and face recognition. The normalized scores are merged as a single score using the weighted sum rule.

In [18], the authors proposed an Iris and face recognition system based on PCA and Discrete Coefficient Transform (DCT) for facial features extraction, while iris features were extracted using 1D Log-Gabor filters and Zernike moments. They also used feature selection with Genetic Algorithms (GA) and scores level fusion with SVM.

In another work [2], the authors proposed a Face-Iris multimodal biometric system based on hybrid level fusion. For the features extraction, they used multi-resolution twodimensional Log-Gabor filter combined with spectral regression kernel discriminant analysis. For the matching, they used the Euclidean distance.

In a recent work [11], the authors presented a Face–Iris multimodal biometric identification system based on multi-resolution 2D Log-Gabor filter for iris features extraction and the Singular Spectrum Analysis (SSA) combined with the Normal Inverse Gaussian (NIG) statistical features derived from wavelet transform for the facial features extraction. The matching was performed using the Fuzzy K-nearest Neighbor (FK-NN). Works in the literature have shown that the combination of biometric templates at different fusion levels and using different feature extraction techniques improves the accuracy of the system. The cited works use different datasets to evaluate their algorithms such as: CASIA iris dataset, IIT Delhi Iris dataset, ORL face dataset, FERET face dataset, and Face94 face dataset. These works generally use two different algorithms for feature extraction of the face and Iris which makes the system more complex. The goal of this work is to propose a single algorithm for the extraction of features on the face and iris.

3. The New Proposed Model for Multimodal Recognition

In this paper, we will focus on face–iris multimodal biometric system based on DWT and SVD for feature extraction and a system having a feature level fusion. The proposed system is described and detailed in this section.

Figure 1 illustrates the proposed system. A dataset containing images of people's faces and irises is required to extract features. The dataset must contain a set of images of the faces and irises of the same person. The images first go through a pre-processing step for image normalization, then feature extraction is done. A feature vector of each person's facial and iris features is stored in a database. This vector will be used later in the matching step. Matching is a comparison operation using the Euclidean distance to find the best match in the dataset.

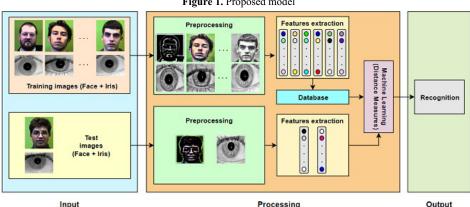


Figure 1. Proposed model

3.1. Preprocessing

Before doing image processing, one usually goes through a pre-processing or data cleaning step before building the intelligent model. The purpose of this step is to prepare the images to facilitate their analysis and computer processing. The choice of preprocessing method depends on the quality and nature of the data. In this work, we used two preprocessing methods, namely contrast and Gamma correction.

3.2. Feature extraction

Feature extraction prior to matching is an essential task. Feature extraction methods take out the most representative and relevant features of images to achieve a high matching score. In this research we have performed a static face and iris recognition based on the Extracted Feature Vectors obtained from the Discrete Wavelet Transform (DWT). In our proposed experiments, the Singular Value Decomposition (SVD) is applied on each sub image and its dominant features are extracted and stored inside the feature vector. We have compared our approach with three other feature extraction techniques: Discrete Cosine Transform (DCT) [19], Hough Transform (HT) [26], and Malakooti Transform (MT) [27].

3.2.1. Discrete Wavelet Transform

Discrete Wavelet Transform (DWT) [20] is an orthogonal transform similar to the Discrete Cosine Transform and an important multiresolution analysis tool. The DWT has been commonly applied to signal processing [21], video compression [22], speech recognition [23], image analysis and feature extraction [24], and various classification systems [25]. The DWT provides sufficient information both for analysis and synthesis of the original image, with a significant reduction in the computation time.

The basic idea of DWT is to provide the time-frequency representation by decomposing the image using DWT into N levels using filtering and decimation to obtain the approximation and detailed coefficients, then, extracting the features from the DWT coefficients. In this study, we used the DWT for feature extraction. The first level DWT divides the input image into four sub-band images. Each sub-band image contains one of high frequency bands and low frequency bands: LL, LH, HL, and HH, where LL denote a low frequency sub-band, LH a horizontal high frequency sub-band, HL vertical high frequency sub-band, and HH a diagonal high frequency sub-band. Here, the second level DWT decomposition LL2 sub-band image is important for us to reduce the input image size and extract features to be saved in the feature vector. Each time we apply DWT on the input image, the LL sub-band will be one-half of the input image. Thus, when we apply two-level DWT on the input image the size of LL sub-band would be 1/4 of the input image dimension. We have selected the LL2 band because it contains the most useful features of the input image while the other three bands can be ignored.

3.2.2. Singular Value Decomposition

Singular Value Decomposition (SVD) is one the strongest mathematical tools that can be used to decompose any square or nonsquare matrix, A, into the multiplication of two unitary matrices U and V and one Diagonal matrix Σ . This technique from the linear algebra can be used to automatically perform dimensionality reduction. SVD is a matrix decomposition method for reducing a matrix to its constituent parts. The SVD is used widely both in the calculation of other matrix operations, such as matrix inverse, but also as a data reduction method in machine learning. SVD can also be used in least squares linear regression, image compression, and denoising data. Given a *nxm* matrix A, the SVD of A is:

$$A = U. \Sigma. V^{\mathrm{T}}$$

Where the diagonal values in the Σ matrix are known as the singular values of the original matrix A. The columns of the U matrix are called the left-singular vectors of A, and the columns of V^T are called the right-singular vectors of A where T is a superscript. SVD can be thought of as a projection method where data with m-columns (features) is projected into a subspace with m or fewer columns, whilst retaining the essence of the original data.

3.3. Matching

Image matching is an important concept in computer vision and object recognition. In face and/or iris recognition, find a matching means that the system recognize the person, otherwise this person is not registered in the database.

In this work, we carried out a comparative study between three matching methods: the Euclidean distance (ED), the Manhattan distance (MD), and the Cosine similarity (CS).

3.4. Face and iris recognition process

In the face and iris recognition process, if the user chooses to do a training on a dataset he must first choose whether or not to do a pre-processing and which method to use (Gamma Correction or Contrast Enhancement). After that, he selects the matrix size, this value is used to divide the input image in several blocks (2, 4, 8, 16, 32, 64, 128, or 256). Then, experiments using some combinations (DWT+SVD, DCT+SVD, MT+SVD, HT+SVD) are done at the end of the pre-processing and the division steps.

For the DWT+SVD, DWT extraction is carried out by a two-level wavelet decomposition by the Haar wavelet. We have applied the SVD on low-frequency LL sub-bands of the input image and extracted the singular values of Red, Green, and Blue Low-frequency LL sub-bands. Once the singular values of each low-frequency LL subband are calculated using the SVD formula, the result will be appended at the end of the corresponding feature vector and the process of the feature extraction is completed. We applied the same dimensionality reduction process on DCT.

For HT and MT extraction, we need to calculate the HT and MT matrices. For the MT, we have used the author's recommended values of a and b (a=1, b=2). The SVD decomposition of MT/HT is calculated by an element-by-element multiplication of the MT/HT matrix with the image block. This whole process is then applied to the face and iris image.

After this feature extraction step, the feature vectors (of the face and the iris) are fused and stored in a database which will be used later in the matching. Each feature vector (face and iris) is obtained by the concatenation of the S, V, and D component).

In the test phase, the images proposed to the system (facial and iris images) go through the same processing and the resulting feature vector will be compared to the stored features database to find the best match using a distance measure (Euclidean, Manhattan, or Cosine).

4. Experimental Results Analysis and Discussions

In this study we used two public datasets widely used in the state of the art, the first is Faces94 which contains facial images [28] and the second is IIT Delhi Iris dataset [29] which contains iris images. We grouped the two datasets so that we had a single dataset containing face and iris images for each person. The irises do not really belong to the people on the faces, but we did this operation to test the different algorithms on multimodal classification (face and iris). The final used dataset contains 150 folders representing individuals. Each folder contains two sub-folders named "Face" and "Iris" which contain the images of faces and irises respectively.

For comparison purposes, we conducted experiments in two stages. The first is a comparison between different feature extraction techniques (DWT+SVD, DCT+SVD, HT+SVD, MT+SVD) with different parameters including matrix size, pre-processing technique, and distance measure. Experiments using 1500 images from the Face and Iris dataset were done. First, we have compared the different combinations of our proposed algorithms (DWT, DCT, HT, and MT combined with SVD), the different matrix sizes (32, 64, 128, and 256), the preprocessing methods (contrast and Gamma correction), and the distance measures (Euclidean, Manhattan, and Cosine).

The goal is to visualize and compare the accuracy of each combination to perform a comparison with the state-of-the-art methods in multimodal biometry.

By analyzing the accuracy of different combinations with different distances, it may be observed that our multimodal algorithm (DWT+SVD) gives the best result in the test especially when the matrix size is equal to 128 and a contrast pre-processing is applied with the Euclidean distance as a matching metric (see Table 1).

Distance	Method	Matrix size	Pre-proc.	Face & Iris
Euclidean	DWT+SVD	128	Contrast	98.90
Manhattan	DWT+SVD	64	Contrast	94.18
Cosine	DWT+SVD	128	Contrast	96.36

Table 1. Best obtained results from different combinations

The DWT gives the best results, this can be explained by the fact that DWT is better than DCT in terms of time and frequency resolution. Its coefficients are calculated by performing the successive Low pass and High pass filter on the Discrete-Time samples. DWT is also better than HT and MT since it selects only the LL2 band which contains the most useful features of the input image. Euclidean distance gives the best results compared to the Cosine and Manhattan distance. Manhattan distance is usually preferred over the Euclidean distance when there is high dimensionality in the data [30].

In our study, the use of SVD with DWT reduces the size of the features vector, which makes the Euclidean distance more suitable for this case.

Method	Feature extraction	Matching	Acc.
G. Huo et al. [15] (2015)	2D Gabor filter with different scales and orientations, then transform them by histogram statistics into an energy- orientation. PCA method is used for dimensionality reduction.	Support Vector Machine (SVM)	97.81
Y. Bouzouina et al. [18] (2017)	PCA and discrete coefficient transform (DCT) for facial features. 1D Log-Gabor filter method and Zernike moment for iris features. Genetic algorithm (GA) is used for dimensionality reduction.		96.72
B. Ammour et al. [2] (2018)	Two-dimensional Log-Gabor filter combined with spectral regression kernel discriminant analysis.	Euclidean Distance	97.45
B. Ammour et al. [11] (2020)	The dataset is pre- processed using the histogram equalization then the features are extracted from face images using singular spectrum analysis (SSA) and normal inverse Gaussian (NIG) combined with statistical features of wavelet. Feature extraction from iris images was performed using multi- resolution 2D Log-Gabor filter and spectral regression kernel discriminant analysis (SRKDA)	Fuzzy K-Nearest Neighbor (FK-NN)	98.18
Our method	The dataset is pre- processed using the contrast method then the feature extraction is made by DWT and SVD with the matrix size equal to 128.	Euclidean Distance	<u>98.90</u>

Table 2. Comparison results

The second stage of the experiments concerns a comparative study with some state- ofthe-art techniques that address the problem of multimodal classification.

Table 2 shows the results of our proposed method compared with four other techniques. We have implemented our proposed methods and the above-described face and iris recognition techniques to judge the outcome of this work. The experiment was done in the same environment and on the same training and test samples (same dataset). From the results, we can see that techniques using Euclidean distance give better results than techniques using Support vector machine (SVM). Fuzzy k-nearest neighbour (FK-NN) also gives good results, which means that, for the matching, techniques based on

distance give better results than techniques based on classical machine learning. For feature extraction, we can notice that DWT combined with SVD ensures a good characterization of the images and thus a good recognition of people. DWT provides simultaneous spatial and frequency domain information of the image and the SVD select the best features obtained from DWT. The results show that our method outperforms the other cited works.

5. Conclusion

In this paper, DWT and SVD-based technique for Multimodal Biometry Recognition have been presented. Experimental results show that the proposed DWT+SVD features are effective and efficient as compared to DCT+SVD, MT+SVD, and HT+SVD because it takes the combined advantages of DWT while estimating the features. It has been proved experimentally that the proposed approach provides an effective (better recognition rate) compared to work carried out in the state-of-the-art techniques on Face/Iris recognition. It has also been proved that distance-based matching techniques gives better results than classical machine learning techniques. In the future work, it is planned to use deep machine learning and compare between the various approach investigated in this ongoing programme.

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