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Analysis and Seismic Design of Tailings Dams and Liquefaction Assessment

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Abstract. The increasing of the mining industry in Latin America, combined with the high seismic conditions of some regions, represents a major challenge for geotechnical engineers in relation to the mining waste disposal design. Earthquakes are one of the principal causes of failure in this kind of structures, which are mainly attributed to liquefaction, whose consequences have been catastrophic such as cases history of Mochikoshi Tailings dams, Japan (1978); Cerro Negro and El Cobre, Chile (1965) and Amatista, Nazca, Peru (1996). Therefore, one of the main aspects in the seismic design of these structures is related to the possible liquefaction of the tailings, due to the characteristics of these materials. This paper presents the design criteria, geotechnical characterization and the seismic stability assessment of a tailings dam. This work is presented from practice approach, with emphasis on considerations that involved the dynamic analysis of a project at the design stage and the evaluation of liquefaction in this structure. The analysis results, interpretation and conclusions are presented based in local and international guidelines.

Keywords. Tailings dams, liquefaction, post-earthquake deformations, flow liquefaction.

1. Introduction

1.1. Background

Tailings dams are retention structures formed by earth materials whose main goal is the storage of mining waste resulting from the mineral benefit process in mining industry. The material stored, called “tailings”, is usually deposited in an aqueous slurry compound, generally called tailing slimes, whose solid particles are made up of silty or clayey materials produced by the crushing of rock and mineral.

Therefore, one of the main concerns in the design of these structures, are related to the possible liquefaction of the materials that make up the geotechnical structure. Therefore, it is necessary to carry out a rigorous analysis and design that guarantees the safety of the structure.

Tailings dams are structures of large dimensions and throughout history have been exposed to different types of failures related to different factors (e.g. operation, natural phenomena and construction). Earthquakes are one of the principal causes of failure in this kind of the structures, which are attributed mainly by liquefaction, whose consequences have been catastrophic such as case histories of El Cobre Old Dam and

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New Dam in Chile (1965), Mochikoshi N. 1 in Japan (1978), Cerro Negro N. 4 in Chile (1964) and Amatista Nasca in Peru (1996), among others. A typical seismic failure in tailings dams is shown in Figure 1.

In tailings dams, the seismic design starts from the conception of the construction method and type of tailings deposition, which in turn are subjected to factors related to site conditions, topography, availability of borrow materials, economics, environmental and operational factors. Therefore, geotechnical engineers must reach the best solution design that addresses these conditions as much as possible.

Figure 1. Seismic failure of Cerro Negro Tailings Dam [1].

1.2. Design considerations

The objective of tailings storage embankment design is to ensure that the structures are able to withstand the potential loading conditions that could be expected during their lifetime to the extent that the risk of failure is acceptably low [2].

The tailings dams design follows certain international and local guidelines, such as the technical bulletins of the International Commission on Large Dams (ICOLD, Bulletin 139) [3], Canadian Dam Association (CDA) [4] and Australian National Committee on Large Dams (ANCOLD) [2]; as well as compliance with local regulations.

These guidelines specify the considerations for management, tailings deposition methods, construction, characterization criteria and considerations for analysis and design. Regarding the latter and as far as seismic conditions are concerned, the guidelines provide suggestions on the target levels for earthquakes in according to the construction stage or phase in the life of mining dam, as well as the safety factors and deformations thresholds that guarantee the adequate behavior of the structure during an earthquake.

1.3. Construction methods

The seismic stability of tailings dams depends strongly on the construction method; within these, the main ones and of greater use in practice are the upstream, downstream, and centerline methods. In addition, there are other methods such as the upstream - downstream construction and solid waste deposition (thickened tailings).

According to the state-of-the-art and practice, the upstream construction method is vulnerable under seismic conditions, reason why, currently its use is prohibited in seismic areas. However, it is important to know and understand the conditions that have led to the vulnerability of these structures, which have been studied by different authors.

The vulnerability of these structures is governed by several factors such as the lack of rigorousness in supervision and construction process, uncertainties during mining operations, tailings deposition process and variations of source material or mineralogical composition. Furthermore, the long construction period of these structures, generally results in inadequate control and modifications of the original design conditions of the dam.

The over-elevation of the deposits for a greater storage, once the maximum elevation of design has been reached is a common practice, which affects the behavior of the structure, mainly in the vibration period, increase of stresses and pore pressure that produce deformations and possible activation of the contractive response of the soil that can trigger flow failure.

The instability of this construction method is because the retention structures are constituted by sand dykes. In addition, the stability of these dykes depends mainly on the tailing slimes for support [5], which generally become saturated and can liquefy under dynamic loading. The characteristics and configuration of the upstream method are shown in Figure 2(a).

Generally, it is known that during a severe earthquake, the material deposited will liquefy, so the principal concern is ensured that the retention dykes (sand dykes) are stable and do not liquefy. Nevertheless, the liquefaction of the tailings deposited (tailing slimes) can produce the failure, due to the saturation of the area near the contact between upstream dykes and tailing slimes. That was the case of the tailings dam of Moshicoshi in Japan, in which the 1978 Izu-Ohshim-Kinkai earthquake caused a failure due to the liquefaction of the materials behind the dam (Byrne et al. [6] and Ishihara, [7]).

The downstream construction method consists in the sequential construction of dykes using tailings sands, waste rock or borrowed fill (rockfill embankments). This option involves the use of internal drains or filters. The characteristics and configuration of this method are shown in Figure 2(b). Although this method has greater stability, it represents an expensive solution due to the volume required for the construction of retention structure.

The center-line method is a combination of upstream and downstream methods. The construction and growth of the dyke are carried out sequentially by keeping the vertical axis of the point of discharge, Figure 2(c). This method has an acceptable seismic stability in addition to a moderate cost.

Other methods of construction are deposits of solid tailings (Thickened tailings) and mixed construction methods such as the upstream-downstream method with rockfill embankment (Figure 3), the latter is specified in Mexican regulations (NOM-141-SEMARNAT-2003 [8]). The principal difference of this structure and conventional upstream method is that instead of using a starter dam, a rockfill embankment is used, which represents the principal retention structure. Currently, the use of this construction method in seismic zones in Mexico is not limited by regulations.

This type of structure must be carefully designed because the upstream dykes are supported on tailing slimes. Although the upstream dykes have a lower height than conventional upstream dams, these will be exposed to construction and operational factors that affect this kind of growth.

The disadvantages of this type of construction are the availability of materials in the area in order to build the rock fill embankment, as well as the inherent risks associated with the construction and operation of the dam, which requires a rigorous monitoring of the designer in order to evaluate the geotechnical conditions during its different stages. The adequate and rigorous design of this construction method, coupled with an appropriate construction supervision and compliance with the operation processes, can be a feasible option in cost and security that allows a greater volume of storage.

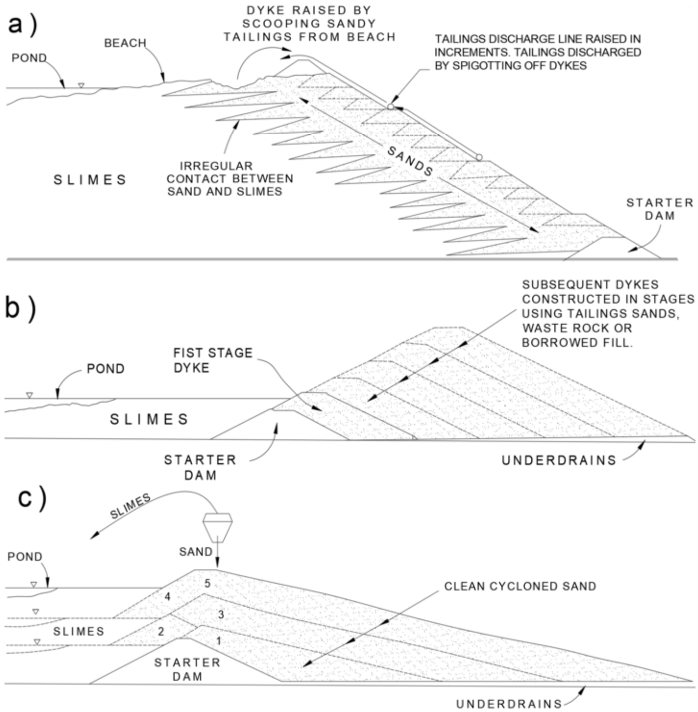


Figure 2. Methods of construction: a) Upstream, b) Downstream and c) Center-line (Figure modified from [5]).

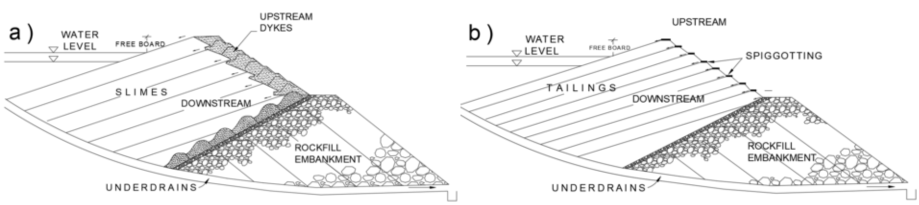


Figure 3. Scheme of upstream-downstream method with rockfill embankment: a) cyclone sand and b) spigotting (Figure modified from [8]).

1.4. Liquefaction in tailings dams

As mentioned before, an aspect of great importance to be considered in the engineering of tailings dams is related to the studies to evaluate liquefaction vulnerability. In practice, it is generally assumed that the tailings are liquefied during an earthquake; both the fine faction of the tailings (slimes) and tailings sands (dykes) are susceptible to liquefaction. State-of-the-art and practice (Troncoso [9], Ishihara [7], Phukunhaphan *et al*. [10], Moriwaki [11], Verdugo [12] and Hu *et al*. [13]) indicate that the predominant factors in the behavior of liquefaction in mine tailings are influenced by the type of source material, grain size distribution (fine or coarse) and the materials properties (relative density and plasticity).

For the study of liquefaction, a standard has been defined in the professional practice, which includes the following stages: 1) Susceptibility and liquefaction potential, 2) Stability analysis or flow slide and 3) Displacement analysis.

Susceptibility evaluation in mine tailings is usually performed according to the criteria defined by Andrews and Martin [14] and Bray *et al*. [15], developed for soils with a significant amount of non-plastic fines. These criteria evaluate if the soil meets the physical characteristics to be liquefiable regardless of the trigger mechanism, for that purpose, the index properties of the soils are used.

Once the susceptibility of the soil to be liquefied is determined, the liquefaction potential and soil behavior (contractive or dilative response) are evaluated. The evaluation of the liquefaction potential can be carried out using semi-empirical methods based on the Seed and Idriss method [16], laboratory tests, site response analysis (time or frequency domain) or using numerical analysis with advanced constitutive models.

It is important to note that liquefaction assessment in tailings dams should not be limited to the estimation of its potential. Hence, for design purposes it is important to know the cyclic behavior of the tailings in terms of cyclic resistance, strains, generation of dynamic pore pressure and strength loss.

In geotechnical earthquake engineering the term of liquefaction can be divided into two main categories: flow liquefaction and cyclic liquefaction [17].

Both phenomena can occur in tailings deposits with different manifestations; therefore, it is important to understand and distinguish between these phenomena, as well as studying their characteristics and triggering mechanism in order to carry out a design capable of mitigating them.

According to Robertson and Wride [18] the cyclic (seismic) liquefaction is associated with the dilative response or strain hardening of the soils, generally soils of a rigid nature such as dense sands. Nevertheless, a condition of "cyclic liquefaction" or "cyclic mobility" can be reached depending on the state of shear stress reversal under cyclic loading in undrained conditions. Cyclic (seismic) liquefaction behavior is shown in Figure 4.

Cyclic liquefaction has as main characteristics, the development of shear stress reversal, which allow reaching a condition of zero effective stress, in this state the soil has very little stiffness and large deformations can occur during cyclic loading. In terms of pore pressure, it means 100% pore pressure excess ratio (r_u), frequently called "initial liquefaction". For cyclic liquefaction, the deformations stop when cyclic loading has concluded; however, flow liquefaction can active if there is a pore pressure redistribution.

In contrast to cyclic liquefaction, cyclic mobility does not present shear stress reversal, so a condition of zero effective stress will not be achieved, resulting in small deformations [18].

Casagrande [19] was the first in define the term cyclic mobility, as the progressive softening of saturated sand specimens when subjected to cyclic loading at constant water content [20]. Castro and Poulos [21] specified that cyclic mobility is distinguished from liquefaction by the fact that a liquefied soil exhibits no appreciable increase in shear resistance regardless of the magnitude of deformation. During cyclic mobility, the residual shear resistance remains greater than the driving static shear stress and deformations accumulate only during cyclic loading [22].

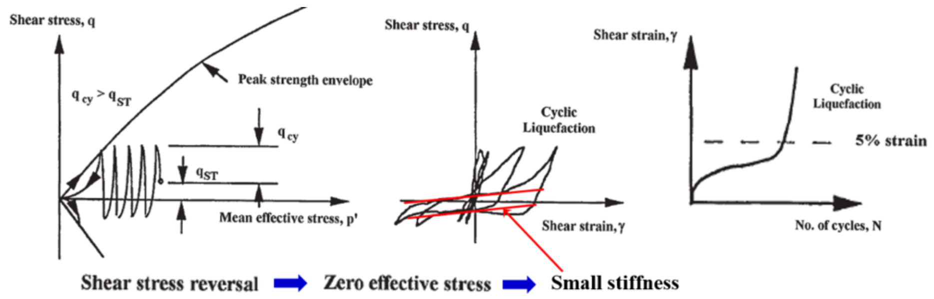


Figure 4. Cyclic (seismic) liquefaction behavior [18].

Flow liquefaction is also referred to as static liquefaction. However, the phenomenon can be triggered by either static or cyclic loading [23].

Flow liquefaction occurs when the static shear stress that maintains the static equilibrium is greater than the undrained shear strength or residual/liquefied shear strength, thus generating large deformations activated by monotonic loads. This phenomenon is related to the contractive response and strain softening behavior of the soil, such as loose sands and non-plastic silts.

Flow liquefaction is of great importance in tailings dams because the deposition of them is in a loose and saturated state, whose behavior under undrained loading conditions tends to be contractive. The risk for flow liquefaction in contractive soils depends on brittleness (sensitivity), which is the measure of the strength loss under the effect of static or seismic loading. Flow Liquefaction behavior is shown in Figure 5.

The case histories of static liquefaction in tailings dams have signified catastrophic failures as the cases of the Kolontar tailings dam failure in Hungary (Figure 6a) and Harmony, Merriespruit, South Africa (Figure 6b), as well as recently cases of failures in Brazil (Samarco dam in 2015 and Brumadinho dam in 2019).

The flow failure can occur suddenly during the earthquake or immediately after the earthquake has ceased, even at long after (few hours or up to 1 day), which is often called post-earthquake deformations, for which the estimation of residual strength is essential in order to evaluate flow liquefaction.

According to Robertson [23] the sequence to evaluate flow liquefaction is: 1. Evaluate susceptibility for strength loss, 2. Evaluate stability using post-earthquake shear strengths and 3. Evaluate trigger for strength loss.

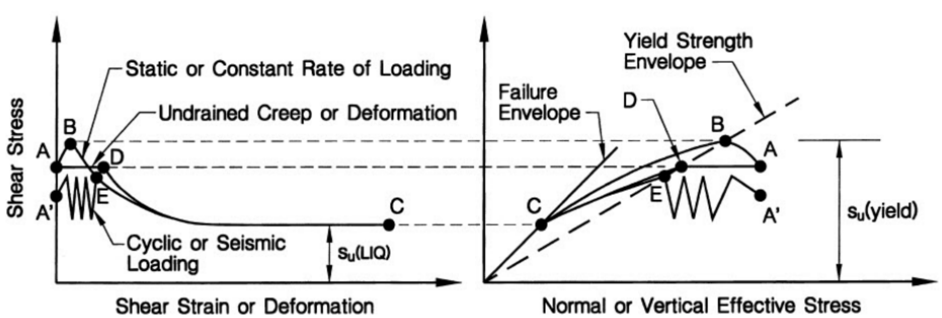


Figure 5. Flow Liquefaction behavior [23].

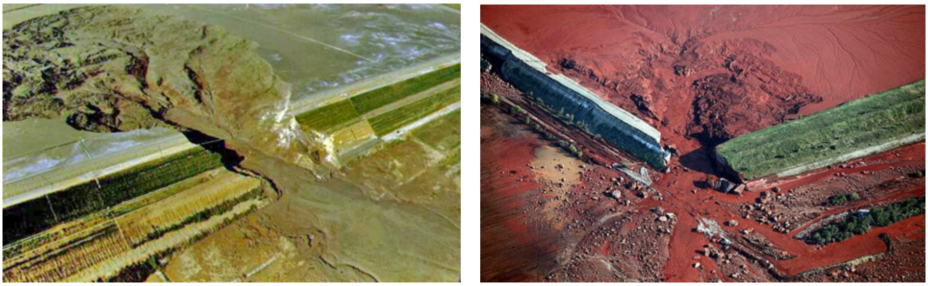


Figure 6. Flow liquefaction failures: a) Kolontar Tailings Dam failure in Hungary [24] and b) Harmony, Merriespruit, South Africa [25].

1.5. Analysis methods

The dynamic analysis in tailings dams aims to evaluate the response and behavior of the structure subjected to earthquake shaking. The dynamic behavior of the structure must include the estimation of permanent deformations due to the earthquake in order to evaluate the seismic stability and the service state of the dam.

There are different analytical and numerical methods for the evaluation of these aspects. Generally, the estimation of permanent deformations is carried out using semi-empirical methods (e.g. Newmark, [26], Yegian *et al*. [27] and Makdisi and Seed [28]), which have been widely accepted in practice. However, currently the dynamic analyses for tailings dams are solved using numerical methods, since it is possible to consider and integrate conditions of the dynamic behavior of the structure.

The selection of analysis methods to evaluate static and dynamic stability depends on the conditions of the problem. Therefore, in those cases, in which the estimation of deformations and generation of excess pore pressure are the principal aspects to evaluate, it is necessary to carry out a numerical analysis. Usually, the dynamic analysis in tailings dams considers the simulation of the liquefaction of the tailing slimes, which represents a conservative scenario for the design.

The main objectives in the simulation of the liquefaction are to estimate the dynamic pore-pressure generation and the prediction of deformations during and after an earthquake. For this purpose, different approaches and methods of analysis have been developed.

The finite element and finite difference methods are of great use for the solution of geotechnical problems associates to tailings dams, since these allow considering the non-linear behavior of the soil and dynamic pore pressure build up through advanced constitutive models.

The most used pore pressure generation models in numerical methods, which are incorporated in commercial codes or programs, are: Seed and Idriss Model (cyclic stress approach), Finn Model – Martin *et al*. [29] formulation, Finn Model – Byrne formulation [30], UBCTOT model, UBCSAND model and recently the PM4SAND model developed by Boulanger and Ziotopoulou [31].

In this paper a dynamic analysis is presented in order to evaluate the seismic stability of a tailings dam for a case study in the design stage. The liquefaction was simulated by Seed and Idriss model. In addition, the design criteria and the geotechnical characterization that involved the evaluation are presented for this case.

2. Case study

2.1. General and geotechnical conditions of the dam

The case study corresponds to the design of a tailings deposit located in an area of high seismicity in Mexico. The structure will be founded primarily on andesitic rock that predominates in the entire impoundment. According to the volume of storage required for the project, operation times and site restrictions, the deposit will reach a height of 80 meters on an approximate surface of 19 hectares. In addition, the design considered a length of beach of 60 meters, in order to avoid the saturation of tailings near the retention structure.

As background for this project, two construction alternatives were evaluated, which were within the local regulatory framework for the conditions of the zone. For the first alternative, a tailings dam constructed by the upstream-downstream method with rock fill embankment was considered, while the second alternative consisted in the design by the downstream method with rock fill embankment. In this article, the evaluation of the seismic stability of the first alternative is presented, given that it represents the most unfavorable condition.

Figure 7 shows the section analysis of the tailings dam, which includes a rock fill embankment (starter dam) with a height of 50 meters and 1.5:1 for upstream slope and 1.8:1 for downstream slope. The upstream dykes projected by upstream method will constructed by borrow material (clay sand with gravel). The height of each dyke is 5 meters with 2:1 (H: V) slopes, achieving a general slope of 3:1 (H:V). The total height of the tailings dam is approximately of 80 meters in its maximum section.

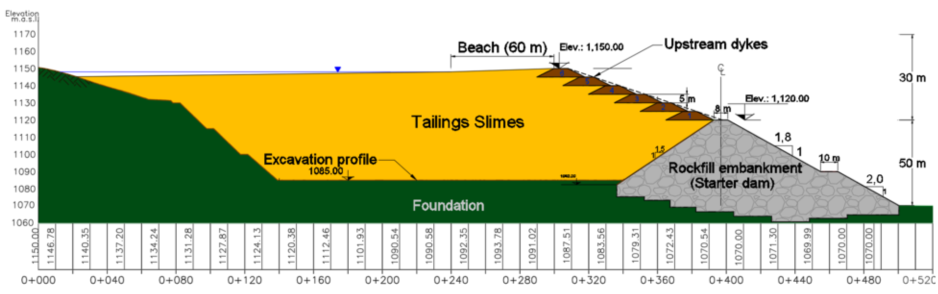


Figure 7. Cross-section of tailings dam (case study).

2.2. Seismic conditions

In order to evaluate the long-term seismic condition of the deposit, the maximum credible earthquake (MCE) was used, which was defined by a seismic hazard study that included the modeling of seismic sources close to study site. This condition was defined considering the recommendations of international guidelines (CDA [4]), which suggest the use of the MCE for a very high to extreme dam classification.

Figure 8 shows the record of maximum accelerations and response spectra associated with the designed earthquake (MCE), which will be used for the dynamic analysis. It should be noted that the earthquake was corrected by baseline.

a)

b)

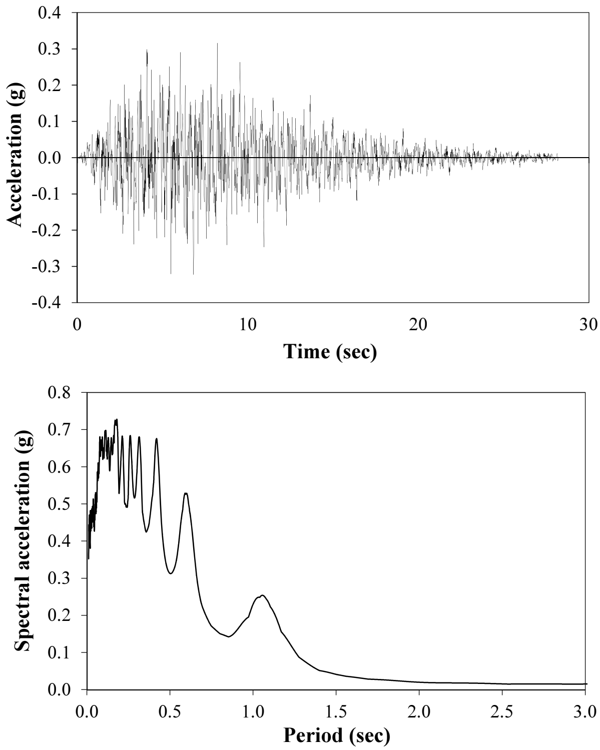


Figure 8. a) Input earthquake acceleration record and b) response spectra.

3. Geotechnical characterization

3.1. Foundation

From a geotechnical exploration campaign and geological studies, it was determined that in the study site, the predominant stratigraphy is composed of clayey sands with gravel and altered rock for the first 10 to 20 meters of depth, which is underlined by rock in a healthy state.

Simple compression tests were performed to determine the resistance parameters based on the Hoek and Brown [32] criterion, and Menard Pressuremeter tests (PMT) were performed to estimate the elastic modulus. The coefficient of permeability of the foundation was defined by in-situ tests, Nasberg tests for shallow depths (0 to 25 m) and Lugeon tests for greater depths (> 25 m).

The in-situ shear wave velocity (Vs) of foundation was defined by dispersion analysis of surface wave testing. Figure 9 shows the profiles of shear wave velocities.

3.2. Retention structures

The representative parameters for the rock fill embankment structure were taken according to the CIGB ICOLD [33]. In order to consider the stress-strain behavior of the dam, a hyperbolic model determined from the elasticity data of the La Yesca dam published by Aleman [34] was used. Strength parameters of upstream dikes were determined from UU triaxial tests in reconstituted samples that represent the compaction conditions of the dam.

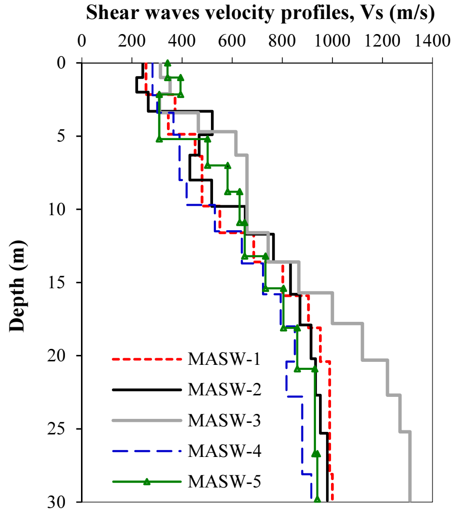


Figure 9. Shear wave velocity (Vs) profiles in foundation.

4. Tailings properties

4.1. Static properties of tailings

The mining waste material to be deposited in the impoundment is integrated of non-plastic fines with sand and minerals, mainly the result of lead and zinc recovery. On average, tailings have 45% water content, 27% of a liquid limit, and 11.9% of plasticity index.

After estimating these properties, consolidation tests were carried out simulating the process of deposition of the tailings in situ, under the influence of different effective confining stress (50, 100, 200, and 400 kPa) and different initial water contents (33, 43, 54, 66%). The purpose of this is to monitor the effect of the variation of water content in the dry unit weight and permeability (Figure 10). Figure 10(a) shows that water content at five meters of depth is kept below 30%, reaching 20% for a depth of 40 meters, in addition to a decrease in permeability in one order of magnitude at the depth of 10 m (Figure 10b). Figure 10(c) shows that in general, there are variations of the order of 100 kg/m^3 for the different water contents. Based on these results, samples were prepared for testing and obtaining its static and dynamic mechanical parameters.

Resistance parameters were obtained from consolidated undrained triaxial test with pore water pressure measurement. Permeability was defined by variable load permeameter test, obtaining permeability values of the order of 9.8E10^{-8} and 8.2E10^{-8} cm/s. In order to represent the increase of tailings stiffness in function of depth, it was assumed that the modulus of elasticity increase with the effective vertical stress.

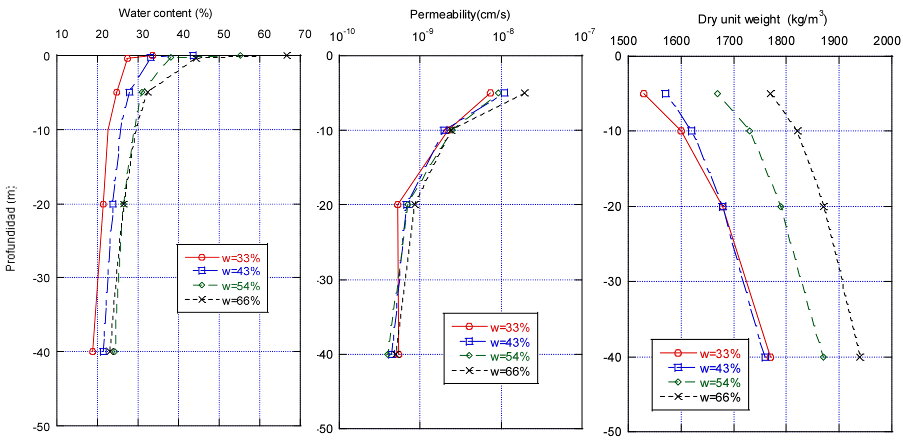


Figure 10. Simulation of the deposition process of tailings in situ: a) variation of water content, b) variation of permeability and c) variation of dry unit weight due to tailings deposition.

4.2. Dynamic properties of tailings

Dynamic behavior of the tailings was represented with curves of degradation of the shear modulus and damping curves versus shear strain amplitude. The degradation curves were obtained from dynamic laboratory test using a cyclic triaxial test and resonant column for large and low strains, respectively. Likewise, the pore pressure was measured from the cyclic triaxial test in order to evaluate the ability of the tailings to generate excess pore pressure under cyclic loading.

For cyclic triaxial test, 10 stages of constant amplitude cyclic shear stress were applied for 20 cycles under the undrained loading condition. The amplitude of shear stress was increased at each stage.

The properties of the reconstituted and tested samples were defined with the consolidation tests by simulating the in-situ tailings deposition process under different effective stresses.

Figure 11 shows the degradation curves of tailings for effective confining pressure of 50, 100, and 200 kPa and their comparison with conventional soils and other tailings. This figure shows that for these tailings at low confining pressure, the material tends to the lower limit of the sands; while at higher confining pressure, its dynamic behavior approaches the upper limit of the sands. Degradation curves of tailings are below the degradation curve for clays with a plasticity index (PI) of 11.6% determined by Dobry and Vucetic [35], which according to the Unified Soil Classification System (USCS) would correspond to tailings materials.

For the damping ratio of tested tailings, it is observed that at low strains and lower confining stresses, the material tends to behave as a fine non-plastic material (Curve of Dobry and Vucetic), but at large strains, the curves are found within the range of damping of the sands. Therefore, despite the fines content and soil classification of tailings, their dynamic behavior in terms of stiffness degradation and damping ratio tends to the behavior of the sand.

For the dynamic analysis, it was necessary to determine the shear wave velocity (Vs) values. However, given that it is a project at the design stage; dynamic properties of tailings were estimated from in situ measurements of shear-wave velocity (Vs) in a tailings dam with similar characteristics. This criterion is an adequate approximation to represent the conditions to which the structure will be subjected during its operation phase. Nevertheless, considering that the construction is gradual, it is possible to carry out measurements of the dynamic properties in order to corroborate the design parameters. Figure 12 shows the profile of Vs for tailings.

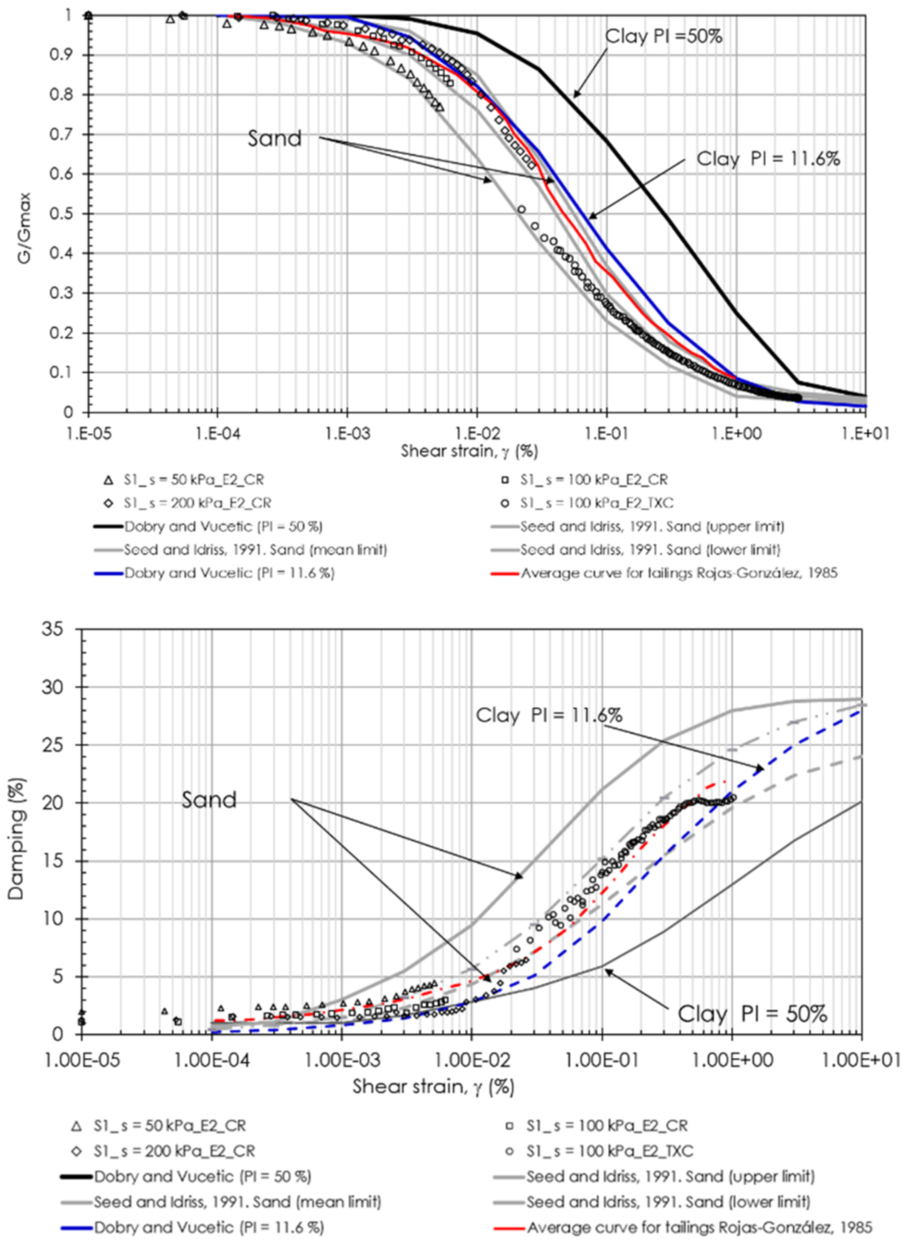


Figure 11. Curves of degradation of the shear modulus G/Gmax and damping curves versus shear strain amplitude under different effective confining pressure in tailings.

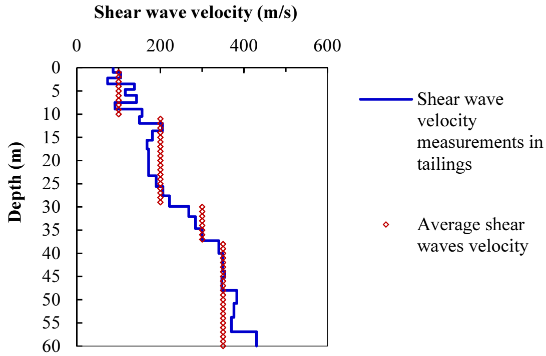


Figure 12. Shear wave velocities (Vs) profile in tailings.

5. Liquefaction

The evaluation of the tailings liquefaction consisted as a first stage in the estimation of liquefaction susceptibility and assessment of the likelihood of "triggering" or initiation of soil liquefaction, while the second stage consisted in the evaluation of flow liquefaction.

5.1. Liquefaction susceptibility assessment

Figures 13(a) and (b) show the liquefaction susceptibility assessment for four samples of tailings using the Bray *et al*. [15] and Andrews and Martin [14] criteria, the properties for this evaluation are presented in Table 1.

According to the criterion of Bray *et al*. [15], samples 2 and 4 are within the Zone A (susceptible to liquefaction), while samples 1 and 3 fall within the Zone B (water content and liquid limit ratios greater than 0.8), see Figure 13(a), so they will be materials with a moderate susceptibility or susceptible to cyclic mobility.

Figure 13(b) shows the susceptibility assessment using the Andrews and Martin criteria; the results indicate the same behavior as that defined with the criteria proposed by Bray *et al* [15]. The results of both criteria are in the limit between susceptible to liquefaction and the zone of uncertainty; therefore, this type of material is found in the transition of clay and sandy behavior.

According to these results, it was decided to evaluate the capacity of the tailings to generate excess pore pressure from the results of the cyclic triaxial tests.

Figure 14(a) presents the results of dynamic laboratory tests in terms of excess pore pressure ratio (r_u) versus the number of cycles for confining stresses of 50 kPa, 100 kPa and 200 kPa, and maximum stress ratios of 0.25, 0.3 and 0.4, respectively.

The results of these tests indicate that tailings material reaches excess pore pressure ratios less than unit 1.0. This means that total liquefaction or "initial liquefaction" does not occur. However, the material may experience significant strength loss and shear deformations under these values of r_u. It may be noted that the results presented are associated at the last stage of cyclic triaxial tests. Likewise, Figure 14(b) shows the history of excess pore water pressure ratio and axial stress-strain hysteresis loops for cyclic triaxial test.

Table 1. Geotechnical properties of tailings.

Sample	w (%)	S (%)	F (%)	L_L (%)	L_P (%)	PI	G_S	USCS	Hydrometer test: Fine sand (%)	Hydrometer test: Silt (%)	Hydrometer test: Clay (%)
1	49.7	3.5	96.5	29	16.1	13	3.58	CL	3.5	91.2	5.3
2	44.0	5.3	94.7	25	13	12	3.58	CL	5.3	89.0	5.7
3	48.9	3.5	96.5	29	16	13	3.58	CL	3.5	88.1	8.4
4	40.5	5.5	94.5	25	14	11	3.56	CL	5.5	88.2	6.3

Water content, w. Percentage of sand, S. Percentage of fines, F. Liquid limit, LL. Plastic limit, LP, Plasticity Index, PI. Specific gravity, Gs.

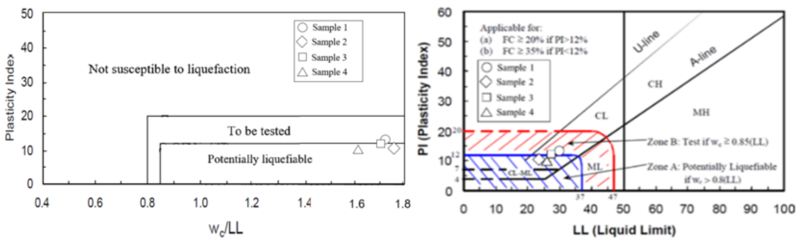


Figure 13. Assessment of the liquefaction susceptibility of the four samples of tailings: (a) Andrews and Martin criteria [14] and (b) Bray *et al*. criteria [15].

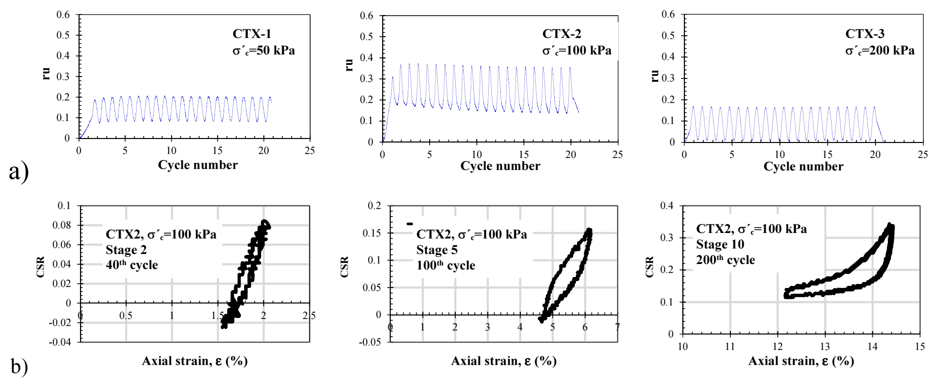


Figure 14. (a) Excess pore pressure ratio versus the number of loading cycles (10th stage) and (b) time histories of axial stress-strain hysteresis loops for different stages and cycle number from cyclic triaxial test CTX-2.

5.2. Flow liquefaction

The flow liquefaction is characteristic of steeply sloping ground, which applies to tailings structures. For the case study, and as part of the flow liquefaction evaluation process, the susceptibility for strength loss and soil behavior was evaluated using the cone penetration tests (CPT) data from a tailing deposit with similar characteristics. Figure 15 shows the results of a CPT test in tailings, which presents a predominant classification between silt mixtures – clayey silt to silty clay and sand mixtures – silty sand to sandy silt (Figure 15b).

The behavior of the soil was determined from the estimation of the concept of "clean sand equivalent" ($Q_{tn,cs}$). Robertson and Wride [18] defined the value of $Q_{tn,cs}$ = 70 as borderline between contractive and dilative behavior, which is associated with the state parameter (ψ) of Jefferies and Been [36]. CPT results show a contractive behavior in the superficial part from zero to five meters, as well as intercalations between the contractive and dilative behavior (Figure 15c).

Likewise, sensitivity was evaluated in these materials using CPT data. Tailings materials with a contractive response presented a medium to high sensitivity (Figure 15d). Furthermore, tailings are located within the “FC zone” of the potential liquefaction chart, which indicates a possible strength loss (Flow-liquefaction) and cyclic softening [23] (Figure 16).

According to the characteristics evaluated with CPT in tailings, the soil can strain soften in undrained shear; therefore, the post-earthquake stability and residual shear strengths will be the most relevant issue to estimate the seismic stability of the tailings dam.

The following sections present the dynamic analysis that considers the development of dynamic pore water pressure and the estimation of post-earthquake deformations.

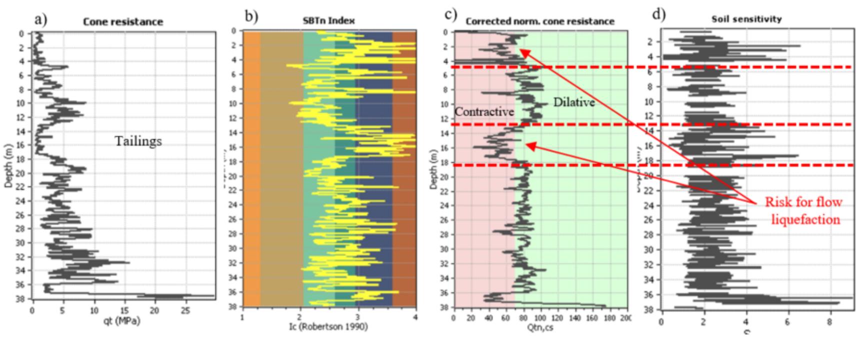


Figure 15. Cone penetration test results in tailing slimes.

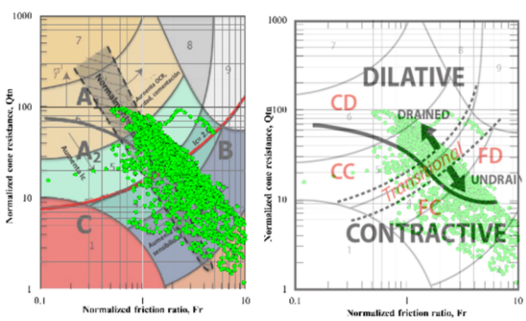


Figure 16. Evaluation of soil response in the CPT soil behavior chart.

6. Dynamic Analysis (Excess pore-pressures from CSR)

6.1. Stages of numerical modelling

The dynamic response and seismic stability of the tailings dam was computed by dynamic two-dimensional finite element analysis. The dynamic analysis considered the following stages: 1) Determination of groundwater level through the dam by transient water flow analysis, 2) Static equilibrium calculation in order to define the initial in-situ stresses, 3) Dynamic analysis (earthquake generation of excess pore pressures in dam) and 4) Post-earthquake deformations analysis.

The numerical simulation was performed for long-term condition, in which the project has reached the maximum operation level. The parameters used for the different analysis are presented in Table 2.

Table 2. Materials properties used in the analysis.

Material	γ kN/m^3	c kPa	ϕ (°)	k m/s	ν	E MPa	S_{uliq}/σ'_v kPa	Vs m/s	G_{max} kPa
Tailings (Depth. 0 - 5 m)	f(z)[1]	2	30	f(z)[1]	0.33	5	0.06	100	16310
Tailings (Depth. 5- 15 m)	f(z)[1]	2	30	f(z)[1]	0.33	10	0.1	200	65240
Tailings (Depth. 15 - 30 m)	f(z)[1]	2	30	f(z)[1]	0.33	18	0.15	300	146789
Tailings (Depth. 30 - 60 m)	f(z)[1]	2	30	f(z)[1]	0.33	18	0.15	400	260958
Upstream dykes	16.4	21	23	1.00E-08	0.3	26.76	-	250	104485
Rock fill embankment	21	5	42	1.00E-06	0.28	f(σ′$_c$)[2]	-	400	342508
Foundation (clayey sands with gravel and altered rock)	22	-	-	1.00E-07	0.3	2000		450	330275
Foundation (rock)	24	-	-	5.90E-08	0.25	4000	-	>620	1198777

[1] Parameters obtained from functions (Figure 10).
[2] Parameter obtained from function [34].

6.2. Water flow analysis

A transient water flow analysis was performed using saturated and unsaturated hydraulic properties. A total head was applied as hydraulic boundary condition at the maximum level of tailings deposit (Elevation = 1150 m.a.s.l). The length of the beach was 60 meters (Figure 17), whose objective is to avoid the saturation of tailings close to the retention structure, this condition is essential for the seismic behavior of the structure, since it mitigates the risk of liquefaction in the tailing slimes for support. The predicted phreatic line or seepage line with numerical model is shown in Figure 17.

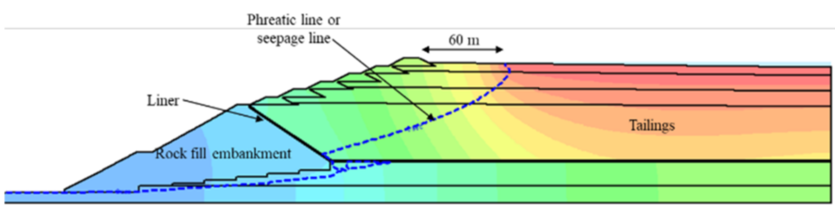


Figure 17. Seepage through the dam.

6.3. Dynamic analysis

Seismic response of the model was obtained from a 2D linear equivalent analysis using the QUAKE/W finite element program [37]. Hence, shear modulus and damping ratio curves determined in section 4.2 were used for this analysis (Figure 12). The numerical model consisted of 12191 quadrilateral elements. The boundary conditions in the model consisted in the restriction of the vertical and horizontal displacement in the base of the model "rigid base"; while the vertical displacement for the lateral boundaries were restricted.

Since the input motion must be applied at the base of the model, a deconvolution analysis was performed through a one-dimensional site response with the code SHAKE-91 [38].The element size was defined considering that the spatial element size (Δx), must be smaller than approximately one-tenth of the wavelength associated with the highest frequency component of the input wave [39].

$$\Delta x < \frac{\lambda}{10} \tag{1}$$

The numerical model mesh was calibrated by comparing its response against the one-dimensional site response analysis (SRA-1D). Figure 18 shows the surface acceleration records computed with site response analysis (SRA-1D) and the finite element model (FEM-2D).

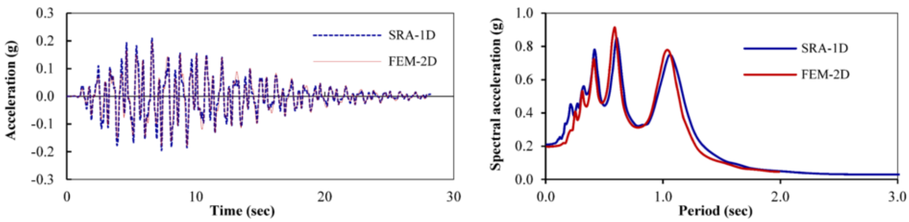


Figure 18. Comparison of acceleration record and response spectrum for SRA-1D and FEM-2D.

The liquefaction simulation was carried out using a pore pressure generation model based on the cyclic stress approach. Under this approach, the pore pressure is computed from the cyclic stresses (CSR) developed during the earthquake shaking and through the pore pressure ratio function (r_u). Therefore, it is necessary to determine the specific cyclic resistance curve and pore pressure function of materials.

The pore-pressures generated during earthquake shaking are a function of the equivalent number of uniform cycles, N, for a particular earthquake and the number of cycles, N_L, which will cause liquefaction for a particular soil under a specific set of stress conditions [37]. Liquefaction will occur once r_u=1 has been reached; that is, when excess pore pressure (Δu) is equal to the confining effective stress (σ_{c3}).

Lee and Albaisa [40] and DeAlba *et al.* [41] found that the pore pressure ratio, r_u, is related to the number of loading cycles by:

$$r_u = \frac{1}{2} + \frac{1}{\pi} sin^{-1}\left[2\left(\frac{N}{N_L}\right)^{\frac{1}{\alpha}} - 1\right] \tag{2}$$

This function is dependent on soil properties and test conditions, in the case of tailings; its behavior is governed by grain size and mineralogical composition. The particularities of this function are important, because the amount of pore pressure determined in the simulation will have a consequence in the strength loss and post-earthquake deformations. Figure 19 presents the pore water pressure function for the numerical analysis, which are representative for fine tailings [42]; additionally, it shows the range of pore pressure behavior of tailing slimes determined by Moriwaki *et al.* [11].

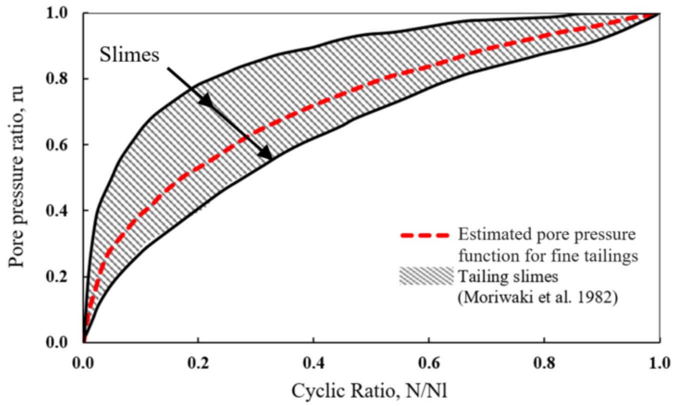


Figure 19. Pore pressure function for fine tailings.

6.4. Post-earthquake deformation analysis

One of the principal issues in the seismic stability assessment in tailings dam is to evaluate the service state of the structure during and after an earthquake. According to the international guidelines, the principal objective is that the permanent deformations generated by the earthquake are not such as to cause the loss of freeboard, that these are not greater than the total height of the structure or that are not enough to cause the failure.

Post-earthquake effects are commonly due to liquefaction phenomenon, presenting large deformations owing to softening or strength loss of soil.

The effect of delayed behavior has been associated with the fact that the soils are brought to the collapse surface by redistribution of stresses or excess pore-water pressures rather than directly by the earthquake shaking [43].

Figure 20 shows the effective stress path that illustrates the delayed behavior caused by the redistribution of stresses. As shown in this figure, during an earthquake shaking the pore pressure increases so that there will be a decrease in the effective stress, which mobilizes the ultimate shear strength or "steady state strength". If the soil is very loose and the driving static shear stress is large enough, the soil grain structure can collapse to the steady-state and strain softening behavior occurs. This strain softening causes stress redistribution within the soil mass [43].

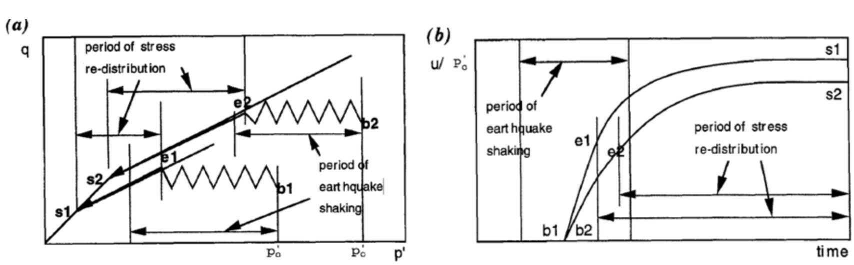


Figure 20. Delayed behavior caused by stress redistribution [43].

The Post-earthquake deformations analysis was performed using the SIGMA/W program, which uses an elastic-plastic constitutive model coupled to stress-redistribution model. Collapse surface angle and ultimate shear strength were used in the model in order to simulate the soil behavior during the collapse. The undrained shear strength $s_{u(liq)}$ were determined using CPT´s data.

7. Results of the analysis

The dynamic response of the tailings dam is presented in terms of accelerations, cyclic stresses, excess pore pressure and deformations. For this purpose, monitoring points were used in the model, which allow monitoring the results during dynamic time.

Additionally, the results are presented in the model as a shaded plot at the end of the earthquake shaking, in order to review the overall behavior of the structure.

7.1. Dynamic response

Figure 21 shows the estimated acceleration time histories at the surface of the saturated tailings and in points located at different depths. It may be seen that maximum acceleration at the surface is of the order of 0.18g. Furthermore, Figure 21 shows that there is a slight amplification of the input movement through the tailings dam; this effect may be due to the development of low excess pore pressure, since the site response is generally de-amplified when liquefaction occurs.

For the monitoring points located at the crest of the starter dam and upstream dikes, the predicted maximum accelerations were of the order of 0.28, 0.23 and 0.24g for points A, B and C, respectively (Figure 22).

The maximum acceleration at the base of the dam (Point H) was of the order of 0.16g, the comparison between this acceleration and that obtained in the crest indicates that there is an amplification level of 1.75. The natural period of the structure was of the order of 0.5 seconds, which was determined from the evaluation of the spectral ratio between seismic response at crest and base level. This period resulted independent of the seismic solicitation.

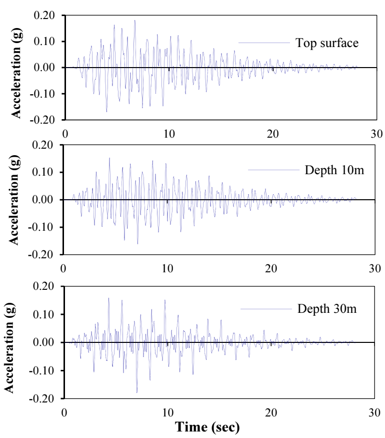


Figure 21. Acceleration time history at different depths in tailings.

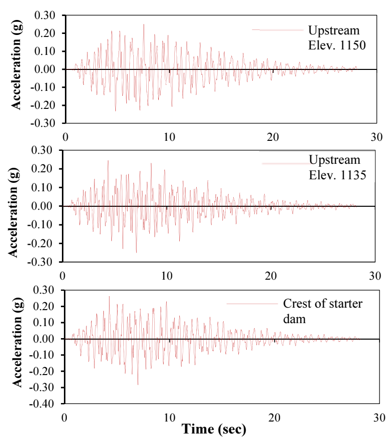


Figure 22. Acceleration time history at the crest of the starter dam and upstream dikes.

7.2. Liquefaction and cyclic stress ratio

Predicted excess pore pressures at the end of the earthquake are presented in Figure 23. It may be seen that tailings deposited superficially at 10 m of depth present low r_u values, between 0.15 to 0.35. However, there is a small zone at 5 m of depth that reach high excess pore pressures (r_u = 0.7). According to these results, in terms of excess pore pressure ratio, the initial liquefaction is not achieved. However, the pore pressure generated during the dynamic analysis can lead to some elements reach the collapse surface and then liquefied.

The cyclic shear stress (CSR) contours computed in the model are presented in Figure 24. It may be observed that in most of the saturated tailings show CSR values in the range of 0.1 to 0.25. Furthermore, high values of cyclic shear stress (CSR = 0.5) are observed in a small zone near the surface.

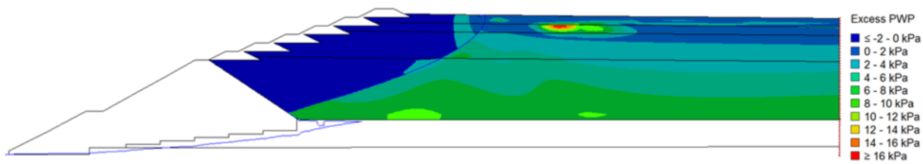


Figure 23. Excess pore pressures at the end of the earthquake.

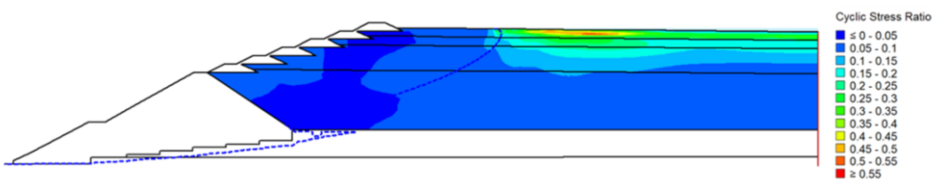


Figure 24. Cyclic stress ratio (CSR) contours computed in the numerical model.

7.3. Post-earthquake deformations

As seen in the previous sections, the post-earthquake deformation condition becomes the principal issue to evaluate in the seismic stability of the tailings dam.

The post-earthquake deformations of the dam are illustrated in Figure 25 in terms of displacement vectors and shading contours; in addition, the maximum values of permanent horizontal and vertical displacements for four monitoring points are presented. The monitoring points are located at the crest of the starter dam (Point A), the crest of the upstream dikes to the elevation of 1135 m.a.s.l (Point B), the crest of maximum elevation of upstream dikes (Elev. = 1150 m.a.s.l) (Point C) and tailings surface (Point D).

According to these results, the predicted deformation at the crest of starter dam (Point A) is less than 3% of the height of the structure, so they are considered admissible.

For point B, the predicted horizontal and vertical displacements were about of 6 and 1.5 cm, respectively. In point C, the maximum settlement computed was approximately of 6 cm and lateral displacements about of 4 cm. The predicted deformations were less than 3% of the height of the structure; in addition, the estimated vertical displacements represent a loss of free board of 2%, which is acceptable. Likewise, the vertical displacements on the crest of dikes must not be greater than 50% of the free board.

According to these results, the deformations determined in the retention structure are not significant for its global or local stability.

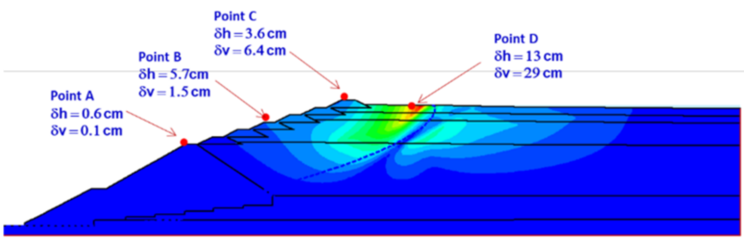
For point D located on the tailings surface, the permanent deformations at the end of the earthquake were of the order of 13 cm (horizontal) and 29 cm (vertical). The deformations were estimated to 60 m of distance from the dam.

The estimated results with the numerical model indicate that the maximum deformations computed are associated with tailings stored in the reservoir, also the permanent displacements induced by the earthquake are admissible for the tailings dam, considering as a threshold value of 3% of the height of the structure at different elevations.

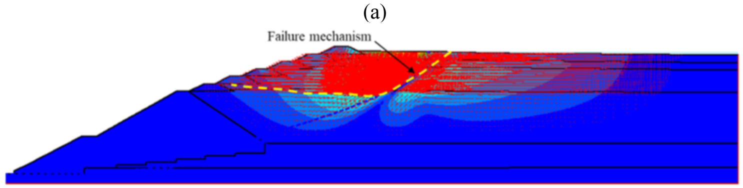
The structure will present post-earthquake displacements, which are not enough to cause its failure or instability. Figure 25 shows that the saturated material will present considerable displacements, predominantly settlements. However, these are not of interest for the global stability of the tailings dam.

The tailings that develop high pore pressure produce significant deformations, which cause the tailings move towards the retention upstream dikes. Figure 25(b) shows that the mechanism of failure starts at the border of the beach. This indicates that despite not achieving the liquefaction during the earthquake, important deformations may occur.

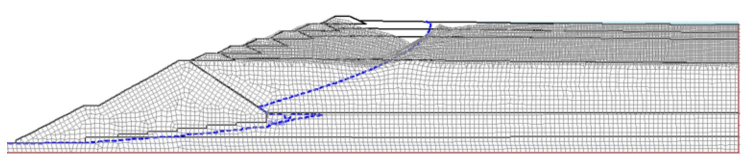
Furthermore, the dynamic analysis was carried out considering the saturation in the beach zone, which resulted in important deformations that compromise the stability of the dam. Therefore, the length of the beach and the appropriate design of pumping and drainage system of the dam play an important role in the seismic behavior of the structure, in which it is pursued that the permanent deformations do not affect the serviceability of the tailing dam.



(a)



(b)



(c)

Figure 25. a) Post-earthquake deformations contours, b) Post-earthquake deformations in terms of displacement vectors b) and c) Distorted mesh of the model (50 times magnified).

8. Conclusion

The analysis and design of tailings dam signified an important challenge due to the seismic conditions of the site and project requirements, as well as compliance with local regulations. The case study allowed assessing the analysis stages that must be carried out for a tailings dam by upstream-downstream method, as well as knowing the behavior and particular properties of tailing slimes.

The following conclusions are based on the studies and analysis developed in the case study.

The project under study considered two construction alternatives, which are within the local regulatory framework for the conditions of the zone. The alternative presented in this paper involves the design of a tailings dam by the upstream-downstream method with rock fill embankment. The design of this alternative does not represent the conventional design of the upstream method; since it has a rock fill embankment (starter dam) that represents the main retaining structure of the dam; in addition, the upstream dykes are made up by borrow material.

The simulation of tailings deposition process through consolidation tests, allowed representing approximately the in situ conditions of tailings and defining the properties for the static and dynamic laboratory tests. Likewise, the use of geotechnical field tests in an active tailings deposit (young deposits) of similar characteristics allowed to determine the parameters that are sensitive to deposition conditions. This was focused on determining the shear wave velocities and behavior of tailing slimes.

The dynamic behavior of tailing slimes in terms of degradation shear modulus and damping ratio tends to the sand behavior. Tailing slimes at low confining pressure tend to the lower limit of the sands; while at higher confining pressure, their dynamic behavior approaches the upper limit of the sands.

The results of susceptibility analysis from Bray *et al*. [15] and Andrews and Martin [14] criteria indicate that tailing slimes are susceptibility to liquefaction.

The pore pressure measurements by cyclic loading show that the tailing slimes has a low pore pressure generation capacity, as well as a high cyclic resistance. These results agree with the studies that have been published by several researchers [7] [11] about the dynamic behavior of fine tailings.

According to the characteristics evaluated by CPTu, tailing slimes are in the transition between clay-like and sand-like behavior; also, these materials present a contractive response and they are susceptible to strength loss (Flow-liquefaction) and cyclic softening.

Tailing slimes can strain soften in undrained shear; therefore, the post-earthquake stability and residual shear strengths will be the most relevant issue to consider in the seismic behavior of the tailings dam.

The stability and dynamic behavior of the tailings dam in study were acceptable; also, the post-earthquake deformations do not affect the service state of the structure. For this construction method, the length of the beach and drainage system were determinant for the good seismic behavior of the structure.

However, the approach of this construction method entails inherent risks that require a rigorous monitoring of the designer and the appropriate supervision during construction and operation.

The following is a summary of comments and recommendations for the analysis and design of this kind of structures.

- The dynamic properties of soils are usually represented by curves of degradation of the shear modulus and damping ratio as a function of the shear strain, and in the particular case of tailings, these cannot be assumed depending on the type of soil, as is generally done in practice, that in many cases, predictive curves are used for sands and clays available in the literature but that do not correspond to the materials with which the tailings are constituted. Therefore, it is important to have an adequate dynamic characterization in order to obtain the specific tailings properties.
- Tailings that present an important participation in the global stability of the structure (such is the case of conventional tailings dams) are susceptible to flow liquefaction, which can be triggered by either static or cyclic loading, therefore it is important to evaluate this phenomenon. For flow liquefaction is necessary to evaluate the soil behavior, susceptibility for strength loss, stability using post-earthquake shear strengths, trigger for strength loss and deformations.
- An important aspect to consider in the dynamic analysis is to evaluate the generation of excess pore pressures during earthquake shaking, which in turn can lead to some permanent deformations that affects the behavior and seismic stability of the dam.
- The liquefaction assessment should consider the susceptibility to liquefaction analysis through their index properties. It is recommended that the liquefaction potential in tailings should be evaluated in terms of excess pore pressure.
- The upstream-downstream method with rockfill embankment may be an adequate solution in areas of high seismicity; however, it requires an appropriate supervision during its construction and operation stage.

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References

[1] Castro, G., and Troncoso, J. 1989. Effects of 1985 Chilean earthquake on three tailing dams. In Proceedings of the Fifth Chilean Conference on Seismology and Earthquake Engineering, Santiago, Chile.

[2] ANCOLD 2012a, Guidelines on Tailings Dams-Planning, Design, Construction, Operation and Closure, Australian. National Committee on Large Dams Incorporated, May 2012.

[3] ICOLD, Bulletin 139, 2011. Improving tailings dam safety-Critical aspects of management, design, operation and closure.

[4] Canadian Dam Association, 2014. Application of Dam Safety Guidelines to Mining Dams. technical bulletin. ISBN 978-0-9936319-2-4A.N.

[5] Finn, 1982. Fundamental aspects of response of tailings dams to earthquakes. Dynamic stability of tailings dams. ASCE National conventions. New Orleans, USA.

[6] Byrne, P. M and Seid-Karbasi, 2003. Seismic Stability of Impoundments. 17th Annual Symposium, VGS.

[7] Ishihara, K. 1984. Post-earthquake failure of a tailings dam due to liquefaction of the pond deposit. In Proc., Inter. Conf. on Case Histories in Geotechnical Engineering.

[8] NOM -141-SEMARNAT-2003. Norma oficial Mexicana que establece el procedimiento para caracterizar los jales, así como las especificaciones y criterios para la caracterización y preparación del sitio, proyecto, construcción, operación y postoperación de presas de jales. en Diario Oficial de la Federación, 2004.
[9] Troncoso, J. 1995. Design and Behavior of Tailings Dams Under Seismic Conditions. International Conferences on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics.
[10] Phukunhaphan, C.F. Tsai and G.R. Thiers, 1982. Dynamic behavior of tailings materials. Dynamic stability of tailings dams. ASCE National conventions. New Orleans, USA.
[11] Y. Moriwaki, M.R. Akky, R. Ebeling, I.M. Idriss and R.S. Ladd, 1982. Cyclic strength and properties of tailing slimes. Dynamic behavior of tailings materials. Dynamic stability of tailings dams. ASCE National conventions. New Orleans, USA.A.N.
[12] Verdugo, R. 2009. Seismic performance based-design of large earth and tailing dams, Performance-Based Design in Earthquake Geotechnical Engineering, ICPBD Conference, Tsukuba, Japan: 41-60.
[13] L. Hu, H. Wu, L. Zhang, P. Zhang and Q. Wen, 2017. Geotechnical Properties of Mine Tailings. Journal of Materials in Civil Engineering, ASCE, ISSN 0899-1561.
[14] Andrews, D. C. A. and Martin, G. R. (2000). Criteria for liquefaction of silty soils. Proc., 12th World Conf. on Earthquake Engineering, Auckland, New Zealand.
[15] Bray, J. D., Sancio, R. B., Riemer, M. F., and Durgunoglu, T. (2004). Liquefaction susceptibility of fine-grained soils. Proc., 11th Int. Conf. on Soil Dynamics and Earthquake Engineering and 3rd Int. Conf. on Earthquake Geotechnical Engineering, Stallion Press, Singapore.
[16] Seed, H. B. and Idriss, I.M. (1971). Simplified procedure for evaluating soil liquefaction potential‖. Journal of the Soil Mechanics and Foundation Division, ASCE, 1971; 97(9): 1249-1273.
[17] Kramer S. L., (1996). Geotechnical earthquake engineering. Prentice-Hall, Inc., Upper Saddle River, New Jersey.
[18] Robertson, P.K. and Wride, C. (1998) Evaluating Cyclic Liquefaction Potential Using the Cone Penetration Test. Canadian Geotechnical Journal, 35, 442-459.
[19] Casagrande, A. 1975. Liquefaction and Cyclic Deformation of Sands-A Critical Review. *V Panamerican Conference on Soil Mech. & Foundation Eng*. Buenos Aires, Argentina.
[20] Castro, G. (1969). Liquefaction of Sands, PhD Thesis, Harvard University, Cambridge, MA, January.
[21] Castro, G. and Poulos, S.J. (1977) Factors Affecting Liquefaction and Cyclic Mobility. Journal of Geotechnical Engineering, ASCE, 103, 501-516.
[22] Rauch, 1997. EPOLLS: An Empirical Method for Predicting Surface Displacements Due to Liquefaction-Induced Lateral Spreading in Earthquakes. Doctoral Dissertations. Virginia Tech.
[23] Robertson, P.K., 2010c. Evaluation of Flow Liquefaction and Liquefied strength Using the Cone Penetration Test. Journal of Geotechnical and Environmental Engineering, ASCE, 136(6): 842-853
[24] Turi, David; Pusztai, Jozsef; and Nyari, Istvan, 2013. Causes and Circumstances of Red Mud Reservoir Dam Failure In 2010 at MAL Zrt Factory Site in Ajka, Hungary. International Conference on Case Histories in Geotechnical Engineering. 10.
[25] M. Davis and T. Martin, 2002. Static liquefaction of tailings-Fundamentals and case histories.
[26] Newmark, N.M., 1965. Effects of earthquakes on dams and embankments. Geotechnique 15, 139-159.
[27] Yegian M.K., Marciano E.A., Ghahraman V.G. (1991) – Earthquake-induced permanent deformations: probabilistic approach. ASCE, Journal of Geotechnical Engineering, vol. CXVII, n. 1, pp. 35- 50.
[28] Makdisi, F.I., Seed, H.B., 1978. Simplified procedure for estimating dam and embankment earthquake-induced deformations. ASCE Journal of the Geotechnical Engineering Division 104, 849-867.
[29] Martin G.R., Finn W.D., Liam y Seed H.B. (1975). ―Fundamentals of liquefaction under cyclic loading‖. Journal of the Geotechnical Engineering Division, ASCE, May 1975, Vol. 101, No GT5.
[30] Byrne P.M. (1991). A Cyclic Shear-Volume Coupling and Pore-Pressure Model for Sand in Proceedings: Second International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, St. Louis, Paper No. 1.24, 47-55.
[31] Boulanger, R. W., and Ziotopoulou, K. (2017). "PM4Sand (Version 3.1): A sand plasticity model for earthquake engineering applications." Report No. UCD/CGM-17/01, Center for Geotechnical Modeling, Department of Civil and Environmental Engineering, University of California, Davis, CA, 113 pp.
[32] Hoek E. Brown E. T. (1968). The Hoek-Brown criterion", 1988, update, Rock Engineering for underground Excavations, Proc 15th Canadian Rock Mech. Symp. Dept. Civil Engineering, University of Toronto.
[33] CIGB ICOLD (2010). Concrete Face Rockfill Dams: Concepts for Design and Construction. Bulletin 141.
[34] Alemán, J.D., 2003. Diseño geotécnico de presas de gran altura de enrocamientos con cara de concreto. Academia de Ingeniería de México.
[35] Vucetic, M. and Dobry, R. (1991) Effect of Soil Plasticity on Cyclic Response. Journal of Geotechnical Engineering, 117, 89-107.

[36] Jefferies, M.G. and Been, K., 2006. Soil Liquefaction – A critical state approach. Taylor & Francis, ISBN 0-419-16170-8 478 pages.
[37] Krahn John (2004). "Dynamic modelling with QUAKE/W - An Engineering Methodology". GEO-SLOPE/W International Ltd., Alberta, Canada.
[38] Schnabel P.B., Lysmer J.L. and Seed H.B. (1972), "SHAKE: A computer program for earthquake response analysis of horizontally layered sites". Report No. EERC 72-12, University of California, Berkeley.
[39] Kuhlemeyer R.L. y Lysmer J. (1973). —Finite Element Method Accuracy for Wave Propagation Problems. J. Soil Mech. & Foundations, Div. ASCE, 99(SM5), 421-427.
[40] Lee, K.L., and Albaisa, A., 1974, Earthquake Induced Settlements In Saturated Sands, Journal of The Soil
Mechanics and Foundations Division, ASCE, Vol.100, No. GT4.
[41] DeAlba, P., Chan, C.K., and Seed, H.B., 1975, Determination of Soil Liquefaction Characteristics by Large-Scale Laboratory Tests, Report EERC 75-14, Earthquake Engineering Research Center, University of California, Berkeley.
[42] Zhang, C., Yang, C. H., and Bai, S. W. (2006). "Experimental study on dynamic characteristics of tailings material." Rock Soil Mech., 27(1), 0035–0040 (in Chinese).
[43] Gu, W.H., Morgenstern, N.R., and Robertson, P.K. (1994). Post-Earthquake Deformation Analysis of Wildlife Site, ASCE Journal of Geotechnical Engineering, Vol.120, No.2, pp. 333-349