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Centrifuge Technology for Characterization of Expansive Clays

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Abstract. In addition to the already common use of geotechnical centrifuges to represent, in a reduced-scale model, the state of stresses corresponding to a full-scale geotechnical structure, centrifuge techniques have increasingly been used for an additional purpose: accelerating flow process through geotechnical systems. This is particularly relevant in geotechnical problems involving low hydraulic conductivity scenarios, including flow through low-hydraulic conductivity shales, through unsaturated soils in general, and through unsaturated soils subjected to volumetric changes during infiltration (i.e. expansive clays). This paper provides an overview of recent analytical and experimental advances involving the use of centrifuge technology for the hydraulic and volumetric evaluation of expansive clays. In particular, a new centrifuge approach is presented for practical characterization of expansive clays aimed at implementation in conventional laboratories rather than research centers. The results indicate that, in spite of the significantly highly practical and expeditious characteristics of the new approach, the predicted swell-stress curve is the same as that obtained using time-consuming conventional experimental techniques.

Keywords. Centrifuge testing, Expansive clays, Unsaturated flow, Soil Water Retention Curve, Swell-stress curve

1. Introduction

The typical philosophy for centrifuge modelling in geotechnical is based upon the requirement of similitude between model and prototype. If a model of a prototype structure is built with dimensions reduced by a factor $1/N$, then an acceleration field of N times the acceleration of gravity, g, will generate stresses by self-weight in the model that are the same as those in the prototype structure. In addition to this approach, centrifuge techniques can also be used for an additional purpose: to accelerate flow processes through geotechnical systems. This may be particularly beneficial in geotechnical problems involving flow through: (1) low hydraulic conductivity materials (e.g. flow through low-hydraulic conductivity shales), (2) unsaturated soils that do not undergo significant volume changes (e.g. sandstones), and (3) unsaturated soils subjected to possibly significant volumetric changes during infiltration (e.g. high plasticity clays).

A practical problem, which is the focus of this paper, is the use of centrifuge principles for hydraulic and volumetric characterization of expansive clays, a particularly costly geotechnical problem. As early as 1973, the annual cost of expansive soil damage in the US was estimated at $2.2 billion [1]. The US Department of Housing and Urban

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Development subsequently estimated that foundation damage caused by swelling clays reached $9 billion per year in 1981 [2]. Estimates by Whiterspoon [3], based on data collected from foundation repair contractors, places this damage at an approximate $13 billion per year. Finally, and allowing for annual inflation and population growth, Wray & Meyer [4] estimate that total annual damage from swelling clays in the US well exceeds $15 billion. Although not life-threatening as compared to other natural disasters, expansive soils constitute a natural hazard with an average annual damage that exceeds that caused by floods, hurricanes, earthquakes and tornados combined [4, 5, 6, 7].

Important advances have been made towards a better understanding of the swelling of clays and their treatment. These include new moisture diffusion and volume change models (e.g. El-Garhy & Wray [8]), verification of numerical simulations (e.g. Houston et al. [9]), the effect of clay structure on its mechanical response [10, 11], evaluation of new stabilizers (e.g. Miller & Zaman [12]), assessment of properties governing the unsaturated characteristics of expansive soils (e.g. Likos & Lu [13, 14], Puppala et al. [15], Lin & Cerato [16]), coupled interactions between clay microstructure and macrostructure (e.g. Likos & Wayllace [17]), and characterization of the mechanical behavior of swelling clays (e.g. Zhan & Ng [18]).

Yet, our ability to measure the one variable that is ultimately responsible for damages exceeding $15 billion per year (i.e. the vertical rise itself) remains, at best, limited. This is because conventional "free-swell" tests often require excessive testing time for their completion in practical applications. Also, since water infiltration into the specimen is solely driven by suction gradients, the termination of swelling tests is ill-defined and constitutes a source of error that compromises the repeatability of test results. Finally, proper design would require a comprehensive battery of swelling tests, as design typically requires the determination of the entire relationship between swelling and confining stresses. Probably because of these shortcomings, the current state of practice relies heavily on (often crude) empirical correlations between vertical rise and clay index properties. In Texas, for example, widespread practice has been to rely solely on the PI of soils to predict vertical heave using correlations developed in the 1950's.

The study discussed in this paper focuses on the characterization of expansive clays using centrifuge technology as an alternative to alleviate the shortcomings associated with using soil index properties. Specifically, centrifuge technology was used for the continuous and expeditious, direct measurement of the swelling of clays. Also, the centrifuge approach is well-suited for characterization of swelling clays because it is expected to be practical to define the entire relationship between swelling and vertical stresses. This is a significant advantage over conventional free swell tests, which are often prohibitively long and provide swelling for a single stress value.

This investigation capitalized on the availability of a centrifuge facility at The University of Texas at Austin, which was specially tailored to investigate flow-related problems in geotechnical applications. This is a high-g centrifuge capable of continuously, non-destructively, and non-intrusively measuring suction, moisture content, and fluid flow rate in a soil specimen during centrifugation. It allowed expedited determination of both the Soil Water Retention Curve (SWRC) and the hydraulic conductivity function (K-function) from a single specimen in a single test [19, 20, 21]. In this new application, the centrifuge capabilities were expanded to determine both the unsaturated hydraulic properties and the swelling response of soils showing significant volume changes.

In addition, a comparatively simple and inexpensive centrifuge environment was developed for direct quantification of swelling. Accordingly, a comparatively small centrifuge, used commercially for multiple industrial purposes, was adapted and also used for expansive soil characterization. Ultimately, this study highlights that the use of centrifuge testing is no longer restricted to research activities, but it has the potential of becoming a conventional equipment in geotechnical laboratories.

2. Background

2.1. Expansive Clays

Expansive clays are soils that increase in volume, often substantially, when given free access to water. One of the most commonly accepted theories to explain the expansion of soils is based on the diffuse double layer (DDL) that develops between clay platelets [22, 23]. Negatively charged soil particles attract cations in order to remain electrically neutral, forming areas of high cation concentration in between particles. When wetted, the higher concentration of cations creates a hydraulic gradient due to differences in osmotic pressure, which in turn induces the flow of water in between the particles, ultimately causing an increase in volume [24].

Studies have been performed to compare the theoretical swell potential (pressure) of pure clays, predicted using the DDL theory, against that measured in the laboratory [25, 26, 27, 28]. These studies have generally found a good relation between theoretical and experimental results, with the exception of soils in which the structure of soil particles affects the compression or expansion (e.g. soils with relatively large particles or with flocculated soil structure). Even in soils for which the experimental observations do not match the theoretical predictions, the trend of experimental results has been found to compare well with theoretical predictions.

As previously mentioned, problems caused by swelling clays throughout the US result in billions of dollars a year in damages [4]. Damages can occur to nearly any structure built on or in expansive soils, including deep and shallow foundations, retaining walls, and roadways. The most problematic areas are those with abundance of swelling clays and a climate that promotes seasonal changes in the water content of clays. In these cases, the key to the design is to predict the magnitude of swelling and/or the magnitude of pressures that are expected from expansive soils. Only with this information can a geotechnical design be conducted to accommodate the expected pressures or movements.

Conventional free swell tests [29] are performed in consolidation frames. Clay samples are compacted in consolidations cells and subjected to overburden pressure. Water is then poured to submerge the sample and vertical deflections are subsequently measured. There is no clear termination point and tests often run for months until the sample height appears to reach equilibrium. Because of their long duration, conventional free swell tests are rarely used in geotechnical practice. This is probably what led to the proliferation of correlations based on soil index properties to predict swelling. This includes correlations with the Plasticity Index (PI) developed by McDowell [30]. Vijayavergiya & Ghazzaly [31] and Nayak & Christensen [32] suggested relations using the dry unit weight and either the PI or Liquid Limit. Rao et al. [33] showed good predictions using a correlation based on dry unit weight, compaction water content, overburden pressure, and free swell index results.

A good example of the imperative need to develop approaches for expeditious, direct measurement of the swelling of clays is the state of the practice throughout Texas (and probably in several other states). Specifically, government agencies and geotechnical designers in Texas rely heavily on the correlations developed by McDowell [30] for their geotechnical designs. The method correlates the PI of a soil with the predicted volume change. Unfortunately, this correlation is based on laboratory swell testing of only three test soils that were subjected to wetting through capillary adsorption. This capillary approach involved placing a specimen atop a water reservoir and allowing swelling by water drawn by capillary rise for a period of time (in days) equal to the PI of the soil. The percent volume change was then corrected for the overburden pressure to yield a predicted vertical rise. While the study by McDowell [30] was valuable at its time, the lack of reliable procedures for direct measurement of the swelling of clays has led to its continued use until today, in spite of the deficiencies of these correlations. First, the method is based on a small number of tests in which all of the samples were remolded. Also, no tests were conducted on soils at comparatively low initial water content. Finally, the correlations have been extrapolated (e.g. Tex-124-E [34]) to PI values as high as 140, which is well beyond the range of testing. Despite these limitations, the method has been used for over fifty years throughout the state of Texas.

2.2. Water Flow through Unsaturated Porous Media

Flow of water in unsaturated soils can be described using three non-linearly related variables, namely the volumetric water content θ (or degree of saturation), the matric suction ψ (or capillary pressure if the air pressure is non-zero), and the hydraulic conductivity K. These variables are quantified experimentally by determining the soil-water retention curve (SWRC) and the hydraulic conductivity-function (K-function), a set of relationships that govern the soil's moisture storage and impedance to water flow, respectively. An important challenge, however, is that measurement of the hydraulic characteristics of unsaturated soils involves lengthy testing periods. Consequently, most projects requiring the use of hydraulic characteristics of unsaturated soils rely heavily on empirical correlations or theoretical models rather than on experimental measurements. Centrifuge testing has been recently used to alleviate the shortcomings, allowing a comparatively expeditious determination of the unsaturated hydraulic characteristics [19, 20, 21].

The SWRC is the relationship between θ and ψ, and represents the energy needed (i.e. ψ) to de-saturate the soil to a given θ. The K-function is the relationship between K and ψ (or θ), and reflects the decrease in available pathways for water flow as a soil de-saturates. The typical SWRC and K-function of a low plasticity clay are shown in Figure 1. The experimental SWRC data shown in this figure were obtained using three of the four approaches described by ASTM D6836 [35] (i.e. hanging column, pressure plate, and thermodynamic tests). It should be noted that each point on the SWRC in Figure 1 was defined using a different water flow mechanism. The hanging column and pressure plate tests involve monitoring the transient outflow of water from the soil specimen during application of ψ (i.e. the difference between the pore air and pore water pressures), while the thermodynamic test involves monitoring the total suction (i.e. the sum of matric and osmotic suction values) during evaporation of water from the soil. Instead of using experimentally-derived data as in the SWRC, the K-function shown in Figure 1 was defined using a theoretical model. Specifically, the van Genuchten-Mualem [36] model was used to define the K-function. This model assumes that the soil behaves as a bundle of capillary tubes having properties described by the parameters of a function fitted to the experimental SWRC of the soil. The predicted K-function is only shown up to 10^{-14} m/s as this value nears the low end of *K* values that have been measured in the laboratory [37]. The K-function represents the proportionality between the hydraulic gradient and water flow rate, and is thus only relevant for conditions in which the water phase in the soil is continuous. When the water phase in the soil becomes discontinuous, hydraulic gradients applied to the water phase will not result in water flow. Instead, vapor transport by diffusion will dominate the migration of water. Because the theoretical prediction of the K-function provides no lower bound on the *K* of a soil, the boundary between liquid and vapor phase transport is difficult to assess. This boundary is likely to occur in the vicinity of the ψ value where the slope of the SWRC starts to decrease, or at about 200 kPa for the data in Figure 1.

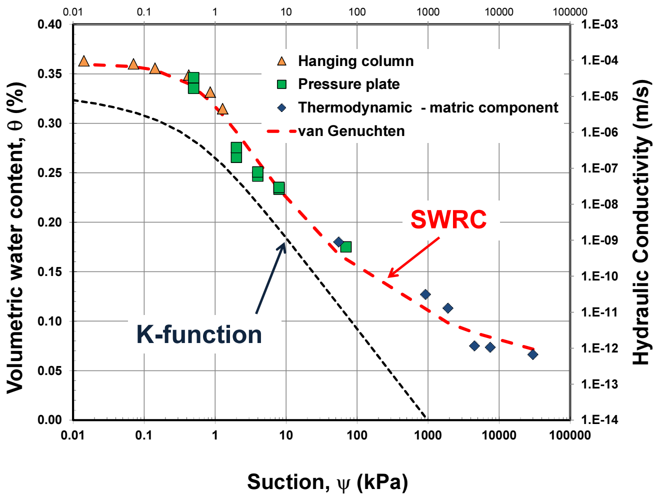


Figure 1. Hydraulic characteristics of an unsaturated clay of low plasticity.

The common practice of predicting the shape of the SWRC and K-function using empirical observations or theoretical models is well documented (e.g. Zapata et al. [38], van Genuchten [36], Brooks-Corey [39]). A SWRC is typically quantified by fitting experimental data to power law, hyperbolic, or polynomial functions [39, 36, 40]. Although the Brooks & Corey [39] model is able to represent a sharp air entry suction, the van Genuchten [36] model is most commonly used in numerical analyses because it is differentiable for the full range of suction values. Preliminary estimates of the SWRC could be obtained using databases that rely on the granulometric distribution of soils [40].

Experimental evaluations to validate the K-functions predicted using theoretical models are rarely conducted in practice. This is especially problematic as the SWRC and K-function are sensitive to soil structure variables such as pore size distribution [41, 42, 43, 44], soil fabric [45], mineralogy [46], compaction conditions [47, 48], use of admixtures [15], volumetric changes [49, 50, 51, 52], and stress state [53]. Khaleel et al. [54] observed that predicted K-functions could be in error by several orders of magnitude due to some of these effects. The broad range of potential impacts and the high magnitude of errors point to the need for direct measurement of the hydraulic characteristics of unsaturated soils.

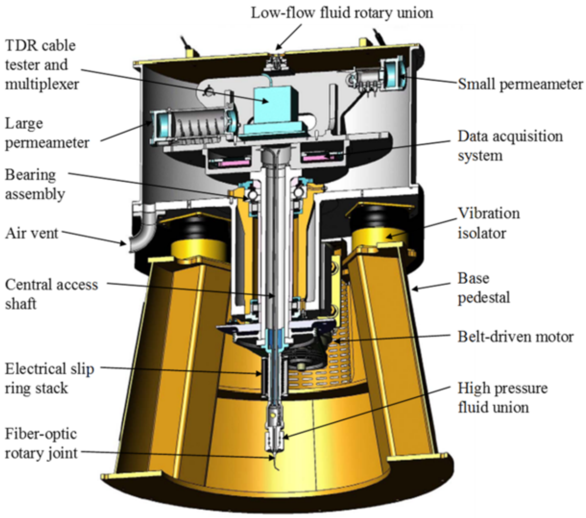
One of the few tests available to directly measure the hydraulic characteristics using controlled infiltration is the column flow test, performed in either a rigid-wall permeameter, with flow controlled by surface infiltration and gravity drainage or in a flexible-wall triaxial permeameter with flow controlled by a pump [55, 47, 56]. Specifically, a column flow test can be used to measure the SWRC and K-function by imposing a known flow rate through the specimen and monitoring the corresponding gradient in hydraulic head, or by imposing a gradient in hydraulic head on the specimen and monitoring the ensuing flow rate. In either approach, there will be a transient period during which θ and ψ will change, followed by steady-state water flow conditions. Although the transient changes in θ and ψ can be used to measure the soil hydraulic characteristics [37], the calculated flow rates and gradients are prone to significant error. Steady-state flow data can be used to measure the hydraulic characteristics with more confidence, but a significantly long time is needed to establish steady flow conditions [57].

2.3. Centrifuge Techniques for Characterization of Unsaturated Hydraulic Properties

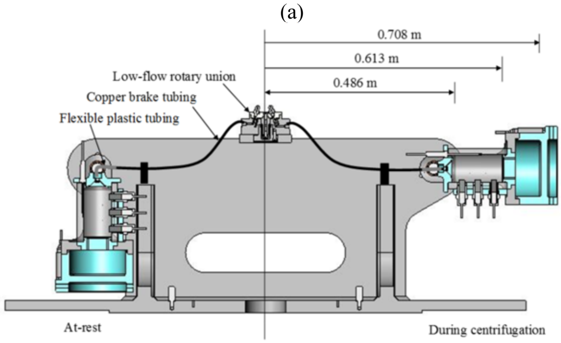
Much of the basis used in this study for characterization of the swelling of clays upon water infiltration relies on recent studies on the characterization of unsaturated flow using centrifuge techniques. This is because centrifugation was recently used to alleviate shortcomings of conventional characterization of the SWRC and K-function of soils. The key benefit stems from the fact that time in flow processes through soil has been shown to decrease quadratically with increasing g-level. The conditions of similarity for geotechnical structures tested in a centrifuge have often been inferred from general scaling relationships. Although modeling limitations are often difficult to overcome when the purpose of the investigation is to compare the performance of model and prototype structures, many of these limitations can be taken into account when the purpose is to identify mechanisms or to determine properties of the tested soils.

Centrifuge modeling has been used in investigations involving contaminant transport phenomena [58, 59, 60, 61, 62, 63], movement of immiscible fluids [64, 65, 66, 67, 68], unstable infiltration [69, 70], and soft soil desiccation [71]. Centrifuge modeling has the distinct advantage of being able to reproduce similitude of the suction-moisture content regime in unsaturated soils. Centrifuges were first used in the early 1930's to define the SWRC by soil scientists and petroleum engineers [72, 73]. The specific scaling relations for unsaturated soils have been investigated both analytically using dimensional analysis and experimentally using "modeling of models" [74, 58, 59, 75, 76]. These investigations showed that unsaturated flow problems should be analyzed using the same scaling relations as (saturated) laminar flow problems. However, only a limited number of studies has focused on centrifuge modeling of expansive clays [77, 78], with focus on the performance of earth structures rather than on clay characterization.

Centrifuge technology has been used in previous studies to decrease testing time when using steady-state infiltration for characterization of unsaturated soils. This was the motivation behind the development of the Steady-State Centrifuge (SSC) by Nimmo et al. [79] and of the Unsaturated Flow Apparatus (UFA) by Conca & Wright [80]. The UFA approach has been employed in geotechnical design to measure the hydraulic characteristics of the soil used in an alternative landfill cover [81]. However, the SSC and UFA use relatively small medical ultracentrifuges, which do not include a data acquisition system that is operational under high accelerations. Consequently, these systems did not permit concurrent measurement of the SWRC and K-function of soils.



(a)



(b)

Figure 2. Centrifuge permeameter at the University of Texas at Austin for characterization of unsaturated flow: (a) Centrifuge detail; (b) Testing environment.

The recent studies at the University of Texas at Austin have focused on the concurrent experimental measurement of both the SWRC and K-function using a new centrifuge permeameter [19, 20, 21]. Figure 2 shows the centrifuge permeameter, which allows expeditious measurement of the three key variables that govern water flow under unsaturated conditions (i.e. θ, ψ and K) in soils that are not subject to significant volume changes during infiltration (e.g. silts, clays with low plasticity). A key aspect of the testing approach developed in that study is that steady-state flow conditions could be achieved in a comparatively short testing time. Reaching steady-state conditions significantly facilitates the experimental measurement of the unsaturated hydraulic properties of soils.

A cross section of the centrifuge permeameter available at The University of Texas at Austin is shown in Figure 2(a). The centrifuge includes a testing environment and data acquisition hub resting atop a spindle and bearing assembly, which is supported by three vibration isolators mounted on a conical base pedestal. A central access shaft in the spindle permits wires and plumbing lines to pass through rotary joints from the data acquisition system and testing environment to the stationary environment. The testing environment is shown with added details in Figure 2(b). Two identical, instrumented permeameters are mounted on a swinging bucket assembly. The swinging buckets permit the longitudinal axis of the permeameter to be aligned with the resultant of the acceleration field. Characterization permeameters at rest (left) and spinning (right) are shown in Figure 2(b). A low-flow rotary union was designed to transmit low flow rates while preventing water loss and minimizing heat generation. The centrifuge is equipped with a solid-state data acquisition board (no moving disc drives).

The centrifuge permeameter at UT Austin has a maximum angular velocity of 875 RPM, which translates to a g-level of 600 at the base of the permeameter. The swinging buckets of the permeameter have a maximum payload of 50 kg for a g-ton rating of 30. This investigation capitalizes on the recent advances in unsaturated soil characterization using the centrifuge permeameter, but incorporates the additional ability to characterize the swelling of clays. This required that the testing environment be capable of measuring volume changes (ε_v) induced during infiltration. Ultimately, the overall goal is to concurrently and expeditiously measure the four variables that continuously change during infiltration of water into swelling clays (i.e. θ, ψ, K, and ε_v).

3. Analytical Framework

The analytical framework for water infiltration under unsaturated conditions in a centrifuge environment provides insight into the experimental approach developed in this study. Water flow occurs in response to a gradient in hydraulic potential, the dominant components of which are gravity and matric suction. These components are independent, as gravity is a body force, while suction is an air-water interface phenomenon. Unlike suction gradient, which varies over orders of magnitude, the gradient of elevation head is constant and equal to 1. Centrifugation is an alternative approach to increase the body force component of the hydraulic potential, the gradient of which may be greater in magnitude than the suction gradient. Centrifugation increases the body forces on a specimen by imposing a centripetal acceleration, a:

$$a = \omega^2 r = N_r g \tag{1}$$

where ω is the angular velocity of the centrifuge, r is the radius at a certain point in a soil specimen, g is the acceleration of gravity, and N_r is the ratio between the centripetal acceleration and g. The r subscript in N_r signifies that N varies with radius. When a single N_r is considered, it corresponds to the value at mid-height of the soil specimen $N_{r,mid}$ (also referred to as "g-level"). A control volume for water flow in the centrifuge is shown in Figure 3. A coordinate z_m is used, with datum at the base of a soil specimen, defined as:

$$z_m = r_0 - r \tag{2}$$

where z_m is the distance from the datum at the base of the specimen, r is the radius from the center of rotation, and r_0 is the radius of the base of the specimen. The cylindrical specimen with length L_m has an inlet face at a radius of r_T, and an outlet face at a radius of r_0. As z_m is defined as positive toward the axis of rotation, the water discharge velocity v_m is positive in the direction of positive z_m, toward the axis of rotation.

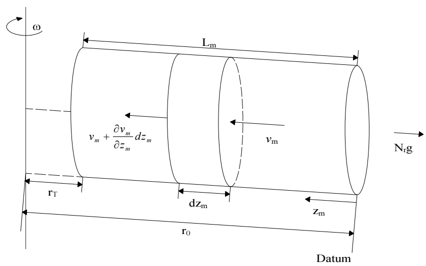


Figure 3. Centrifuge control volume [76].

The self-weight of water in the centrifuge acceleration field increases the driving force for water flow. The hydraulic potential in the centrifuge model Φ_m is quantified as:

$$\Phi_m = -\frac{1}{2}\left(\frac{I\omega^2}{m}\right) + \frac{1}{2}\left(\frac{v_m}{n}\right)^2 + \frac{P_w}{\rho_w} + \frac{P_O}{\rho_w} \tag{3}$$

where I is the rotational inertia of a point mass m in a centrifuge field. The components on the right hand side correspond to the rotational potential energy per unit mass induced by centrifugation, the kinetic energy per unit mass due to the relative linear velocity of the fluid with respect to the solids, the energy per unit mass due to the water pressure, and energy per unit mass due to the osmotic pressure. The sign of the first term is negative because the rotational kinetic energy increases in the opposite direction of the coordinate z_m. For a point mass m in a centrifuge field, the moment of inertia is given by:

$$I = m\left(r_0 - z_m\right)^2 \tag{4}$$

Assuming that centrifugation does not cause turbulent water flow, the kinetic energy term may be neglected. Also, the air pressure is assumed to be negligible, so the suction can be substituted for the water pressure as $P_w = -\psi$. Finally, the osmotic pressure can be assumed not to vary with moisture content for unsaturated soils, so the last term is not considered. Considering these assumptions, the hydraulic potential in the centrifuge is:

$$\Phi_m = -\frac{1}{2}\omega^2\left(r_0 - z_m\right)^2 + \frac{(-\psi)}{\rho_w} \tag{5}$$

Alternatively, the hydraulic potential can be written in terms of hydraulic head:

$$h = \frac{\Phi_m}{g} = -\frac{\omega^2}{2g}\left(r_0 - z_m\right)^2 - \frac{\psi}{\rho_w g} \tag{6}$$

where h is the total hydraulic head having units of length, the suction head equals the suction divided by $\rho_w g$. Similar to water flow under 1-gravity, the discharge velocity through a soil specimen in the centrifuge is proportional to the gradient in the total hydraulic potential. Darcy's law in the centrifuge is given by:

$$v_m = -\frac{K(\psi)}{g}\frac{d\Phi_m}{dz_m} \tag{7}$$

The gradient of the hydraulic potential with respect to z_m in Eq. (7) is:

$$\frac{d\Phi_m}{dz_m} = \omega^2 \left(r_0 - z_m\right) - \frac{1}{\rho_w}\frac{\partial\psi}{\partial z_m} \tag{8}$$

This equation can be incorporated into Eq. (7) to determine the discharge velocity:

$$v_m = -\frac{K(\psi)}{g}\left[\omega^2 \left(r_0 - z_m\right) - \frac{1}{\rho_w}\frac{d\psi}{dz_m}\right] \tag{9}$$

The first term within the brackets is the centrifuge acceleration, while the second term represents the suction gradient. This equation indicates that the discharge velocity varies with the radius of the centrifuge. Flow continuity requires that:

$$\frac{d\theta}{dt} = -\frac{dv_m}{dz_m} \tag{10}$$

Combining the continuity equation with Eq. (9), Richards' equation for one-dimensional flow of water through an unsaturated soil in a centrifuge results in:

$$\frac{d\theta}{d\psi}\frac{d\psi}{dt} = \frac{d}{dz_m}\left[K(\psi)\left[\frac{\omega^2}{g}\left(r_0 - z_m\right) - \frac{1}{\rho_w g}\frac{d\psi}{dz_m}\right]\right] \tag{11}$$

This equation has been solved numerically by Bear *et al.* [82] and Simunek and Nimmo [83]. Alternatively, analytical solutions can be derived for the case in which the K-function is represented using Gardner's [84] model. Dell'Avanzi et al. [76] derived an analytical solution for suction profiles during steady state water flow in the centrifuge, as follows:

$$\begin{aligned}
\psi\left(z_m\right) &= -\frac{1}{\alpha}\ln\left[e^{\left(\ln\left|\left(v_m/N_rK_s\right)+e^{-\alpha\psi_0}\right| - \alpha\omega^2\rho_w z_m\left(r_0 - z_m/2\right)\right)} - \frac{v_m}{N_rK_s}\right] \quad \text{if } \left(v_m/N_rK_s\right)+e^{-\alpha\psi_0} > 0 \\
\psi\left(z_m\right) &= -\frac{1}{\alpha}\ln\left[-e^{\left(\ln\left|\left(v_m/N_rK_s\right)+e^{-\alpha\psi_0}\right| - \alpha\omega^2\rho_w z_m\left(r_0 - z_m/2\right)\right)} - \frac{v_m}{N_rK_s}\right] \quad \text{if } \left(v_m/N_rK_s\right)+e^{-\alpha\psi_0} < 0
\end{aligned} \tag{12}$$

where ψ_0 is the suction at the outflow face of the centrifuge specimen. Suction profiles for a soil layer with an imposed surface discharge velocity and a saturated bottom boundary are shown in Figure 4 for values $\alpha = 1$ kPa^{-1} and $K_s = 10^{-6}$ m/s, and a normalized specimen geometry representative of the centrifuge permeameter used in this study.

The average g-level tends to influence both the distribution of suction with height as well as the suction in the upper portion of the specimen. v_m tends to influence the suction in the upper portion of the specimen. The magnitudes of suction in this figure are not important as they depend on the assumed values of α and K_s. However, a more important feature of these graphs is the distribution of suction with height in the specimen. Independent of the suction (or saturation) condition at the bottom, the suction does not vary significantly in the upper portion of the specimen length, even for low $N_{r,mid}$. Because N_r varies with radius, the suction distribution in the upper portion of the specimen shows a minor gradient. However, for this particular set of hydraulic characteristics and specimen geometry, it may be assumed that the suction is constant in this zone. Accordingly, the suction magnitude at the outflow boundary does not have a significant effect on the shape of the suction profile in the upper zone of the specimen, as indicated by the suction profiles in Figure 4.

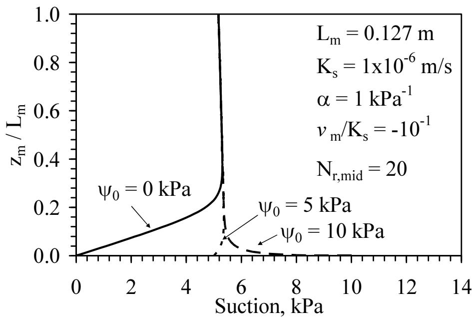


Figure 4. Steady state suction profiles for different: (a) $N_{r,mid}$; (b) v_m; (c) ψ_0.

Given the value of α in the Gardner [84] model, the limiting suction in the upper zone of the soil layer may be predicted as [76]:

$$\psi_\infty = -\frac{1}{\alpha}\ln\left(-\frac{v_m}{N_{r,mid}K_s}\right) \tag{13}$$

Unlike the 1-g case, the limiting suction is valid when $N_r = N_{r,mid}$ throughout the profile. Using the calculated components of the total head gradient, the distribution in K under steady state water flow can be determined using Darcy's law, as:

$$K(\psi, z_m) = \frac{-v_m}{\left[\frac{\omega^2}{g}(r_0 - z_m) - \frac{1}{\rho_w g}\frac{d\psi}{dz_m}\right]} \tag{14}$$

where v_m is an imposed, discharge velocity (equal to Q/A) and ω is an imposed centrifuge angular velocity.

4. A New Centrifuge for Characterization of Expansive Clays

The recent experimental and analytical developments that led to the expeditious determination of the properties governing water flow in soils under unsaturated conditions (i.e. SWRC and K-function) were adopted for the expeditious determination of the swelling of soils. This is because the time involved in the swelling process, which is often significantly long and has compromised its direct determination in practical applications, is directly governed by the timed involved in the water infiltration process. Accordingly, adding monitoring of volumetric changes to the previously described equipment would result in the ultimate goal of concurrently and expeditiously measuring the four variables that continuously change during infiltration of water into swelling clays (i.e. θ, ψ, K, and ε_v). While this ambitious goal was pursued at the University of Texas at Austin (see Quaglia [85]), the focus of this paper is on a new experimental technique that simplifies the experimental approach for cases in which the focus is strictly the characterization of volumetric changes.

4.1. Centrifuge components

The centrifuge used in this research study is a floor-mounted centrifuge used for a variety of industrial purposes. This is a considerably simpler equipment than the one described in Section 2.3, which involves a hydraulic rotary joint and a comprehensive in-flight data acquisition system. The floor-mounted centrifuge contains six hangers that hold freely swinging aluminum centrifuge cups. The setup of the centrifuge is fairly customizable, as the contents of the centrifuge cups were altered to fit requirements of different tests. Plastic permeameter cups that fit inside the centrifuge cups were designed and manufactured specifically for this equipment. A view of the centrifuge cups and the floor-mounted new centrifuge is shown in Figure 5 [86].

(a) (b)

Figure 5. Centrifuge equipment for characterization of expansive clays: (a) Centrifuge permeameters, (b) centrifuge system [86].

The original testing approach involved ponding water on top of a compacted soil sample and spinning the sample at comparatively high g levels (ranging from 25 to 400 g). As previously discussed, the increased g level leads to an increased hydraulic gradient that forces the water through the samples at an increased rate, promoting expeditious swelling of the clay. The centrifuge cups (Figure 6a) hang from the spinning centrifuge arms. The holders have an inner diameter of 63.5 mm and a usable inside depth of 114.3 mm. The base of the specimen holder includes a small vent hole to allow air and water outflow. While in-flight, the distance from the base of a sample to the center of rotation in the small centrifuge is 165 mm.

The permeameter cups (Figure 6b) fit inside the centrifuge cups with an outside diameter of 63.3 mm and a depth of 114.3 mm. The cups have an inside diameter of 57.2 mm at the top, which is reduced to 47.1 mm approximately 25 mm from the base of the cups. This reduction was adopted to form a ledge that allows a porous plate to support soil samples. The base of the cup is removable and is used as a liquid collection system. Outflow can be measured by measuring the increase in weight of the collection cup. A small air vent (visible in Figure 6b) connects the collection cup to the area above the sample to allow equalization of air pressure between the chambers located above the ponded water and at the bottom of the sample [87].

The porous supporting plate (Figure 6c) sits on top of the ledge in the permeameter cup and creates a firm yet pervious surface to place specimens. The plate contains 0.8 mm-diameter holes that allow water to flow freely from the base of the specimen. To avoid migration of soil particles, a filter paper is placed between the porous plate and the soil specimen. A rubber permeameter cap fits inside the top of the permeameter cup in order to minimize evaporation during testing. The rubber cap provides an airtight seal once the centrifuge is in flight.

(a) (b) (c)

Figure 6. Centrifuge components: a) Centrifuge cup b) Permeameter cup c) Porous disc.

An alternative centrifuge cup was developed, which allows for testing undisturbed soil specimens collected in the field using Shelby tube samplers. In order to accommodate testing of field specimens from push samplers, the boundary conditions and permeameter cup design were changed to incorporate cutting rings. A comparison between the original centrifuge cup design and the double infiltration cup design is shown in Figure 7.

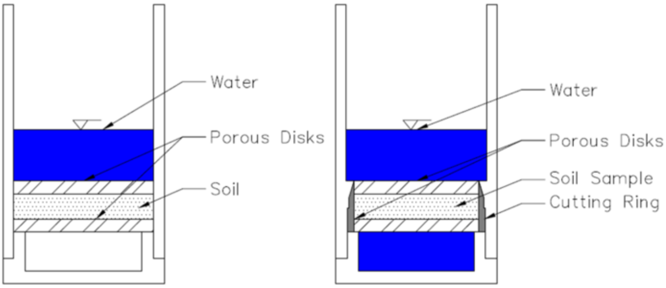


Figure 7. Single infiltration and double infiltration permeameter cups.

In order to allow centrifuge testing of undisturbed soil specimens, a method was developed to moisture condition (dry) the field-collected specimens. The procedure was developed after verifying that a rapid rate of drying results in shrinkage cracks that compromise the subsequent centrifuge testing [88]. To retard the rate of drying, an environmental chamber with a relatively constant, high relative humidity was developed, which involves a glove box and the use of saturated salt solutions. The time to reach the targeted moisture content (e.g. 3 moisture percentage points dry of optimum) may be as long as 36 to 72 hours, but the approach allowed reaching the target initial moisture content without developing shrinkage cracks in the soil specimens.

4.2. Instrumentation and in-flight data acquisition

A data acquisition system (DAS) was developed for the new centrifuge. The system includes a battery housing unit, a linear position sensor (LPS) for each permeameter cup that records changes in height of the clay specimen, a JeeNode (Version 6) Arduino along with the analog-to-digital converter (ADC), and an accelerometer to measure the g-level experienced by the specimen. These components are displayed in Figure 8.

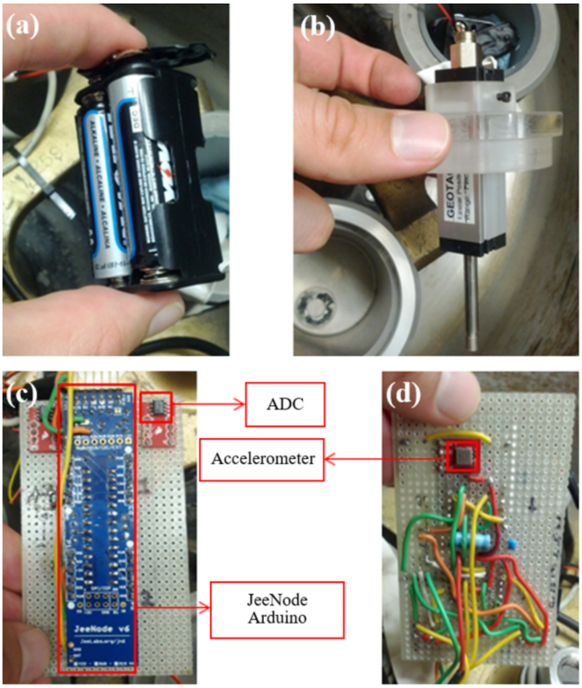


Figure 8. Components of the Data Acquisition System in the Swell Centrifuge: (a) Battery Housing Unit; (b) Linear Position Sensor; (c) JeeNode Arduino and Analog-to-Digital Converter; (d) Accelerometer [89].

The JeeNode Arduino involves a programmable microchip that controls and interfaces with the other DAS components through serial communication (RS232). The microchip sends a signal, notifying the ADC to take displacement and acceleration readings. The ADC converts the voltage to digital readings and sends them to the Arduino along with the digital accelerometer reading. The internal JeeNode Arduino, communicates the readings to an external JeeNode Arduino via wireless radio. The external Arduino transfers the readings to a Labview program, which translate the readings into specimen heights.

5. Testing Procedure

Reconstituted soil specimens are prepared to a target density using kneading or static compaction. The soil specimen is initially subjected to a seating load by raising the g-level to 2 to 3 g's and allowing the compression of the soil at a low seating pressure. This cycle is consistent with the seating load applied in conventional swell tests.

The compression cycle involves raising the g-level on the soil specimen (without access to water) for several minutes and then stopping the centrifuge. This allows measuring the compression due to the increased load during centrifugation. In tests involving undisturbed in-situ soil specimens, the g-level should be selected to replicate in-situ field stresses. In tests involving reconstituted soil specimens, they have been generally tested at the g-levels of 5, 25, and 200. These g-levels result in effective stresses ranging from approximately 0.47 kPa up to 95.76 kPa, which are representative of the stress range typical to the active zone for expansive clays.

After the initial compression cycle, the specimen is removed from the centrifuge, decreasing the applied overburden. Water is then added to the cups, which are subsequently placed in the centrifuge. The centrifuge is then run again, and allowed to spin for 24 to 48 hours. Primary swelling of the sample is typically reached within one day, and the sample is often tested for some additional time to ensure that secondary swelling has begun. After the test is completed, cups are removed from the centrifuge, and the cup and cup base with water are then weighed. Final sample heights are measured to determine the final swell in comparison to that recorded by the sensors while spinning. The soil is then extruded and placed on a metal tray for oven-drying to measure the gravimetric water content of the soil.

6. Typical Centrifuge Swell Test Results

6.1. Swell vs. Time Test Results

Test results from three soils are presented herein: Eagle Ford shale, Houston Black clay, and Black Taylor clay. The initial specimen conditions considered as baseline are an initial moisture corresponding to the soil optimum moisture content and an initial unit weight corresponding to the soil compacted to a relative compaction of 97% in relation to the Standard Proctor test. For example, the Eagle Ford baseline condition consisted of a soil with a moisture content of 24% (± 0.5%) and a dry unit weight of 14.34 kN/m^3. A parametric evaluation was conducted, which involved specimens prepared at a moisture content ranging from the soil optimum moisture content -3% to the optimum moisture content + 3%. The specimens were prepared to relative compaction values of 94%, 97%, and 100% in relation to the dry unit weight as determined by the standard proctor test. The specimens were tested under g-levels of 5, 25, and 200 g's in order to generate the stress-swell curves. These g-levels correspond to confining stresses of approximately 1.44, 4.79, and 47.88 kPa.

For each test, the data recorded is the change in height of a soil at a specified time interval. From this data, the swelling of a specimen can be continuously defined throughout the test, generating a swell versus time curve. A typical swell-over-time curve is shown in Figure 9. The conditions for this Eagle Ford clay specimen are those corresponding to its baseline conditions.

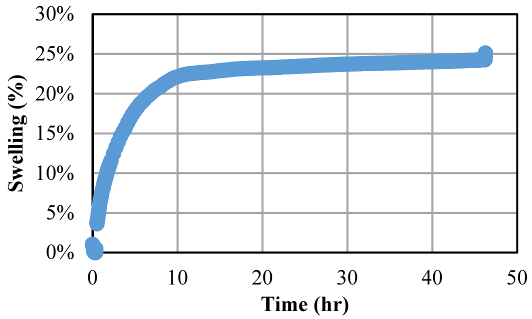


Figure 9. Time-history of swelling in a centrifuge test using a specimen of Eagle Ford clay.

As the figure illustrates, the first portion of the swelling curve is where the primary swell occurs, involving significant swell changes as the water enters the voids. This stage may take some 10 hours, until the swelling reaches an inflection point, and then continues to increase at a slower rate corresponding to the secondary swelling. The inflection point is often used to report the soil swelling.

Table 1 contains the average swelling data at the baseline conditions for the three most tested soils (Eagle Ford, Houston Black, and Black Taylor) for each of the tested g-levels.

Table 1. Baseline conditions for the clays evaluated in this study.

Soil	w_{opt} (%)	Target Dry Unit Weight, (kN/m^3)	Swelling (%) @ 5g	Swelling (%) @ 25g	Swelling (%) @ 200g
Eagle Ford	24	14.79	24.68	15.65	7.26
Houston Black	25.5	14.28	6.13	5.34	1.46
Black Taylor	23.3	14.28	3.92	2.65	2.26

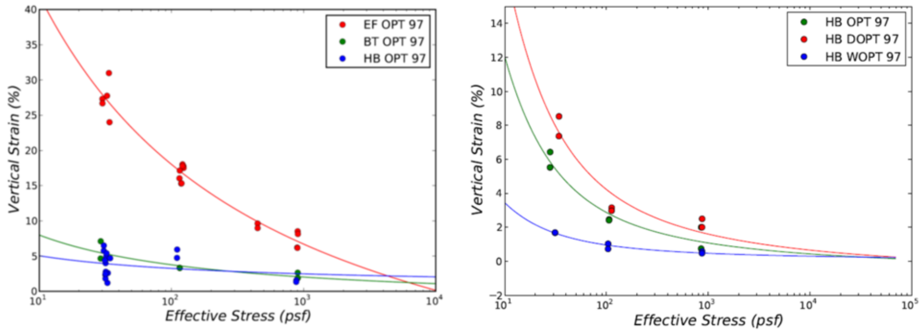
The most highly plastic soil, the Eagle Ford shale, shows the highest average swelling per test. However, the Houston Black clay does show a higher average swelling for the 5-g and 25-g tests than the Black Taylor, even though the Houston Black clay has a higher PI than the Black Taylor clay.

6.2. Swell-Stress Curves

The swell-stress curves were defined for three soils for which a large suite of tests were conducted. They include the Eagle Ford, Black Taylor, and Houston Black clays. The resulting curves are shown in Figure 10. The Eagle Ford clay shows significantly higher swelling potential than the Black Taylor and Houston Black. The Black Taylor and

Houston Black clays show similar swelling, with the Black Taylor having a slightly higher swell-stress curve.

Curves based on samples with varied compaction moisture and density were also obtained. The results in Figure 10b show the effect of compaction moisture content on swelling for the case of the Houston Black Clay.



(a) (b)

Figure 10. Swell stress Curves: (a) for three different soils; (b) for Houston black at three different initial moisture content values. Notes: EF = Eagle Ford; BT = Black Taylor; HB = Houston Black

6.3. Comparison between Centrifuge with Standard Swell Test Results

The swell results obtained from the centrifuge testing were compared against those from standard swell test results, using specimens with the same initial unit weight and water content. The results are shown in Figure 11. The centrifuge results showed excellent consistency with the standard swell test results, as shown in Figure 11 for the case of Eagle Ford clay. Equally consistent results were obtained for the other clays.

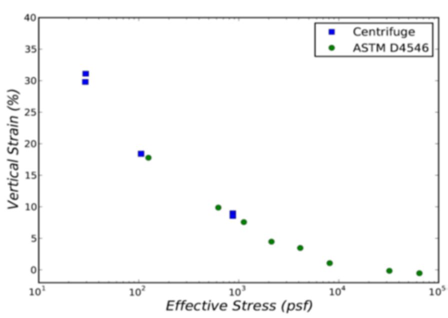


Figure 11. Comparison between centrifuge and standard swell test results (Eagle Ford clay)

7. Conclusions

Characterization of soil volumetric strains as an additional variable within the framework of unsaturated hydraulic characteristics adds complexity to the already intricate, non-linear relationships representing the response of the volumetric water content and hydraulic conductivity as a function of matric suction. From the experimental point of view, proper characterization of the unsaturated hydraulic characteristics of expansive clays is not only complicated by the need to control an additional variable (volume changes), but also because these soils have particularly low hydraulic conductivity values, which adds significant challenges in terms of testing time and accuracy of measurements.

This investigation used centrifuge technology as an alternative to alleviate the shortcomings in the characterization of the unsaturated hydraulic characteristics, and associated volumetric changes, of expansive clays. Specifically, centrifuge technology was adopted for continuous and expeditious measurement of the changing soil moisture content, suction, hydraulic conductivity and void ratio that occur during unsaturated flow processes. The use of centrifuge technology allowed accurate and expeditious determination of the swelling of clays, including the determination of the swell-stress relationship. Determination of this relationship is particularly relevant because the financial losses caused by problems associated with expansive clays correspond to the highest costs associated with natural hazards in the US.

The new centrifuge approach developed in this study for characterization of swelling of clays involves subjecting soil specimens to water infiltration during comparatively small testing periods. The results indicate that, in spite of the significantly more practical and expeditious characteristics of the new approach, the predicted swell-stress curve is the same as that obtained using time-consuming conventional experimental techniques. The centrifuge approach was found to be particularly appropriate for determination of the Potential Vertical Raise, a magnitude that has often be used for the design of roads founded on expansive clays.

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