Advances in Parallel Computing Algorithms, Tools and Paradigms
D.J. Hemanth et al. (Eds.)

doi:10.3233/APC220060

Trident Fuzzy Aggregation Operators on Right and Left Apex-Base Angles in Parallel Computing

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Abstract. In this paper, a new concept called Trident Fuzzy Aggregation Operators in which the operation is made by the grouping of numerous Trapezoidal Trident Fuzzy Numbers (TTriFN) to become a solitary trapezoidal trident fuzzy numeral for parallel computing application is introduced. This proposed operator in computing architecture is applicable in the development of fuzzy networking and ranking concepts. Also, the geometric mean over 'n' TTriFN on the left-side apex-base angles and right-side apex-base angles are introduced in the architecture development process.

Keywords. Trident fuzzy aggregation, trident form, fuzzy number, apex angle.

1. Introduction

The concept of the Fuzzy Set system is presented in the year 1965 [1] and Intuitionistic Fuzzy Aggregation Operators in 2007 [2]. In the year 2012, the novel accumulation operator for trapezoidal fuzzy quantities created on the mathematical earnings of the left and right Apex Angles (AA) is given [3] [4]. A notion of averaging operators which is represented by using generators of the product triangular norm is given through Interval-valued intuitionistic fuzzy aggregation operators [5]. Moreover, the ranking concept of trapezoidal fuzzy numbers based on AAs is proposed by [6]. In 2015, apex based least-cost technique for fuzzy transportation problems is given [7]. A new approach called Trident Fuzzy Number is proposed and its properties are discussed [8] [9]. Further, a fuzzy optimum solution through pascal's triangle graded mean and harmonization of a square fuzzy changeover likelihood relative matrix is proposed by [10] [11] which paves the way for trapezoidal trident fuzzy number. Then, a novel mixture prototype named the multifaceted cubic fuzzy set was introduced by [12], and also the multifaceted cubic fuzzy biased averaging, geometric operators, and its properties are introduced through complex power aggregation operators.

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Archimedian t-norm and t-conorm built combination operatives for hexagonal fuzzy numbers along with the method of refining instruction. Moreover, fuzzy aggregation for ranking in business [13].Methodology for computing applications can be implemented for the networking optimization of various applications [14] [15]. This study includes the following sections: preliminaries, fuzzy aggregation, proposed method: Trident fuzzy aggregation,right andleft apex-base angle for trapezoidal trident fuzzy number, and the Geometric mean over 'n' trapezoidal trident fuzzy numbers for left-side and right-side apex-base angle and lastly, the deduction is constructed.

2. Preliminaries

Mathematical representation of vagueness or impreciseness in our everyday life is given in the fuzzy sets and it is introduced by Lotfi. A. Zadeh.

2.1. Fuzzy Set and Fuzzy Number

Fuzzy Set is represented by the set $'A'$ in X and is considered by its membership value$\mu_A(x)$ from A to [0,1] which signifies the grade of the membership. A generalized fuzzy set is represented by,$A = \{(x, \mu_A(x)) | x \in X\}$.Fuzzy Number (FN)is the set A, on R (real numbers) is named as anFN with membership function, $A: R \to [0, 1]$. Characteristics of FNare: (a) A is convex if $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min \{\mu_A(x_1), \mu_A(x_2)\}\ \forall x_1, x_2 \in X, \lambda \in [0,1]$.(b) A is normal if $\exists$ X $\in R \ni \max \mu_A(x) = 1$, and(c) $\mu_A(x)$is piecewise continuous.

2.2. Trident Fuzzy Number and Trapezoidal Trident Fuzzy Number

Generalized Trident FN'S' of any subset of a fuzzy set of the real line R, where the following conditions are satisfied by its membership valueμ_s:

i. $\mu_s$is a continuous from R to [0, 1].
ii. $\mu_s(x) = 1^{1/3}, -\infty < x \leq a_{T_i}$.
iii. $\mu_s(x) = [L(x)]^{1/3}$, is strictly decreasing on $[a_{T_i}, b_{T_i}]$.
iv. $\mu_s(x) = w^{1/3},\ b_{T_i} \leq x \leq c_{T_i}$.
v. $\mu_s(x) = [R(x)]^{1/3}$, is strictly increasing on $[c_{T_i}, d_{T_i}]$.
vi. $\mu_s(x) = 1^{1/3}, d_{T_i} \leq x \leq \infty$, where $0 \leq w \leq 1$ and $a_{T_i}, b_{T_i}, c_{T_i}, d_{T_i}$ are real numbers.

Trapezoidal Trident FN(TTriFN) denoted by $S = \langle s_1, s_2, s_3, s_4 \rangle$ where all s_1, s_2, s_3, s_4 are real numbers and anFN$\mu_s(x): R \to [0,1]$ is represented by its membership function is represented as follows:

$$\mu_s(x) = \begin{cases} 1^{1/3}, x < s_1 \ ; \left[\frac{s_2 - x}{s_2 - s_1}\right]^{1/3}, s_1 \leq x \leq s_2 \ ; \ 0, s_2 \leq x \leq s_3 \\ \left[\frac{x - s_3}{s_4 - s_3}\right]^{1/3}, s_3 \leq x \leq s_4 \ ; \ 1^{1/3}, x > s_4 \end{cases}$$

3. Fuzzy Aggregation (FAgg)

Operations are done bycombining a few trapezoidal FNs to give single trapezoidal FN and this concept is represented as Fuzzy-Aggregation (FAgg) operations. The Fuzzy-Aggregation Operations are as follows:
Arithmetic-mean -The aggregation-operator called Arithmetic-mean defined on 'n' trapezoidal
FNs $\langle l_1, m_1, n_1, o_1\rangle, \langle l_2, m_2, n_2, o_2\rangle, \langle l_3, m_3, n_3, o_3\rangle \ldots \langle l_i, m_i, n_i, o_i\rangle \ldots \langle l_n, m_n, m_n, o_n\rangle$ is given by $\langle l, m, n, o\rangle$ where$l = \frac{1}{n}\sum_1^n l_i$, $m = \frac{1}{n}\sum_1^n m_i$, $n = \frac{1}{n}\sum_1^n n_i$, $o = \frac{1}{n}\sum_1^n o_i$.
Geometric-Mean -The aggregation-operator called Geometric-mean defined on 'n' trapezoidal
FNs $\langle l_1, m_1, n_1, o_1\rangle, \langle l_2, m_2, n_2, o_2\rangle, \langle l_3, m_3, n_3, o_3\rangle \ldots \langle l_i, m_i, n_i, o_i\rangle \ldots \langle l_n, m_n, m_n, o_n\rangle$ is given by $\langle l, m, n, o\rangle$ where $l = (\prod_1^n l_i)^{\frac{1}{n}}$, $m = (\prod_1^n m_i)^{\frac{1}{n}}$, $n = (\prod_1^n n_i)^{\frac{1}{n}}$, $o = (\prod_1^n o_i)^{1/n}$.

4. Proposed Method

For this study, the following proposedmethod is given in sections:
Trident Fuzzy-Aggregation (TriFAgg) - Operations done on combining few trapezoidal trident FNs to give single trapezoidal trident FN and this concept is represented as TriFAgg operations. The TriFAgg Operations are as follows:

(a)The aggregation-operator called Trident Arithmetic-mean defined on 'n' trapezoidal trident
FNs $\langle l_1, m_1, n_1, o_1\rangle, \langle l_2, m_2, n_2, o_2\rangle, \langle l_3, m_3, n_3, o_3\rangle \ldots \langle l_i, m_i, n_i, o_i\rangle \ldots \langle l_n, m_n, m_n, o_n\rangle$ is given by $\langle l, m, n, o\rangle$ where$l = \frac{1}{3n}\sum_1^n l_i$, $m = \frac{1}{3n}\sum_1^n m_i$, $n = \frac{1}{3n}\sum_1^n n_i$, $o = \frac{1}{3n}\sum_1^n o_i$.

(b) The aggregation-operator called Trident Geometric-mean defined on 'n' trapezoidal trident FNs$\langle l_1, m_1, n_1, o_1\rangle, \langle l_2, m_2, n_2, o_2\rangle, \langle l_3, m_3, n_3, o_3\rangle \ldots$
$\langle l_i, m_i, n_i, o_i\rangle \ldots \langle l_n, m_n, m_n, o_n\rangle$ is given by $\langle l, m, n, o\rangle$ where $l = (\prod_1^n l_i)^{\frac{1}{3n}}$, $m = (\prod_1^n m_i)^{\frac{1}{3n}}$,$n = (\prod_1^n n_i)^{\frac{1}{3n}}$, $o = (\prod_1^n o_i)^{\frac{1}{3n}}$.

Right and Left Apex-Base Angle for TTriFN - Figure 1 depicts a TTriFN $\langle p, q, r, s\rangle$ with the value v, the membership grade or possibility μ along the co-ordinate axes (x and y). The right and left side AAs are givenby the arcs denoted bysingle and double dashes respectively. Also, the points of the trapezium are given by (p,0), (q,1), (r,1), and (s,0) moving in the positive (clockwise) direction.

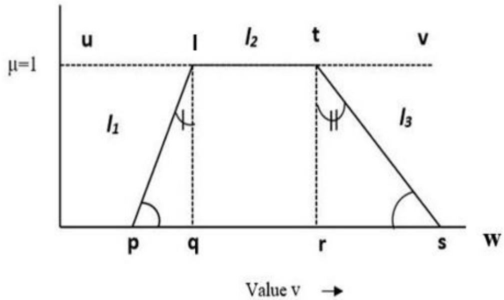

Figure 1. Trapezoidal Fuzzy Number.

The lines between the vertices (p, 0) and (q, 1), also the lines between the vertices (r, 1) and (s, 0) are the membership-function with intervals [p, q] and [r, s]. The membership value of TTriFN is the piecewise-continuous linear-function are l1, l2, and l3. The intuitiveness of TTriFN is given by the trident fuzzy value in between any of the functions inside the interval or the range [q, r]. The trident FN varies in the range [p, s]. The membership grade has a specified value v between p and s, v ϵ [p, s] is denoted by the coordinate of the projection of v on l1, l2, or l3 accordingly as $p \leq v \leq q$ or $q \leq v \leq r$ or $r \leq v \leq s$. Here, [p, q] is the interval or the range which is represented by its membership grade as a specificvalue less than q,[r, s] is the interval or the range which is represented by its membership grade as a specificvalue more than r. Membership grade is taking the value up to the maximum of 1 in between the interval or the range [q, r] and reduction with the increment in the distance on both left-hand and right-hand sides of the interval [q, r] and it becomes 0 beyond the value p at the left-hand and the value s at the right-hand sides. The right and left apex-base angle is represented from the TTriFN shown in figure 1. If the value v ϵ [q, r] in this TTriFN then the corresponding possibility is μ =1.In the above figure 1, the left-hand side apex-angle and the base-angle for trapezoidal trident FNare given by ∠psq and ∠sp. Similarly, the right-hand side apex-angle and the base-angle for trapezoidal trident FNare given by ∠str and ∠tsr.Here, the left-side and right-side AAs are subtended to the left and right side of the interval [q, r] respectively. Here, the left apex and left base angles are given in (1) and the right apex and right base angles are given in (2) as follows:

$$\left.\begin{aligned} \angle plq &= \frac{\Pi}{2} - \angle qpl \text{ (left apex)} \\ \angle lpq &= \frac{\Pi}{2} - \angle plq \text{ (left base)} \end{aligned}\right\} \text{--- (1)} \qquad \left.\begin{aligned} \angle str &= \angle wst - \frac{\Pi}{2} \text{ (right apex)} \\ \angle tsr &= \angle qtv - \frac{\Pi}{2} \text{ (right base)} \end{aligned}\right\} \text{--- (2)}$$

5. Geometric Mean over ‘n’ TTriFN

The Geometric mean oven ‘n’ trapezoidal trident FNs for the right-side AA and the left-side AAs are discussed in this section.
Left side AA - The left-hand side and geometric mean over ‘n’ trapezoidal trident FN is given by.

$$\left(\prod_{1}^{n}(\angle psq)_i\right)^{\frac{1}{3n}} = \left(\prod_{1}^{n}\left(\frac{\Pi}{2} - \angle qps\right)_i\right)^{\frac{1}{3n}} = \frac{\Pi}{2} - \left(\prod_{1}^{n}(\angle qps)_i\right)^{\frac{1}{3n}} \quad - \quad (3)$$

The left-hand side of the above equation (3) is represented by its contributions on the left-side lines (i.e) l1's of all TTriFN to the aggregate-apex-angle. From (3), it can be viewed by,

$$tan\left(\prod_1^n(\angle psq)_i\right)^{\frac{1}{3n}} = \tan\left(\frac{\Pi}{2} - \left(\prod_1^n(\angle qps)_i\right)^{\frac{1}{3n}}\right) = \cot\left(\prod_1^n(\angle qps)_i\right)^{\frac{1}{3n}} = 1/\tan\left(\prod_1^n(\angle qps)_i\right)^{\frac{1}{3n}}$$

Also, it can be shown as,

$$tan\left(\prod_1^n(\angle psq)_i\right)^{\frac{1}{3n}} = tan\left(\prod_1^n \tan^{-1}\tan\angle psq_i\right)^{\frac{1}{3n}} = tan\left(\prod_1^n \tan^{-1}(q_i - p_i)\right)^{\frac{1}{3n}} - \quad \textbf{(4)}$$

and

$$tan\left(\prod_1^n(\angle psq)_i\right)^{\frac{1}{3n}} = q - p - \quad \textbf{(5)}$$

; Equating equations (4) and (5) we get,

$$q - p = tan\left(\prod_1^n \tan^{-1}(q_i - p_i)\right)^{\frac{1}{3n}} \quad \text{and} \quad p = q - tan\left(\prod_1^n \tan^{-1}(q_i - p_i)\right)^{\frac{1}{3n}}$$

Proceeding in the same way,

$$q = \left(\prod_1^n q_i\right)^{\frac{1}{3n}}$$

Repeating the above process along the left side, it is possible to show that the left side apex-base angles are given as follows:

$$\left.\begin{aligned} p &= \left(\prod_1^n q_i\right)^{\frac{1}{3n}} - tan\left(\prod_1^n \tan^{-1}(q_i - p_i)\right)^{\frac{1}{3n}} & r &= \left(\prod_1^n s_i\right)^{\frac{1}{3n}} - tan\left(\prod_1^n \tan^{-1}(s_i - r_i)\right)^{\frac{1}{3n}} \\ q &= \left(\prod_1^n p_i\right)^{\frac{1}{3n}} + tan\left(\prod_1^n \tan^{-1}(q_i - p_i)\right)^{\frac{1}{3n}} & s &= \left(\prod_1^n r_i\right)^{\frac{1}{3n}} + tan\left(\prod_1^n \tan^{-1}(s_i - r_i)\right)^{\frac{1}{3n}} \end{aligned}\right\} \text{---}\textbf{(6)}$$

The value of p, q, r, s is given in the above equation (6).

Right side AA - The right-hand side and geometric-mean over 'n' trapezoidal trident FN is given by,

$$\left(\prod_1^n(\angle str)_i\right)^{\frac{1}{3n}} = \left(\prod_1^n\left(\angle wst - \frac{\Pi}{2}\right)_i\right)^{\frac{1}{3n}} = \left(\prod_1^n(\angle wst)_i\right)^{\frac{1}{3n}} - \frac{\Pi}{2} - \quad (7)$$

The right-hand side of the above equation (7) is represented by its contributions on the right-side lines (i.e) l_3' of all TTriFN for the aggregate-apex-angle. From equation (7), it can be viewed by,

$$tan\left(\prod_1^n(\angle str)_i\right)^{\frac{1}{3n}} = \tan\left(\left(\prod_1^n(\angle wst)_i\right)^{\frac{1}{3n}} - \frac{\Pi}{2}\right) = -\cot\left(\prod_1^n(\angle wst)_i\right)^{\frac{1}{3n}} = -1/\tan\left(\prod_1^n(\angle wst)_i\right)^{\frac{1}{3n}}$$

Also, it can be shown as,

$$tan\left(\prod_1^n(\angle str)_i\right)^{\frac{1}{3n}} = -tan\left(\prod_1^n \tan^{-1}\tan\angle wst_i\right)^{\frac{1}{3n}} = -tan\left(\prod_1^n \tan^{-1}(w_i - s_i)\right)^{\frac{1}{3n}} - \quad \textbf{(8)}$$

$$tan\left(\prod_{1}^{n}(\angle \mathrm{str})_i\right)^{\frac{1}{3n}} = -(w-s) = s-w \quad \text{-- (9)}$$

and, Equating equations (8) and (9) we get,

$$s-w = -tan\left(\prod_{1}^{n}\tan^{-1}(w_i-s_i)\right)^{\frac{1}{3n}} \quad \text{and} \quad s = w - tan\left(\prod_{1}^{n}\tan^{-1}(w_i-s_i)\right)^{\frac{1}{3n}}$$

Proceeding in the same way,

$$w = \left(\prod_{1}^{n}\mathrm{w}_i\right)^{\frac{1}{3n}}$$

Repeating the above process along the right side, it is possible to show that the right-side apex-base angles are as follows,

$$\left.\begin{aligned} s &= \left(\prod_{1}^{n}\mathrm{w}_i\right)^{\frac{1}{3n}} - tan\left(\prod_{1}^{n}\tan^{-1}(w_i-s_i)\right)^{\frac{1}{3n}} \quad & q &= \left(\prod_{1}^{n}\mathrm{p}_i\right)^{\frac{1}{3n}} - tan\left(\prod_{1}^{n}\tan^{-1}(q_i-p_i)\right)^{\frac{1}{3n}} \\ r &= \left(\prod_{1}^{n}\mathrm{s}_i\right)^{\frac{1}{3n}} + tan\left(\prod_{1}^{n}\tan^{-1}(w_i-s_i)\right)^{\frac{1}{3n}} \quad & p &= \left(\prod_{1}^{n}\mathrm{q}_i\right)^{\frac{1}{3n}} + tan\left(\prod_{1}^{n}\tan^{-1}(q_i-p_i)\right)^{\frac{1}{3n}} \end{aligned}\right\} \text{--(10)}$$

The value of p, q, r, s is given in the above equation (10).
Equation (6) and (10) represents the right side and left side apex-base angles.

6. Conclusion

Here the new concept called Trident Fuzzy Aggregation using trapezoidal trident fuzzy numbers (TTriFN) for parallel computing architecture is discussed. The Geometric mean over ‘n’ TTriFN for the right side and left side apex-base angles was found along with the help of TTriFN. This idea can be extended for pentagonal trident fuzzy numbers, hexagonal trident fuzzy numbers, and so on in computing applications. The proposed concept can also be applied in various distributed networking models and ranking concepts to implement these trident fuzzy aggregation operators effectively.

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