Unconventional Methods for Geoscience, Shale Gas and Petroleum in the 21st Century
J. Watada et al. (Eds.)

doi:10.3233/AERD230022

Derivative of Disproportion Functions for Pattern Recognition

Viktor V. AVRAMENKO[2], Nataliya I. KALASHNYKOVA*[,3],
Viacheslav V. KALASHNIKOV[3], Junzo WATADA[1]

[1]*Professor Emeritus, Waseda University, Tokyo, Japan;*
an invited researcher at IPS research center, Faculty of Science and Engineering, Waseda University, 2-2 hibikino, Wakamatsu, Kitakyushu, Fukuoka, 808-0135; +81-90-3464-4929 mobile; watada@waseda.jp
[2] *SumDU, (Sumy State University), Sumy, Ukraine 40007; avr@sumdu.edu.ua*
[3]*Universidad Autónoma de Nuevo León (UANL), ave. Universidad S/N, Ciudad Universitaria, San Nicolás de los Garza, Nuevo León, Mexico 66455; nataliya.kalashnykova@uanl.edu,mx; viacheslav.kalashnikovx@uanl.edu.mx*
* *Corresponding author: Viacheslav V. Kalashnikov, Universidad Autónoma de Nuevo León (UANL); viacheslav.kalashnikovx@uanl.edu.mx*

Abstract. In this chapter, we explore the application of the derivative of disproportion functions in developing a cryptographic system and a pattern recognition technique. Firstly, we present an algorithm that utilizes focal functions, known as functions of disproportion, in a cryptographic system. The transmitted symbols are encrypted using the weighted sum of at least two of these functions with randomly generated coefficients. Numerical experiments demonstrate the robustness and reliability of the proposed procedure.
Furthermore, we demonstrate how the derivative of disproportion functions can govern a dynamic process, serving as a tool to identify the form of a real-valued function. This process enables us to determine the class to which the function belongs, independently of its unknown parameter values.

Keywords: Pattern recognition, cryptographic systems, derivative of disproportion functions, decoding algorithms, recognition schemes.
Key function -> focal function; derivative of disproportion function -> derivative of disproportion function.

Acknowledgments:
The authors would like to acknowledge the support and financial assistance provided by Universidad Autónoma de Nuevo León (UANL) through the corresponding author, Viacheslav V. Kalashnikov.

1: Introduction

Pattern recognition plays a crucial role in industries such as Geology, Petroleum, and Shale Gas, where analyzing and classifying patterns is essential (confer, for example [9]-[13]). This paper aims to provide an effective mathematical tool for pattern recognition in these industries.

Cryptography is indispensable for protecting valuable information against unauthorized access, given the high value of information in industries and society. Cryptographic systems, which are part of information systems, employ techniques such as the Data Encryption Standard (DES), Advanced Encryption Standard (AES), and the Rivest-Shamir-Adleman (RSA) cryptosystem. However, with the advancement of powerful computers, network technologies, and neural computing, it becomes necessary to reconsider and develop new cryptographic systems that can withstand modern challenges.

Modern cryptographic systems typically utilize integer numbers as keys, with longer keys considered more reliable and difficult to break. However, using real numbers or real functions as keys can enhance the security and resistance to hacking in cryptographic systems.

Pattern recognition problems share similarities with cryptographic systems. There is often a need to recognize the form of a numerical function that describes a technological process. For instance, in the chemical industry, the recognition of membrane oscillations in a vibration granulator is crucial, as deviations from sinusoidal mode or the appearance of additional harmonics indicate damage or quality issues in production. Similarly, recognizing the exponential form in chemical reactions is essential to identify irregularities or deviations from expected behavior.

The problem of function recognition can be challenging, especially when the function parameters are unknown or represented by a random variable with an unknown distribution. In this chapter, we address two problems: (i) encoding/decoding a signal represented by a sequence of symbols using real-valued functions, and (ii) efficient recognition of function patterns using the derivative of disproportion functions.

In problem (i), symbols are encrypted by summing real-valued functions (keys) multiplied by randomly chosen factors. By leveraging the properties of derivative of disproportion functions, we can decipher the encoded symbols by recognizing the combined functions used in the encryption process, even when the randomly chosen multipliers are unknown.

Problem (ii) is addressed using the same derivative of disproportion functions and their key properties for efficient recognition of function patterns.

The structure of the chapter is as follows: Sections 2 and 3 introduce the derivative of disproportion functions and explain problem (i) in detail. In Section 4, the decryption method is presented. Section 5 provides numerical examples and computational experiment results. Sections 6 and 7 demonstrate the robustness of the developed cryptographic system and discuss the requirements for the disproportion of focal functions, respectively. Section 8 discusses problem (ii) and presents its solutions. Finally, Section 9 provides the conclusions of the chapter.

2: Derivative of Disproportion Functions

The derivative of disproportion functions, introduced in previous works [5]-[8], provides a basis for developing new encoding procedures and characterizing real-valued functions. It also enables estimation of the deflection of a real-valued function from a power function, denoted as $y = k \cdot x^n$, at a specific value of the variable *x*, regardless of the factor *k*. In this case, $n \geq 1$ is a fixed integer number.

Definition 1: We are given the disproportion function $y = y(x)$. The following expression

$$@d_x^{(n)} y = \frac{y}{x^n} - \frac{1}{n!} \cdot \frac{d^n y}{dx^n}. \tag{1}$$

is called the *n*-th order *derivative* of the given function with respect to *x* ($x \neq 0$)
For the case $n = 1$ (order 1) the formula (1) takes the following form:

$$@d_x^{(1)} y = \frac{y}{x} - \frac{dy}{dx}. \tag{2}$$

It is evident that for a linear function $y = kx$, its first-order derivative of disproportion is always zero for any value of the coefficient *k*. In formula (2), the derivative of the function as the main element of the computed disproportion is represented by the symbol "*d*" and the symbol "@" is used to denote the operation of computing the disproportion.

Assume that the function is given in a parametric form. In this case, , the *n*-th order derivative of the disproportion function described by (1) can be obtained by the use of the parametric dependence of *y* on *x* and the well-known rules of the computation of $d^n y / dx^n$.

If $y = \psi(t)$ and $x = \varphi(t)$ (where *t* is the parameter and $\varphi(t) \neq 0, \varphi'(t) \neq 0$ for all *t*), then the first-order derivative of the disproportion function is calculated as follows:

$$@d_x^{(1)} y = @d_{\varphi(t)}^{(1)} \psi(t) = \frac{y}{x} - \frac{y'_t}{x'_t} = \frac{\psi(t)}{\varphi(t)} - \frac{\psi'(t)}{\varphi'(t)}. \tag{3}$$

When a certain value of *k* satisfies $\psi(t) = k\varphi(t)$, the derivative of the disproportion (3) becomes zero over the common domain of the functions $y = \psi(t)$ and $x = \varphi(t)$.

Lemma 1: Let $R^{\pm} := R \backslash \{0\}$. The operator for taking the derivative of the disproportion of order *n* is linear and possesses the following properties:

1. If a function $y \in C^n\left(R^{\pm}\right)$ is multiplied by any scalar *k* its derivative of disproportion is multiplied by the same scalar.
2. The order *n* derivative of the difference or sum of disproportion functions equals the difference or sum of their derivative of disproportions.
3. The first-order derivative of disproportion for a linear function $y = kx$ for any value of the coefficient *k* is equal to zero.

Proof: The properties can be easily verified through simple algebraic manipulations using Definition 1.

Remark 1: Consequently, the operator $@d_x^{(n)}$ defined on the space $C^n\left(R^{\pm}\right)$ of n times continuously differentiable real functions is linear within this space.

3: Description of Problem (i)

In this section, we discuss a communication system, referred to as the C-system, which incorporates a cryptosystem K for encoding and transmits symbols (signals) using n-times continuously differentiable focal functions $f_i = f_i(t)$, defined on a time interval $t \in [0, T_i], T_i > 0, i = 1,\ldots,m$. At a specific time moment t, a transmitted symbol is encoded as the sum of at least two focal functions with possible time delays (shifts) $\tau_i \in [0, T_i], i = 1,\ldots,m.$

Consider the following example: when a symbol is transmitted, it is encoded as the weighted sum of two focal functions f_p and $f_q, 1 \le p, q \le m,$. The encoded signal transmitted through the communication channel can be represented as:

$$y(t) = k_p f_p\left(t + \tau_p\right) + k_q f_q\left(t + \tau_q\right), k_p > 0, k_q > 0. \tag{4}$$

Now, let's assume that an unauthorized invader (hacker) gains access to the channel and does not obtain information about the focal functions, their time delays (shifts) , or the coefficients .

On the C-system (channel) side, the list of focal functions and their delays is completely known. However, it is unknown which functions (and with what coefficients) are detected as the received signal coded as in equation (4). These functions and their coefficients are recognized in (4) as decoded and represented as the received symbol. The problem is to solve this issue using the method described in the next section.

4: Explanation of the Decoder

Solving the problem of decoding the received signal is challenging since the focal functions and their coefficients can only be approximately known. The received message, denoted as $y(t)$, is expanded in time, requiring the exact or approximate derivatives of this function. In the case of discrete data, such as $y(t) = \left(y(t_0), y(t_1), \ldots, y(t_{N-1})\right)^T$, the desired approximate "derivative" of the discrete function can be obtained using a certain method, similar to the Gregory-Newton interpolation method (*cf.*, [4]).

The proposed decoding algorithm is complex, and due to space limitations, we provide an explanation here for the case $m = 3$. For the complete version, refer to [6].

The general algorithm can be illustrated as follows: assuming the known time delays (shifts) $\tau_i, i = 1\ldots, m$ for the focal functions, the received message can be expressed as the sum of all focal functions with the yet-unknown coefficients k_i :

$$y(t) = \sum_{i=1}^{m} k_i f_i\left(t + \tau_i\right). \tag{5}$$

The objective is to determine these coefficients at the current time moment t. Some of these coefficients may be zero for functions that are not actually present in the encoded signal (5).

Let's outline the algorithm for the case $m = 3$, which involves three steps:

Step 1: Arbitrarily select one of the focal functions, for example, the first one $f_1 = f_1(t+\tau_1)$. Calculate the derivative of the disproportion for the signa $y(t)$ using equation (3) and denote it as $F_{01}(t) := @d_{f_1}^{(1)} y(t)$. Compute the derivatives of the disproportions $F_{21}(t)$ and $F_{31}(t)$ for the focal functions $f_2(t+\tau_2)$ and $f_3(t+\tau_3)$ with respect to $f_1(t+\tau_1)$. Due to the linearity of the operator @ (as mentioned in Remark 1), equation (5) simplifies to (for $m = 3$):

$$
\begin{aligned}
F_{01}(t) &\equiv @d_{f_1}^{(1)} y(t) = \frac{y(t)}{f_1(t+\tau_1)} - \frac{y'(t)}{f'_1(t+\tau_1)} = \\
&= k_1 \cdot 0 + k_2 \left[\frac{f_2(t+\tau_2)}{f_1(t+\tau_1)} - \frac{f'_2(t+\tau_2)}{f'_1(t+\tau_1)} \right] + \\
&\quad + k_3 \left[\frac{f_3(t+\tau_2)}{f_1(t+\tau_1)} - \frac{f'_3(t+\tau_2)}{f'_1(t+\tau_1)} \right] = \\
&= k_2 \, @d_{f_1}^{(1)} f_2(t+\tau_2) + k_3 \, @d_{f_1}^{(1)} f_3(t+\tau_2) \equiv \\
&\equiv k_2 F_{21}(t) + k_3 F_{31}(t).
\end{aligned} \tag{6}
$$

The first term on the right-hand side of equation (6) is zero due to condition 3 of Lemma 1.

Step 2: Randomly choose another derivative of the disproportion, say $F_{21}(t)$. Calculate the derivatives with respect to $F_{21}(t)$ of the disproportion functions $F_{01}(t)$ and $F_{31}(t)$, which we denote $F_{0121}(t)$ and $F_{3121}(t)$, respectively. Apply the derivative operator of the disproportion of order 1 to both sides of equation (6) and use the linearity and condition 3 of Lemma 1 to obtain:

$$
\begin{aligned}
F_{0121}(t) &\equiv \frac{F_{01}(t)}{F_{21}(t)} - \frac{F'_{01}(t)}{F'_{21}(t)} = \\
&= k_2 \cdot 0 + k_3 \left[\frac{F_{31}(t)}{F_{21}(t)} - \frac{F'_{31}(t)}{F'_{21}(t)} \right] \equiv k_3 F_{3121}(t).
\end{aligned} \tag{7}
$$

Step 3: Equation (7) shows that the function F_{0121} is linearly dependent on the function F_{3121}. As condition 3 of Lemma 1 implies, the derivative of the disproportion function $F_{01213121}(t)$ of the function F_{0121} with respect to F_{3121} is zero for all feasible t:

$$
\begin{aligned}
F_{01213121}(t) &\equiv @d_{F_{3121}}^{(1)} F_{0121}(t) = \frac{F_{0121}(t)}{F_{3121}(t)} - \frac{F'_{0121}(t)}{F'_{3121}(t)} = \\
&= k_3 - k_3 = 0.
\end{aligned}
$$

From equations (6) and (7), the relationships can be used in reverse order to compute the desired values of the unknown coefficients k_i. Starting from (7), the following can be obtained:

$$k_3 = \frac{F_{0121}}{F_{3121}}; \quad (8)$$

Combining (8) with (6) results in:

$$k_2 = \frac{F_{01} - k_3 F_{31}}{F_{21}}. \quad (9)$$

Finally, by substituting the found coefficients k_2 and k_3 into (5), we get the following:

$$k_1 = \frac{y(t) - k_2 f_2(t+\tau_2) - k_3 f_3(t+\tau_3)}{f_1(t+\tau_1)}. \quad (10)$$

After decoding the received message $y(t)$, the algorithm stops, having determined the unknown coefficients and their association with the focal functions. Coefficients related to non-used focal functions should be set to zero.

Remark 2: Knowing the involved functions and their delay (shift) values τ_i enables the use of this simplified version of the decoding algorithm. However, the research [6] provides more sophisticated procedures to decipher the received message even in the absence of such crucial information.

5: Numerical Experiments and Examples

In the operation of the cryptosystem, let's illustrate the binary coding using an arbitrary sequence of symbols: a transition to a new line (""), space "_", "0" and "1". In this model three real focal functions are used. The symbols are coded by the weighted sum of at least two of these functions with randomly assigned coefficients. The time shifts (delays) of the standard functions are assumed to be zero from the current time *t*.

The C-system (channel) can only generate binary-coded symbols. If any other symbol appears that is not listed above, it is received as a paragraph return.

In order to numerically explain the methods for computing approximate derivatives, the signal $y(t)$ must be controlled within an interval containing at least 10 discrete points of the time variable *t*. In this case, the number of points in this interval is selected as a constant value of 75 (refer to [6] for more details). The stability (resistance) of the cryptosystem increases with a higher number of points in the interval.

During the simulation of the cryptosystem's operations, the focal functions are assumed to be the following three functions:

$$f_1(t) = 100 sin((\alpha_1 - \beta_1)t) cos(15\beta_1 t);$$

$$f_2(t) = 100 exp(-0.1\alpha_2 t) sin(10\beta_2 t) cos((\alpha_2 + \beta_2)t);$$

$$f_3(t) = 100 exp(-0.1\alpha_3 t) sin(400\beta_3 t),$$

where $\alpha_1 = 1$; $\alpha_2 = 0.12$; $\alpha_3 = 0.5$; $\beta_1 = 0.1$; $\beta_2 = 1.2$; $\beta_3 = 0.7$.

When the focal functions encode the signal $y(t)$ using equation (5) before being ejected, the coefficients k_1, k_2, and k_3 are randomly selected using a pseudo-random number generator with a uniform distribution from zero to 10 for each symbol. Only when encoding a symbol '1', $y(t)$ includes the entire weighted sum of all three focal functions, so their coefficients k_1, k_2, and k_3 are not equal to zero. When encoding '0', we set $k_1 = 0$, and for encoding a space, we set $k_3 = 0$. If another symbol or the "paragraph return" is encoded, then $k_2 = 0$.

The formulas (8) – (10) are used by the receiver in order to calculate $k_i,\ i = 1,2,3$ and determine the key functions involved in the formula for the coded message and after that, the received message must be decoded.

Numerical examples are provided to illustrate the decoding algorithm:

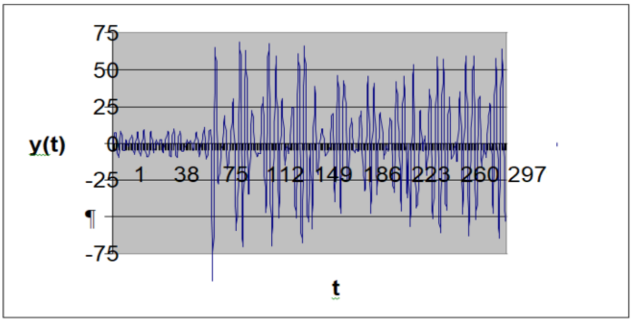

Figure 1 shows the signal corresponding to the serial transmission of four symbols '0'.

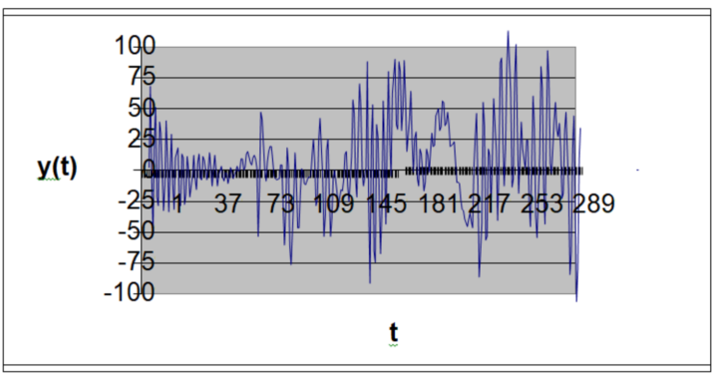

Figure 2 shows the signal corresponding to the serial transmission of four symbols '1'.

In all considered cases, the received message was deciphered exactly as what was transmitted. However, as can be seen from the figures, it is challenging to understand the message produced by the transmitted signal through the communication channel without using the decoding algorithm.

6: The Cryptosystem Is Robust

The robustness or stability of the developed cryptosystem depends on the selection of focal functions and their total quantity. The more components are involved in the signal, the more challenging it becomes to decode the signal in case of hacking. It is not only necessary to identify the type and number of focal functions but also to determine the coefficients involved.

Fitting the coefficients is a difficult task. We illustrate the said words using a simple example. In the numerical example presented in Section V in the function $f_3(t)$ the numerical parameter 400 is replaced with the numerical parameter 400.0001, or in the function $f_2(t)$ we replace $sin(10\beta_2 t)$ by $sin(9.9999\beta_2 t)$, then as a result of these changes, the codeword consisting of four 0's will be decrypted as the codeword consisting of four 1's.

The considered example shows that even after having found the key functions involved any venture of "guessing" the ciphered word by a search for the coefficients-weights, will most probably fail. Thus, the random search for the coefficients of the focal functions is almost impossible, let alone determining the number of focal functions and their analytical forms. The attempts to detect the number of key functions and their forms can be considered as completely senseless.

Furthermore, it is important to note that the same character may be encrypted with different symbols depending on its position. Frequency analysis is also rendered useless for unauthorized access and deciphering of the messages in this case.

All of the properties mentioned above demonstrate that the codes utilizing the weighted sums of real focal functions provide strong resistance against hacking. They are reliable, robust, and cryptographically secure.

7: Description and Solution of Problem (ii)

Problem (ii) involves analyzing a smooth function and determining which class it belongs to based on the set of sample functions given in Equation (11):

$$\begin{aligned} &y = ca^{bx},\ y = A\,sin(\omega x+\varphi),\ y = A\,ln\,x, \\ &y = \sum_{i=1}^{n} A_i\,sin(\omega x+\varphi_i),\ y = A\,sin^2(\omega x+\varphi), \end{aligned} \tag{11}$$

the parameters of the above-mentioned functions are assumed to be unknown. We have to analyze a given smooth function $y = f(x)$ and answer the question: to which class it does belong.

The function may not have the same parameters as those in Equation (11), but it is known to belong to one of the classes specified in Equation (11). The goal is to determine, based on the function's values and its derivatives at a certain point t, which form from Equation (11) the function belongs to.

Proposition 1: *We are given $y = f(x)$ and the calculated by formula (3) the derivative disproportion of this function with respect to $y' = f'(x)$ equals zero then this function is the exponential one.*

The proof of Proposition 1 follows the application of formula (3) having set $\psi(x) \equiv f(x)$ and $\varphi(x) \equiv f'(x)$, equalizing the lefthand side of formula (3) to zero, in order to determine the function $y = f(x)$, we get the following differential equation

$$\frac{d^2 y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2 . \tag{12}$$

It is well-known that the general solution of this equation is given with the exponential function

$$y = ca^{bx}, \tag{13}$$

for some $a > 0,\ a \neq 1$.

Thus, Proposition 1, in the case the derivative disproportion equals zero guarantees that the considered function is an exponential function.

Proposition 2: *We are given* $y = f(x)$ *and the calculated by formula* (*3*), *putting* $\psi(x) \equiv f(x)$ *and* $\varphi(x) \equiv f'(x)$, *the derivative disproportion of the function with respect to its second derivative* $y'' = f''(x)$ *equals zero then this function is the combination of sinusoidal ones.*

In order to prove Proposition 2, we use formula (3) having set $\psi(x) \equiv f(x)$ an $\varphi(x) \equiv f''(x)$ and equalizing the lefthand side to zero we get the following differential equation to find the desired function $y = f(x)$

$$\frac{d^3 y}{dx^3} = \frac{1}{y}\left(\frac{d^2 y}{dx^2}\right)\left(\frac{dy}{dx}\right). \tag{15}$$

Equation (15) is evidently reduced to the following second-order differential equation

$$\frac{d^2 y}{dx^2} = \omega y , \tag{16}$$

the solution of which is given by the function

$$y = \sum_{i=1}^{n} A_i \, sin\left(\omega t + \varphi_i\right). \tag{17}$$

Thus, Proposition 2 establishes an important fact that if the derivative of the disproportion function with respect to its second derivative is zero, then the function is a combination of sinusoidal functions.

In addition to exponential and sinusoidal functions, there are two other classes of signals

$$y = \sum_{i=1}^{n} A_i \, sin^2\left(\omega x + \varphi_i\right), \tag{18}$$

and

$$y = A \, ln \, x; \tag{19}$$

described by equations (18) and (19), that are frequently encountered in technological applications for nonlinear transformation detection.

Proposition 3: *Let the derivative disproportion calculated by formula (3) of the function's first derivative* $y = f'(x)$ *with respect to its third derivative* $y''' = f'''(x)$ *equal zero, hence this function is from class (18).*

The proof of Proposition 3, as previously, follows the application of formula (3) and equalizing the lefthand side to zero, as a result we get the following differential equation

$$\frac{d^4 y}{dx^4} = \frac{\left(\frac{d^3 y}{dx^3}\right)\left(\frac{d^2 y}{dx^2}\right)}{\frac{dy}{dx}}. \tag{20}$$

Making the change of variables, analogical to that used when proving Proposition 2, we come to the fact that the solution of (20) can be found in the following form:

$$y = \sum_{i=1}^{n} A_i \sin^2\left(\omega t + \varphi_i\right), \tag{21}$$

Therefore, Proposition 3 says that if the derivative of the disproportion function's first derivative with respect to its third derivative is zero, then the function belongs to class (18).

The next Proposition 4 formulates the conditions under which the function belongs to class (19).

Proposition 4: *Let the derivative disproportion calculated by formula (3) of a function* $y = f(x)$ *with respect to the logarithmic function* $y = \ln x$ *be zero, thus the examined function has the form (19).*

The proof of Proposition 4, as previously, follows the application of formula (3) and equalizing the lefthand side to zero, as a result we get the following differential equation

$$\frac{dy}{dx} = \frac{y}{x \ln x}. \tag{22}$$

The solution of equality (22) is easily obtained as follows:

$$\frac{dy}{y} = \frac{dx}{x \ln x} = \frac{d \ln x}{\ln x} \Rightarrow y = A \ln x,\ x > 0. \tag{23}$$

To summarize, the derivative of the disproportion technique allows for the recognition of all elementary functions from the set (11), and the properties of the derivative of the disproportions enable the recognition of not only the original function but also its inverse ratio function, expanding the set of functions that can be analyzed.

8: Conclusions

In this chapter, we have explored the derivative of disproportion functions and their applications in engineering and real-life problems.

The first application focused on solving problem (i), where we developed a cryptosystem that utilizes scalar functions of real variables as key components. We demonstrated the encryption and decryption processes using the (weighted) sum of focal functions and the first-order derivative of disproportion functions. We also highlighted the importance of considering the division by small numbers or the ratio of numbers close to zero during the decryption process to avoid information distortion. Additionally, we emphasized the need to re-encrypt the message with different coefficients if necessary.

The second application addressed problem (ii), which involves pattern recognition tasks. The goal is to determine the class to which a function belongs, regardless of its parameters. We showed that the derivative of disproportion functions can be employed to solve this recognition problem. Theoretical results and proofs were presented regarding the solution of this problem.

In conclusion, the derivative of disproportion functions offers valuable insights and practical applications in various fields. It can be utilized in encryption and decryption processes, as well as in pattern recognition tasks. This research was partly supported by the Mexico SEP-CONACYT grants FC-2016-01-1938.

References:

[1] U.S. Department of Commerce/National Institute of Standards and Technology Data Encryption Standard (DES) Federal Information, Processing Standards Publication, 46-3, October 25 (1999).

[2] Federal Information Processing Standards Publication 197, November 26 (2001), Specification for the ADVANCED ENCRYPTION STANDARD (AES).

[3] R. Rivest, A. Shamir, and L. Adleman, "A method for obtaining digital signatures and public – key cryptosystems," Communications of the ACM, vol. 21, no. 2, pp. 120-126, 1978.

[4] I.R. Khan, R. Ohba, and N. Hozumi, "Mathematical proof of closed-form expressions for finite difference approximations based on Taylor series", Journal of Computational and Applied Mathematics, vol. 150, no. 3, pp. 303-309, 2003.

[5] V.V. Avramenko, "Characteristic properties of disproportionality functions and their application to solving diagnosis problems," Transactions of Sumy State University (SumDU), No. 16, pp. 24-28, 2000.

[6] V.V. Avramenko, M.I. Zabolotny, "A Way of Data Coding," Patent UA H04L 9/00 №42957, Ukraine, 2009.

[7] V.V. Avramenko, A.P. Karpenko, "Recognition of fragments of given standards in an analyzed signal with the aid of disproportionality functions," Transactions of Sumy State University (SumDU), No. 34, pp. 96-101, 2002.

[8] V.V. Kalashnikov, V.V. Avramenko, N.I. Kalashnykova, and V.V. Kalashnikov-Jr., "A cryptosystem based on sums of focal functions," International Journal of Combinatorial Optimization Problems and Informatics, vol. 8, No. 1, pp. 31-38, 2017.

[9] Jinchuan Zhang, Lamei Lin, Xuan Tang, Shengling Jiang, Yuxi Li, "The distribution patterns of organic-rich shale and exploration prospect of shale gas in China", Advances in Resources Research, Online ISSN 2436-178X, Vol. 2, No. 1 (January, 2022), pp. 16-30. 2022.

[10] Xiaozheng Liu, "Geological characteristics and concentrated zones evaluation of tight oil reservoirs: A case study of the sedimentary basins in China", Advances in Resources Research, DOI https://doi.org/10.50908/arr.1.1_33, Vol. 1, Issue 1, pp. 33-47, 2021.

[11] Shuangfang Lu, Wenbiao Huang, Fangwen Chen, Jijun Li, Min Wang, Haitao Xue, Weiming Wang, Xiyuan Cai, "Classification and evaluation criteria of shale oil and gas resources: Discussion and application", Petroleum Exploration and Development, https://doi.org/10.1016/S1876-3804(12)60042-1, Vol. 39, Issue 2 (April 2012), pp. 268-276, 2012.

[12] Seyyed Mohammad Amin Partovi, Saeid Sadeghnejad, "Geological boundary detection from well-logs: An efficient approach based on pattern recognition", Journal of Petroleum Science and Engineering, Vol. 176 (May 2019), pp. 444-455, 2019.

[13] Xiaolin Wang, Guosheng Zhang, Wei Tang, Donghui Wang, Kun Wang, Jiayi Liu, Dong Du, "A review of commercial development of continental shale oil in China", Energy Geoscience. Vol. 3, Issue 3 (July 2022), pp. 282-289, 2022.