*Unconventional Methods for Geoscience, Shale Gas and Petroleum in the 21st Century*
*J. Watada et al. (Eds.)*

*doi:10.3233/AERD230017*

# Estimating the Radius of Investigation and Drainage Area by Reservoir Simulation

Fadzli Sajali[1], Jang Hyun Lee[1] and Naqiuddin Mahyuddin[1]

[1] Geosciences and Petroleum Engineering Department, Universiti Teknologi PETRONAS, 32610, Bandar Seri Iskandar, Perak Darul Ridzuan, Malaysia

fadzlisajali@yahoo.com (F.S.); lee.janghyun@utp.edu.my (J.H.L.); naqiuddinham@gmail.com (N.M.)

**Abstract.** The radius of investigation is still ambiguous and there is uncertainty in radius of investigation calculation. Every changes of pressure in the reservoir will change the radius of investigation. Thus, these variations will make the maximum radius of investigation difficult to define. To analyze this uncertainty, the pressure changes in a reservoir is evaluated by using the Ei-Function equation to plot the pressure profile which is pressure versus distance of the well graph. Furthermore, the pressure profile graph can be used to set a cut off of pressure difference at the end of transient effect that can be defined as maximum radius of investigation. This project required Matlab software for analytical approach and Eclipse Simulator software for numerical approach. The numerical method is used to prove the analytical method. The analytical method will provide the pressure profile which indicate the pressure of reservoir reading further away from the well. Similarly, the numerical method will generate the pressure of reservoir numerically to indicate the same as analytical method. The homogeneous reservoir is used to analyze this ambiguity where the manipulated variable is the flowrate and production time. The preliminary interpretation showed that different flowrate will not affect the radius of investigation while different production time will affect the radius of investigation.

**Keywords:** Radius of investigation · Pressure transient analysis · Reservoir pressure profile · Homogeneous reservoir · Drainage area · Solution of diffusivity equation

## 1 Introduction

Radius of investigation is often used in pressure transient testing. However, the concept is still inconclusive and there is no standard description in the petroleum literature. The radius of investigation is very important to identify how far the pressure transient may propagate, how big is the drainage area of the reservoir and how much volume can be identified (Kuchuk F. J., 2009). Theoretically, the radius of investigation can be obtained by using analytical approach of the radius of investigation equation. In reality, the radius of investigation from production period is difficult to define and not clearly explained. The reservoir pressure profile shows during transient flow, the pressure in the reservoir increases further away from the wellbore until the pressure reach the initial reservoir pressure. The radius of investigation can be estimated when the pressure transient propagation is stabilized (reach initial reservoir pressure).

Currently, there is standard equation for estimating the radius of investigation with several assumptions. The current equation used can be known as conservative equation with numerous different constants. The constant used in the conservative equation is very sensitive and give enormous effect to the radius of investigation and pressure difference at the end of the transient effect. Although, the radius of investigation can be obtained, but at which pressure difference does the radius of investigation lies. Whether the radius of investigation is overestimated or underestimated based on the pressure difference to be initial reservoir pressure. The smaller the pressure difference at the end of the transient effect, the further the radius of investigation.

The Matlab software has been used as a main tool to generate the radius of investigation plot. The pressure versus distance of the well graph has been generated by using Ei-function with dimensionless pressure, time and radius. This result is used to analyze the radius of investigation uncertainty. On the other hand, the drainage area of the reservoir can be established once the radius of investigation is defined. Furthermore, Eclipse Simulator is used to prove the analytical method by using numerical method. The simulation interpretation has been carried out to observe the radius of investigation and the drainage area behavior. The objective of this study is to compare the radius of investigation with different pressure difference and to set a cutoff for how smallest the pressure difference can be defined as an initial reservoir pressure to get maximum radius of investigation. The preliminary interpretation showed that different flowrate will not affect the radius of investigation while different production time will affect the radius of investigation. The pressure difference cut off for different flowrate would not be the same because the pressure profile showed for every different flowrate has slightly shifted up and down with the same radius of investigation while the cut off for different production time could be the same because it has the same behavior. Thus, this study is focused on the radius of investigation ambiguity in vertical oil well with homogeneous infinite acting reservoir by using analytical and numerical method.

## 2 Radius of Investigation Theory

There are several definitions of the radius of investigation however the definition is not precisely truthful. Radius of investigation is defined as the distance that the pressure transient has moved into the formation (Bourdet D., 2002). Other definition from Oilfield Glossary SLB (n.d.), the calculated maximum radius in a formation in which pressure has been affected during the flow period of a transient well test. In theory, the radius of investigation can be derived from solution of the diffusivity equation. A solution of the diffusivity equation (in field unit) is given by

$$P_i - P(r, \Delta t) = -70.6 \frac{q\beta\mu}{kh} E_i\left(-\frac{948\phi\mu C_t r^2}{k\Delta t}\right) \quad \textbf{(1)}$$

There will be two important equations to be used in the solution of the diffusivity equation. The first equation is diffusivity coefficient

$$\eta = \frac{k}{\phi\mu C_t} \quad \textbf{(2)}$$

The second equation is the Ei-Function

$$E_i(-x) = -\int_x^{+\infty} \frac{e^{-u}}{u} du \quad \textbf{(3)}$$

Based on Matthews and Russel (1967), the following assumptions need to be considered in order to use the Ei-Function equation

- Infinite acting reservoir.
- Well is producing at a constant flow rate.
- Uniform reservoir pressure, Pi when production begins.
- Wellbore radius, $r_w$ is centered in a cylindrical reservoir radius, re.
- No flow across the outer boundary

The solution of the diffusivity equation can be rewritten in term of conservative definition of dimensionless variables as (Variables without any physical unit and sometimes it said to be zero dimension.)

$$P_D(r_D, t_D) = \frac{1}{2} E_i\left(\frac{r_D^2}{4t_D}\right) \quad \textbf{(4)}$$

Where the dimensionless variables (in field unit) are

$$P_D = \frac{kh}{141.2q\mu}\left[P_o - P_{wf}\right] \quad \textbf{(5)}$$

$$r_D = \frac{r}{r_w} \quad \textbf{(6)}$$

$$t_D = \frac{0.0002637kt}{\pi\phi C_t r_w^2} \quad \textbf{(7)}$$

This solution defines the important of radial flow regime in the vertical well testing with infinite acting radial and no flow boundary. When the well starts producing, the fluid flow towards the well with a radial geometry

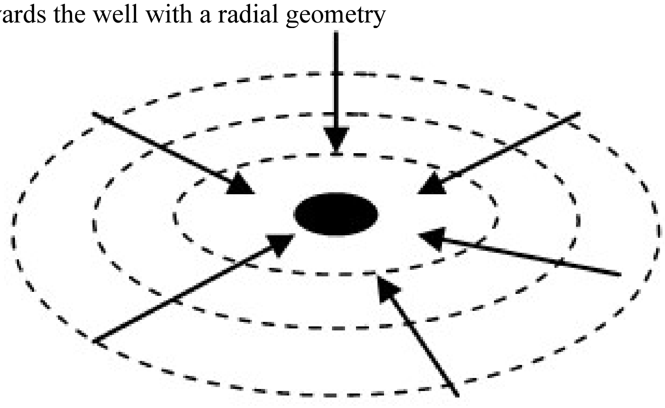
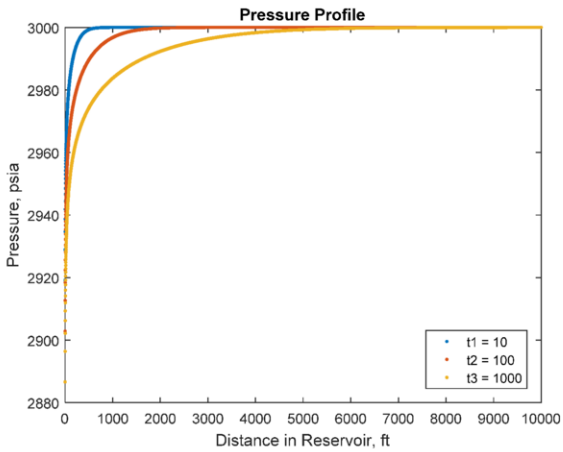


**Fig. 1.** Side View of Radial Flow Regime in Vertical Well

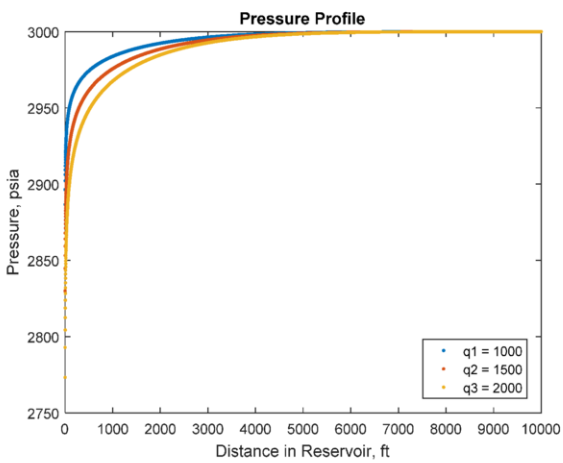
Within the reservoir, as the well production established, the pressure around the wellbore decreases with the following pressure change, ΔP

$$P_i - P(r, \Delta t) = -70.6\frac{q\beta\mu}{kh}E_i\left(-\frac{948\phi\mu C_t r^2}{k\Delta t}\right) \quad \textbf{(8)}$$

This pressure change, ΔP increase with time and decrease as the distance from well increase. The further the distance from the well, the pressure will lean towards initial reservoir pressure. Figure 2(a) below shows the longer the production time, the pressure profile will be shifted downward, the further for the pressure to reach initial pressure and the further the radius of investigation. In addition, Figure 2(b) shows when the well is flowing at different flowrate with the same 1000 hours production time, the radius of investigation will be the same with slightly different pressure behavior where the higher the flowrate, the pressure profile will shift downward.




(a)




(b)

**Fig. 2.** Pressure Profile in the Reservoir (a) at Different Production time and (b) at Different Flowrate.

Furthermore, a solution of diffusivity equation can be derived to pressure with respect to time. The derivative pressure with respect to time will define the pressure variation which can demonstrate the radius of investigation definition as below

$$\frac{dP(r,\Delta t)}{d\Delta t} = -70.6\frac{q\beta\mu}{kh}\frac{e^{-\frac{948\phi\mu C_t r^2}{k\Delta t}}}{\Delta t} \quad \textbf{(9)}$$

(dp/dt=dp/du.du/dt)
The maximum pressure variation is defined as

$$\frac{d^2P(r,\Delta t)}{d\Delta t^2} = 0 = -70.6\frac{q\beta\mu}{kh}\frac{e^{-\frac{948\phi\mu C_t r^2}{k\Delta t}}}{\Delta t^2}\left(\frac{948\phi\mu C_t r^2}{k\Delta t} - 1\right) \quad \textbf{(10)}$$

Hence:

$$\frac{948\phi\mu C_t R_i^2}{k\Delta t} = 1 \quad \textbf{(11)}$$

The radius of investigation is defined as: Muskat (1937), Poolen (1964), Lee (1982) and Streltsova (1988)

$$R_i = 0.0324\sqrt{\frac{k\Delta t}{\phi\mu C_t}} \quad \textbf{(12)}$$

Another definition of radius of investigation from Poolen (1964)

$$R_i = 0.029\sqrt{\frac{k\Delta t}{\phi\mu C_t}} \quad \textbf{(13)}$$

## 3 Method

The reservoir characteristics used in this paper is similar to the Ei-Function assumptions which already explained in the introduction part. The reservoir characteristics used to analyze the radius of investigation are as follows

**Table 1.** Reservoir Characteristics Input Parameters.

| | |
|---|---|
| Wellbore Radius, $r_w$ (ft) | 0.35 |
| Flowrate, q (bbl/d) | 1000, 1500, 2000 |
| Production Time, t (hr) | 10, 100, 1000 |
| Reservoir Thickness, h (ft) | 100 |
| Permeability, k (md) | 100 |
| Porosity, Ø (%) | 20 |
| Viscosity, μ (cp) | 1 |
| Total Compressibility Factor, $C_t$ (psi-1) | 0.00006 |

The analysis is done with different production time and different flowrate to see the effect of radius of investigation.

The Matlab software is used to plot the pressure profile which indicate the pressure behavior in the reservoir. Several equations have been used in order to proceed with the analysis. To demonstrate the radius of investigation used in this paper, the dimensionless solution of diffusivity equation has been applied in the Matlab calculation. The equation used in the Matlab are listed below

$$P_D(r_D, t_D) = \frac{1}{2} E_i \left( \frac{r_D^{\,2}}{4t_D} \right) \tag{14}$$

$$P_D = \frac{kh}{141.2q\mu} \left[ P_o - P_{wf} \right] \tag{15}$$

$$r_D = \frac{r}{r_w} \tag{16}$$

$$t_D = \frac{0.0002637kt}{\pi\phi C_t r_w^{\,2}} \tag{17}$$

These dimensionless equations are coded in the Matlab where the final output would be the pressure (Pwf) versus distance of the well (r). The Matlab output will compute every single pressure at every single distance. Hence, this make the result to be able to interpret the radius of investigation at different pressure difference.

Some more, the conservative equations are also be used in this study to observe the calculated radius of investigation lies at which pressure difference. The conservative equations used in this paper are by Muskat (1937), Poolen (1964), Lee (1982) and Streltsova (1988)

$$R_i = 0.0324\sqrt{\frac{k\Delta t}{\phi\mu C_t}} \tag{18}$$

And Poolen (1964)

$$R_i = 0.029\sqrt{\frac{k\Delta t}{\phi\mu C_t}} \quad \textbf{(19)}$$

On the other hand, the conservative equation somehow is not precisely accurate. Thus, this study has made some modification to get a tolerable radius of investigation. According to Kuchuk (2009), several constants have been presented by different authors for radius of investigation equation. The standard radius of investigation equation without a constant value is as below

$$R_i = C_r\sqrt{0.0002637\frac{k\Delta t}{\phi\mu C_t}} \quad \textbf{(20)}$$

Where Cr has various constant values. The constant 0.029 in radius of investigation equation is based on Poolen (1964) where Cr is given by 1.78. Additionally, Lee (1982), Muskat (1937) and Streltsova (1988) used Cr as 2 to obtain the constant of 0.0324. Based on equation 13, the new constant, Cr will be used to get a better tolerable radius of investigation.

Besides, the radius of investigation is analyzed by using simulation to prove the analytical method. The static model is constructed in the Petrel with 200 x 200 x 1 grid block where each block size about 100 feet. There is no gas or water in the static model. There are 3 simulation cases have been run with different production time and different flowrate. Hence, the pressure distribution will be assessed in the simulation software to analyze the radius of investigation.

# 4 Result

## 4.1 Radius of Investigation for 10 Hours Production Time

As the radius of investigation at 10 hours production time is small, it is better to analyze the ambiguity of radius of investigation with longer production time to get clearer result and observation. Therefore, the radius of investigation will be proceeded with 100 hours and 1000 hours production time with reservoir properties and characteristics as in Table 1.

## 4.2 Radius of Investigation for 100 Hours Production Time

**Table 2.** Exponential Integral Radius of Investigation at 100 Hours Production Time

| | | | | |
|---|---|---|---|---|
| Reservoir Pressure, Pe (psia) | 3000.0000 | 3000.0000 | 3000.0000 | 3000.0000 |
| Bottom Hole Flowing Pressure, Pwf (psia) | 2999.9900 | 2999.8997 | 2998.9977 | 2989.9983 |
| Pressure Difference, ΔP (Pe-Pwf) (psia) | 0.01 | 0.10 | 1.00 | 10.00 |
| Radius of Investigation, rinv (ft) | 2845.00 | 2219.00 | 1460.00 | 516.00 |

Based on the Table 2 above, the cut off of pressure difference for exponential integral used in Matlab is 1 psia because this pressure difference is acceptable. Based on the idea from Thompson and Reynolds (1997) the radius of investigation is the distance from the well to the region of the reservoir which has the greatest impact on the pressure data being measured at the wellbore. Therefore, the 1 psia can be defined as the portion of the reservoir region which has the impact on the pressure. As can be observed in the table above, the smaller the pressure difference, the further the radius of investigation. Thus, the acceptable pressure difference to the reservoir pressure is 1 psia compared to the smallest as we might overestimate the radius of investigation if the pressure difference is smaller than 1 psia.

**Table 3.** Radius of Investigation with constant 0.029, 100 Hours Production Time

| | |
|---|---|
| Reservoir Pressure, Pe (psia) | 3000.0000 |
| Bottom Hole Flowing Pressure, Pwf (psia) | 2999.8249 |
| Pressure Difference, ΔP (Pe-Pwf) (psia) | 0.1751 |
| Radius of Investigation, rinv (ft) | 2050.61 |

Based on the calculated radius of investigation with conservative equation in Table 3, the pressure different is obtained by back calculation. Hence, the pressure difference is about 0.1751 psia with 2050.61 ft of radius of investigation.

**Table 4.** Radius of Investigation with constant 0.0324, 100 Hours Production Time

| | |
|---|---|
| Reservoir Pressure, Pe (psia) | 3000.0000 |
| Bottom Hole Flowing Pressure, Pwf (psia) | 2999.9215 |
| Pressure Difference, ΔP (Pe-Pwf) (psia) | 0.0785 |
| Radius of Investigation, rinv (ft) | 2291.03 |

The interpretation from this analysis can be clearly seen that, the pressure difference of 1 psia from Matlab give shorter radius of investigation while the conservative equation with different constant gives further radius of investigation. The bigger the constant value, the further the radius of investigation. The pressure difference for constant 0.029 is 0.1751 psia while for constant 0.0324 is 0.0785 psia which show the pressure difference is unrealistic to rely on and this equation might overestimate the radius of investigation. By changing to smaller constant value for the conservative equation might possibly increase the pressure difference to 1 psia with the same radius of investigation to Matlab analysis.

Figure 3 is the summary of the radius of investigation from exponential integral in Matlab and conservative equation where can be observed the smaller the pressure difference, the further the radius of investigation. Furthermore, the higher the constant for conservative equation, the smaller the pressure difference, the further the radius of investigation.

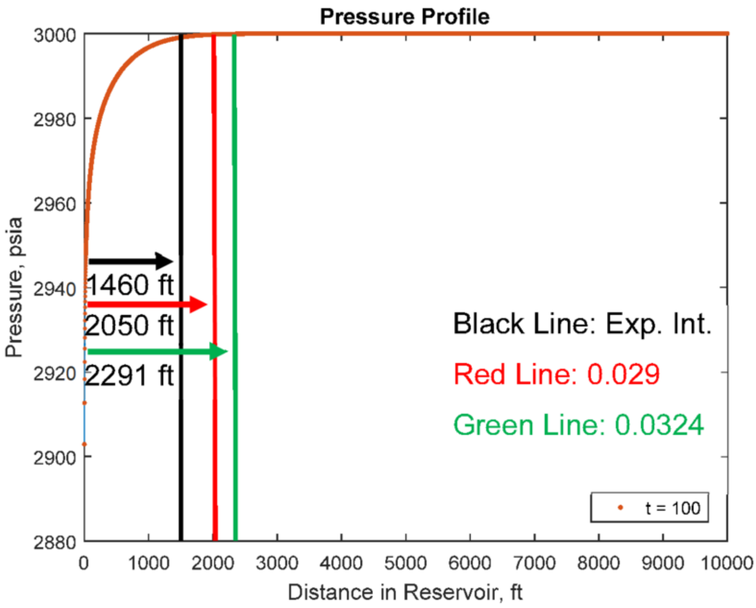



**Fig. 3.** Pressure Profile in the Reservoir at 100 Hours Production Time

## 4.3 Radius of Investigation for 1000 Hours Production Time

**Table 5.** Exponential Integral Radius of Investigation at 1000 Hours Production Time

| | | | | |
|---|---|---|---|---|
| Reservoir Pressure, Pe (psia) | 2999.9973 | 2999.9973 | 2999.9973 | 2999.9973 |
| Bottom Hole Flowing Pressure, Pwf (psia) | 2999.9873 | 2999.8973 | 2998.9969 | 2989.9929 |
| Pressure Difference, ΔP (Pe-Pwf) (psia) | 0.01 | 0.10 | 1.00 | 10.00 |
| Radius of Investigation, rinv (ft) | 8808.00 | 6995.00 | 4616.00 | 1631.00 |

The acceptable cutoff for pressure difference is decided to be 1 psia to obtain the tolerable radius of investigation. Based on Table 5, the radius of investigation at 1 psia cutoff pressure difference is 4616 ft.

**Table 6.** Radius of Investigation with constant 0.029, 1000 Hours Production Time

| | |
|---|---|
| Reservoir Pressure, Pe (psia) | 2999.9973 |
| Bottom Hole Flowing Pressure, Pwf (psia) | 2999.8251 |
| Pressure Difference, ΔP (Pe-Pwf) (psia) | 0.1722 |
| Radius of Investigation, rinv (ft) | 6484.60 |

By using the constant of 0.029 in conservative equation, the pressure difference obtained is 0.1722 which is almost the same which previous pressure difference at 100 hours production time.

**Table 7.** Radius of Investigation with constant 0.0324, 1000 Hours Production Time

| | |
|---|---|
| Reservoir Pressure, Pe (psia) | 2999.9973 |
| Bottom Hole Flowing Pressure, Pwf (psia) | 2999.9214 |
| Pressure Difference, ΔP (Pe-Pwf) (psia) | 0.0759 |
| Radius of Investigation, rinv (ft) | 7244.86 |

However, for constant 0.0324 in conservative equation give the pressure difference about 0.0759 with further radius of investigation of 7244 ft compared to 6484 ft (Constant 0.029).

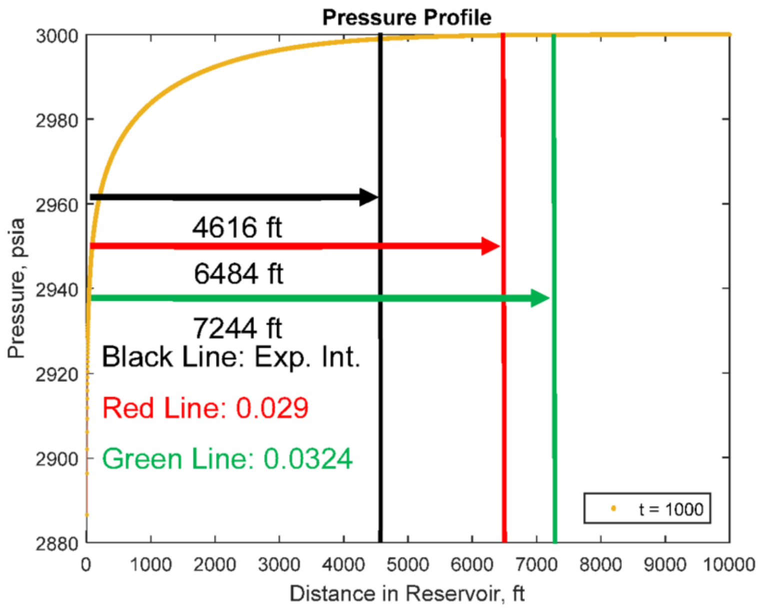



**Fig. 4.** Pressure Profile in the Reservoir at 1000 Hours Production Time

Figure 4 is the summary of radius of investigation by using exponential integral in Matlab and conservative equation. The smaller the pressure difference, the further the radius of investigation. However, in the conservative equation, the pressure difference is unrealistic, and the radius of investigation is quite over estimated. Even though there is a pressure difference about 0.172 (Constant 0.029) and 0.075 (Constant 0.0324), there is not much contribution in oil production on that drainage area. Thus, the acceptable pressure difference to be used is 1 psia which is more realistic and tolerable.

### 4.4 Radius of Investigation for 1000, 1500 and 2000bbl/d at 1000 Hours Production Time

The pressure difference cutoff for difference production time has been decided to be 1 psia. However, when the pressure difference is fixed for different flowrate, the radius of investigation will be different due to the shifted pressure profile trend. Based on the conservative equation and the Matlab Pressure Profile graph, for the difference flowrate, the radius of investigation should be the same. Table 8 below shows fixed pressure difference of 1 psia with different radius of investigation.

**Table 8.** Exponential Integral Radius of Investigation with Fixed 1 psia Pressure Different Cutoff

| Flowrate, q (bbl/d) | 1000 | 1500 | 2000 |
|---|---|---|---|
| Reservoir Pressure, Pe (psia) | 2999.9973 | 2999.9960 | 2999.9947 |
| Bottom Hole Flowing Pressure, Pwf (psia) | 2998.9969 | 2998.9959 | 2998.9939 |
| Pressure Difference, ΔP (Pe-Pwf) (psia) | 1.00 | 1.00 | 1.00 |
| Radius of Investigation, rinv (ft) | 4616.0000 | 5078.0000 | 5394.0000 |

However, this fixed pressure difference is not replicate the radius of investigation in conservative equation and definition. Therefore, some modification is made in order to replicate the same radius of investigation even if the flowrate is changing with the same production time.

**Table 9.** Exponential Integral Radius of Investigation with Difference Cutoff

| Flowrate, q (bbl/d) | 1000 | 1500 | 2000 |
|---|---|---|---|
| Reservoir Pressure, Pe (psia) | 2999.9973 | 2999.9960 | 2999.9947 |
| Bottom Hole Flowing Pressure, Pwf (psia) | 2998.9969 | 2998.4954 | 2997.9938 |
| Pressure Difference, ΔP (Pe-Pwf) (psia) | 1.00 | 1.50 | 2.00 |
| Radius of Investigation, rinv (ft) | 4616.0000 | 4616.0000 | 4616.0000 |

To get the same radius of investigation, the pressure difference should be changed based on the initial flowrate and pressure difference. For example, the initial flowrate is the reference for the pressure difference, when the flowrate is changed to 200%, the pressure difference should change 200% as well. Table 9 shows the same radius of investigation with different flowrate with changing of pressure difference.

Figures 5 and 6 show the clearer representation for fixed pressure difference and altered pressure difference. As a conclusion, different flowrate has the same radius of investigation equivalent with the conservative equation and definition.

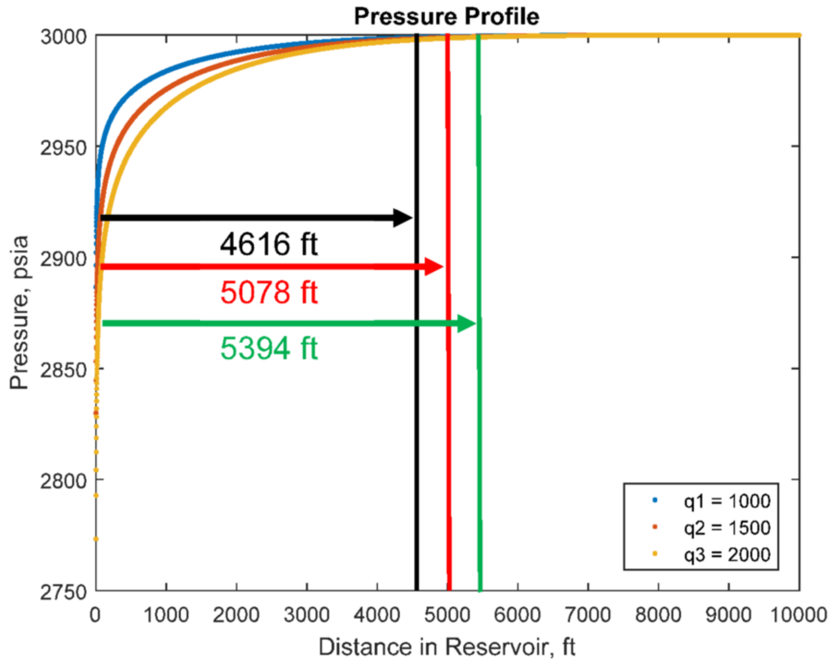



**Fig. 5.** Pressure vs Distance of the Well Graph for Different Flowrate (Fixed 1 psia Pressure Difference)

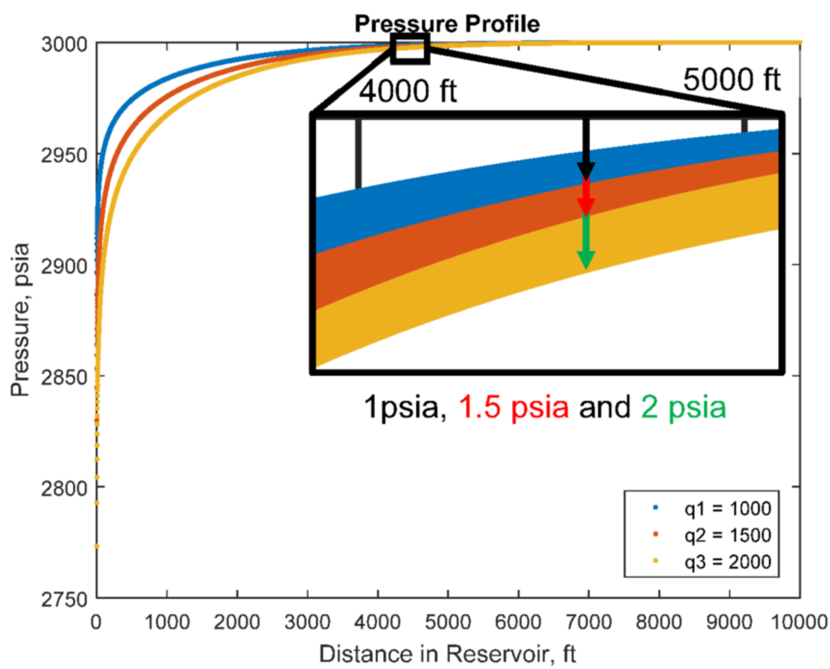



**Fig. 6.** Pressure vs Distance of the Well Graph for Different Flowrate and Cutoff (Altered Pressure Difference)

### 4.5 Radius of Investigation Constant

According to Kuchuk (2009), several constants have been presented by different authors for radius of investigation equation.

$$R_i = C_r \sqrt{0.0002637 \frac{k\Delta t}{\phi \mu C_t}} \quad \mathbf{(21)}$$

Where Cr has various values presented in Table 10. The constant 0.029 in radius of investigation equation is based on Poolen (1964) where Cr is given by 1.78. Additionally, Lee (1982), Muskat (1937) and Streltsova (1988) used Cr as 2 to obtain the constant of 0.0324.

**Table 10.** Exponential Integral Radius of Investigation with Difference Cutoff

| Author | Cr |
|---|---|
| Brownscomble and Kern (1951) | 1.783 |
| Chatas (1953) (Linear Flow) | 1.41 |
| Daughkaew et al. (2000) | 0.379-1.623 |
| Finjord (1988) | 2.82 |
| Hurst (1961) | 2.8284 |
| Hurst et al. (1969) | 2.64 |
| Johson (1988) | 2.81 |
| Jones (1962) | 4 |
| Kutasov and Hejri (1984) | 2.03-2.14 |
| Lee (1982) | 2 |
| Muskat (1937) | 2 |
| Streltsova (1988) | 2 |
| Tek et al. (1957) | 4.29 |
| Van Poolen (1964) | 2 |
| Van Poolen (1964) | 1.78 |

However, these two constant values (0.029 and 0.0324) are having a pressure difference about 0.17222 psia and 0.07587 psia which are actually overestimate the radius of investigation. As can be observed, the higher the constant used in the equation, the smaller the pressure difference and the further the radius of investigation.

Therefore, based on the analysis, the suitable constant can be used to get 1 psia pressure difference is by using the Cr = 1.2715 which will give the equation as below

$$R_i = 0.020648 \sqrt{\frac{k\Delta t}{\phi \mu C_t}} \quad \mathbf{(22)}$$

**Table 11.** Exponential Integral and Conservative Equation Comparison with Constant 0.020648 (100 Hours Production Time)

| Method | Exponential Integral | Conservative Equation |
|---|---|---|
| Reservoir Pressure, Pe (psia) | 3000.0000 | 3000.0000 |
| Bottom Hole Flowing Pressure, Pwf (psia) | 2998.9977 | 2998.9977 |
| Pressure Difference, ΔP (Pe-Pwf) (psia) | 1.00 | 1.00 |
| Radius of Investigation, rinv (ft) | 1460.00 | 1460.03 |

**Table 12.** Exponential Integral and Conservative Equation Comparison with Constant 0.020648 (1000 Hours Production Time)

| Method | Exponential Integral | Conservative Equation |
|---|---|---|
| Reservoir Pressure, Pe (psia) | 2999.9973 | 2999.9973 |
| Bottom Hole Flowing Pressure, Pwf (psia) | 2998.9969 | 2998.9969 |
| Pressure Difference, ΔP (Pe-Pwf) (psia) | 1.00 | 1.00 |
| Radius of Investigation, rinv (ft) | 4616.00 | 4617.03 |

The new constant 0.020648 give a similar radius of investigation with pressure difference of 1 psia. Even though this new constant is not giving exactly similar result, but the tolerance is acceptable.

## 4.6 Radius of Investigation in Simulation Model

The simulation model is a numerical method which is different with analytical method as the calculation made in the simulation is totally different. Although, the radius of investigation still can be obtained in the simulation model by observing the pressure changes along the reservoir. This project has successfully done to observe the pressure behavior in the reservoir from time to time to obtain the acceptable radius of investigation.

### 4.6.1. Radius of Investigation at 100 and 1000 Hours of Production Time (Model)

Based on, Figure 7 and Table 13 as a legend, the radius of investigation at 100 hours production time with the pressure difference of 1 psia (Red Color Circle) is 2233 ft while when the pressure reach the reservoir pressure of 3000 psia, the radius of investigation is 7513 ft. This shows the radius of investigation from simulation model is overestimated more than conservative equation and Matlab even though the pressure difference is set to 1 psia. For that reason, the pressure difference is chose to be 4 psia (Peach Color Circle) different for only in reservoir simulation to get the closer radius of investigation with Matlab and conservative equation. Hence, the radius of investigation with pressure difference of 4 psia is 1349 ft which is closer with Matlab.

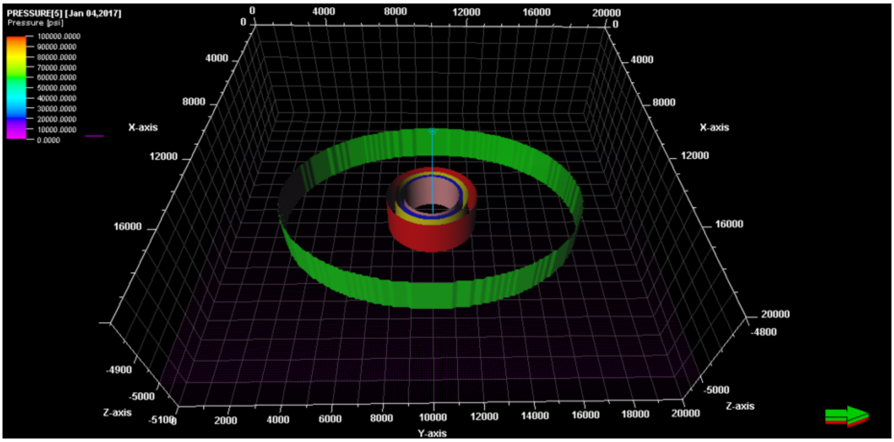



**Fig. 7.** Radius of Investigation at 100 Hours Production Time in Simulation Model

**Table 13.** Pressure Legend in Simulation Model at 100 Hours Production Time

| Color | Green | Red | Yellow | Blue | Peach |
|---|---|---|---|---|---|
| Reservoir Pressure, Pe (psia) | 3000 | 3000 | 3000 | 3000 | 3000 |
| Bottom Hole Flowing Pressure, Pwf (psia) | 3000 | 2999 | 2998 | 2997 | 2996 |
| Pressure Difference, ΔP (Pe-Pwf) (psia) | 0 | 1 | 2 | 3 | 4 |
| Radius of Investigation, rinv (ft) | 7513 | 2233 | 1804 | 1507 | 1349 |

For 1000 hours production time based on Figure 8 and Table 14, the pressure and radius of investigation trend look similar with 100 hours production time case. The radius of investigation is getting further as the production time increasing as mentioned earlier previously. Thus, the radius of investigation with pressure difference of 4 psia is 4913 ft (Peach Color Circle) which is closer with Matlab.

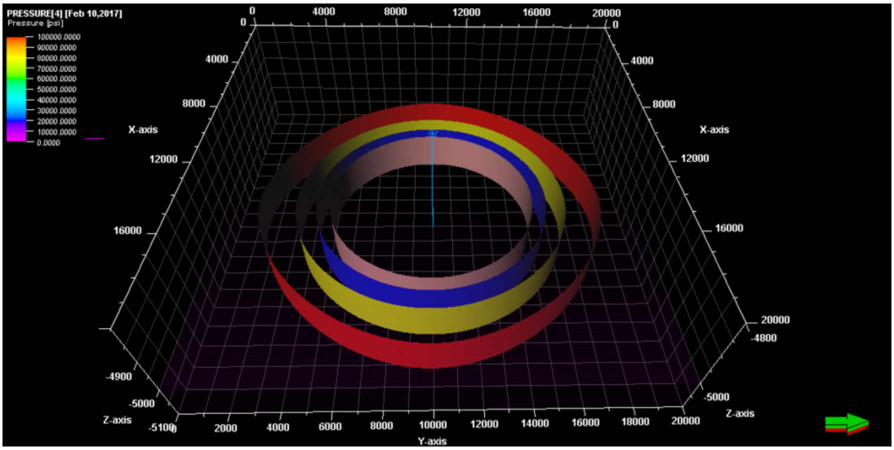



**Fig. 8.** Radius of Investigation at 1000 Hours Production Time in Simulation Model

**Table 14.** Pressure Legend in Simulation Model at 1000 Hours Production Time

| Color | | | | |
|---|---|---|---|---|
| Reservoir Pressure, Pe (psia) | 3000 | 3000 | 3000 | 3000 |
| Bottom Hole Flowing Pressure, Pwf (psia) | 2999 | 2998 | 2997 | 2996 |
| Pressure Difference, ΔP (Pe-Pwf) (psia) | 1 | 2 | 3 | 4 |
| Radius of Investigation, rinv (ft) | 8306 | 6614 | 5606 | 4913 |

**4.**6.2. Radius of Investigation for 1000, 1500 and 2000bbl/d at 1000 Hours Production Time (Model)

**Table 15.** Pressure Legend in Simulation Model at 1000bbl/d

| Color | | | | |
|---|---|---|---|---|
| Reservoir Pressure, Pe (psia) | 3000 | 3000 | 3000 | 3000 |
| Bottom Hole Flowing Pressure, Pwf (psia) | 2999 | 2998 | 2997 | 2996 |
| Pressure Difference, ΔP (Pe-Pwf) (psia) | 1 | 2 | 3 | 4 |
| Radius of Investigation, rinv (ft) | 8306 | 6614 | 5606 | 4913 |

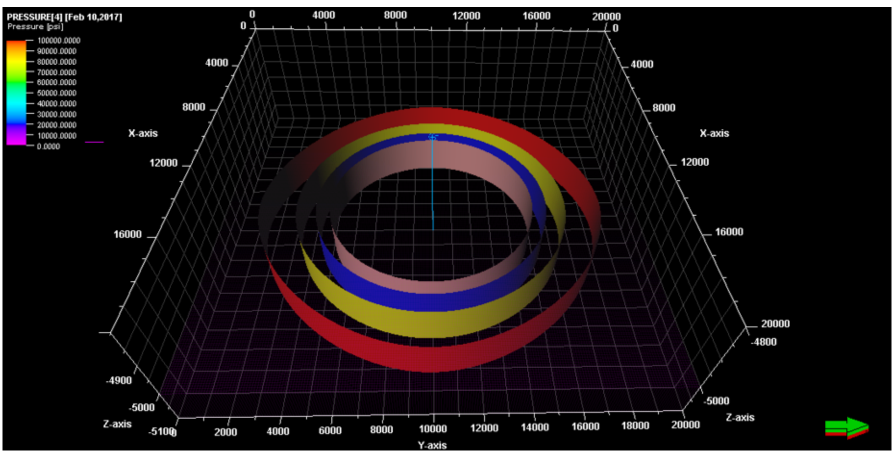



**Fig. 9.** Radius of Investigation at 1000bbl/d in Simulation Model

For different flowrate, the radius of investigation should be the same regardless the flowrate is changing with the same production time. As already decided, the pressure difference for simulation model is selected to be 4 psia different and the flowrate is 1000bbl/d which also be a reference pressure for the initial radius of investigation and reference flowrate. For 1000bbl/d, the radius of investigation with the pressure difference of 4 psia, the radius of investigation is 4913 ft (Peach Color Circle).

**Table 16.** Pressure Legend in Simulation Model at 1500bbl/d

| Color | | | | | |
|---|---|---|---|---|---|
| Reservoir Pressure, Pe (psia) | 3000 | 3000 | 3000 | 3000 | 3000 |
| Bottom Hole Flowing Pressure, Pwf (psia) | 2999 | 2998 | 2997 | 2996 | 2994 |
| Pressure Difference, ΔP (Pe-Pwf) (psia) | 1 | 2 | 3 | 4 | 6 |
| Radius of Investigation, rinv (ft) | 8927 | 7618 | 6614 | 5981 | 4942 |

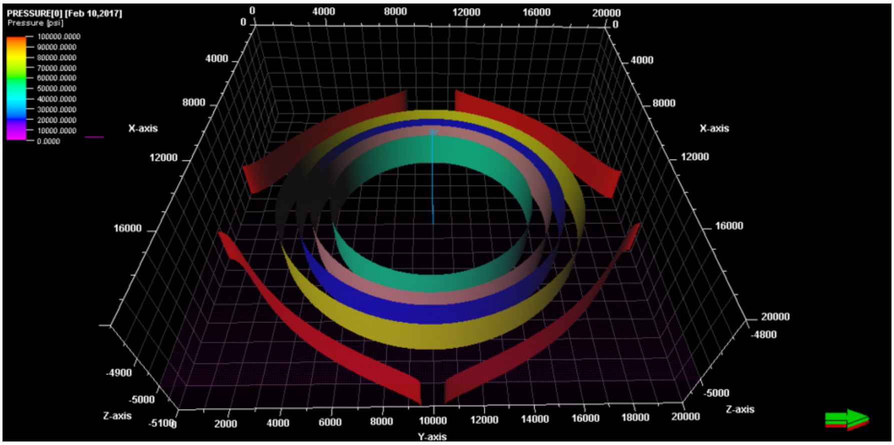



**Fig. 10.** Radius of Investigation at 1500bbl/d in Simulation Model

Furthermore, the flowrate is increased to 1500bbl/d which the increment is about 150% from the initial flowrate. In addition, the pressure difference needs to be increased 150% as well to get the same radius of investigation. As can be observed, if the pressure difference is chose to be 4 psia (With 1500bbl/d), the radius of investigation would be 5981 ft which is over estimated than the flowrate of 1000bbl/d. Thus, with the increment of 150% from the initial pressure difference, the current pressure difference would be 6 psia with 4942 ft radius of investigation. This shows, the difference of radius of investigation between 1000bbl/d and 1500bbl/d is fairly the comparable. Obviously, the radius of investigation will be the same with difference flowrate where the pressure difference should have the same percentage increment or decrement to the flowrate.

**Table 17.** Pressure Legend in Simulation Model at 2000bbl/d

| Color | | | | | |
|---|---|---|---|---|---|
| Reservoir Pressure, Pe (psia) | 3000 | 3000 | 3000 | 3000 | 3000 |
| Bottom Hole Flowing Pressure, Pwf (psia) | 2999 | 2998 | 2997 | 2996 | 2992 |
| Pressure Difference, ΔP (Pe-Pwf) (psia) | 1 | 2 | 3 | 4 | 8 |
| Radius of Investigation, rinv (ft) | 9787 | 8848 | 7562 | 6902 | 5231 |

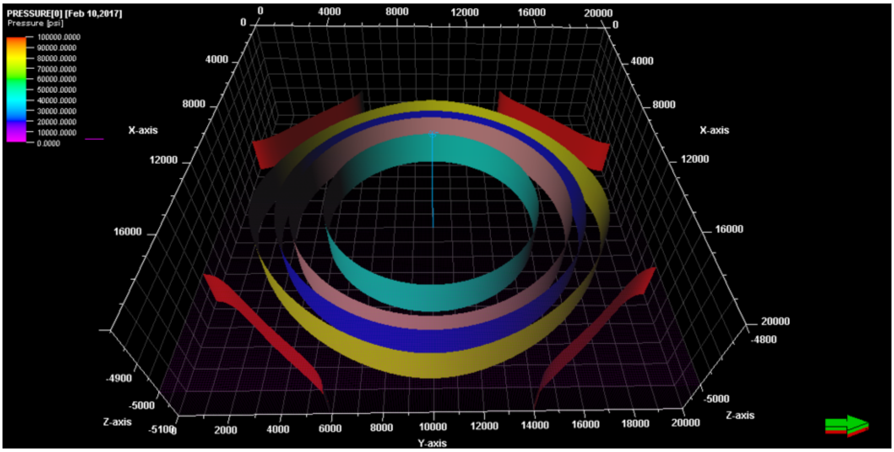



**Fig. 11.** Radius of Investigation at 2000bbl/d in Simulation Model

For 2000bbl/d case, the concept is similar to 1000bbl/d and 1500bbl/d. With the flowrate increased about 200% from the initial flowrate, the pressure difference cutoff will be increased to 200% from the initial pressure difference. As a result, the 2000bbl/d will have 5231 ft radius of investigation which give slightly greater radius of investigation than previous flowrate. This is because, the numerical method calculation is different with the definition of radius of investigation. However, the simulation model can still be comparable with the analytical method as the radius of investigation is a unique solution. There is no fixed value for the radius of investigation and the radius of investigation might slightly varies to other method.

## 4.7 Radius of Investigation Comparison

**Table 18.** Radius of Investigation Comparison at 100 Hours Production Time

| Method | Exp. Int. | 0.0029 | 0.00324 | 0.020648 | Simulation |
|---|---|---|---|---|---|
| Reservoir Pressure, Pe (psia) | 3000.0000 | 3000.0000 | 3000.0000 | 3000.0000 | 3000.0000 |
| Bottom Hole Flowing Pressure, Pwf (psia) | 2998.9977 | 2999.8249 | 2999.9215 | 2998.9977 | 2996.0000 |
| Pressure Difference, ΔP (Pe-Pwf) (psia) | 1.0023 | 0.1751 | 0.0785 | 1.0023 | 4.0000 |
| Radius of Investigation, rinv (ft) | 1460.00 | 2050.61 | 2291.03 | 1460.03 | 1349.00 |

**Table 19.** Radius of Investigation Comparison at 1000 Hours Production Time

| Method | Exp. Int. | 0.0029 | 0.00324 | 0.020648 | Simulation |
|---|---|---|---|---|---|
| Reservoir Pressure, Pe (psia) | 2999.9973 | 2999.9973 | 2999.9973 | 2999.9973 | 3000.0000 |
| Bottom Hole Flowing Pressure, Pwf (psia) | 2998.9969 | 2999.8251 | 2999.9215 | 2998.9969 | 2996.0000 |
| Pressure Difference, ΔP (Pe-Pwf) (psia) | 1.0004 | 0.1722 | 0.0759 | 1.0004 | 4.0000 |
| Radius of Investigation, rinv (ft) | 4616.00 | 6484.60 | 7244.86 | 4617.03 | 4913.00 |

The radius of investigation for difference production time is comparable where the difference between each method is very small and tolerable. However, the conservative equation with constant 0.029 and 0.0324 are slightly overestimated the radius of investigation due to very small pressure difference. The small pressure difference could not contribute much to the reserve calculation and might not actually drain the oil at those area for the well. In general, the 1 psia pressure difference cutoff is the tolerable pressure difference to get the maximum drainage area while in simulation, the pressure difference is slightly higher (4 psia) than conservative equation.

**Table 20.** Radius of Investigation Comparison at 1000bbl/d

| Method | Exp. Int. | 0.0029 | 0.00324 | 0.020648 | Simulation |
|---|---|---|---|---|---|
| Reservoir Pressure, Pe (psia) | 2999.9973 | 2999.9973 | 2999.9973 | 2999.9973 | 3000.0000 |
| Bottom Hole Flowing Pressure, Pwf (psia) | 2998.9969 | 2999.8251 | 2999.9215 | 2998.9969 | 2996.0000 |
| Pressure Difference, ΔP (Pe-Pwf) (psia) | 1.0004 | 0.1722 | 0.0759 | 1.0004 | 4.0000 |
| Radius of Investigation, rinv (ft) | 4616.00 | 6484.60 | 7244.86 | 4617.03 | 4913.00 |

**Table 21.** Radius of Investigation Comparison at 1500bbl/d

| Method | Exp. Int. | 0.0029 | 0.00324 | 0.020648 | Simulation |
|---|---|---|---|---|---|
| Reservoir Pressure, Pe (psia) | 2999.9973 | 2999.9973 | 2999.9973 | 2999.9973 | 3000.0000 |
| Bottom Hole Flowing Pressure, Pwf (psia) | 2998.4954 | 2999.8251 | 2999.9215 | 2998.4954 | 2994.0000 |
| Pressure Difference, ΔP (Pe-Pwf) (psia) | 1.5006 | 0.1722 | 0.0759 | 1.5006 | 6.0000 |
| Radius of Investigation, rinv (ft) | 4616.00 | 6484.60 | 7244.86 | 4617.03 | 4942.00 |

**Table 22.** Radius of Investigation Comparison at 2000bbl/d

| Method | Exp. Int. | 0.0029 | 0.00324 | 0.020648 | Simulation |
|---|---|---|---|---|---|
| Reservoir Pressure, Pe (psia) | 2999.9973 | 2999.9973 | 2999.9973 | 2999.9973 | 3000.0000 |
| Bottom Hole Flowing Pressure, Pwf (psia) | 2997.9938 | 2999.8251 | 2999.9215 | 2997.9938 | 2992.0000 |
| Pressure Difference, ΔP (Pe-Pwf) (psia) | 2.0008 | 0.1722 | 0.0759 | 2.0008 | 8.0000 |
| Radius of Investigation, rinv (ft) | 4616.00 | 6484.60 | 7244.86 | 4617.03 | 5231.00 |

For different flowrate, the conservative equation has a very straight forward answer for the radius of investigation where the equation does not consider the flowrate. However, the radius of investigation is tricky for exponential integral in Matlab calculation and Simulation model. Although, the radius of investigation is still can be obtained by modified the pressure difference cutoff according to the increment or decrement of the flowrate. Hence, the radius of investigation will be the same for different flowrate.

## 5 Discussion

In this section, we suggested to construct a reservoir model with smaller grid size to get a better and accurate result in term of numerical calculation. There might be a numerical error around the wellbore when using a bigger grid size as the pressure around the wellbore has the greatest impact on the pressure changes during production. Furthermore, to practice this estimation of radius of investigation, we pointed out that to expect a shorter radius of investigation compared to the estimated calculation. This is because we believed, the only trusted zone in the radius of investigation has shorter radius with greatest impact on pressure changes. Thus, the only trusted zone is when the pressure changes has the highest different with initial reservoir pressure.

# 6 Conclusion

The propagation of radius of investigation has successfully studied by using analytical and numerical method. The dimensionless terms for radius of investigation equation are used as well as the conservative equation by Muskat (1937) and Poleen (1964) for the analytical calculation in Matlab. Furthermore, the Eclipse Simulator is used to generate the radius of investigation numerically and to prove the analytical method. The homogeneous infinite reservoir model is created to run the simulation. In reality, the radius of investigation can be fairly different for other non-homogeneous reservoir. It should be appreciated that strictly the analysis presented only applies to homogeneous, isotropic reservoirs that have been completed across their full interval. The radius of investigation and the time to reach semi-steady state flow can be quite different for situations which differ from this highly idealized arrangement. Thus, this project demonstrated a homogeneous reservoir with constant flowrate and different flowrate. Based on this analysis, the following conclusions are offered

1. The further the radius of investigation, the smaller the pressure difference in the reservoir.
2. The tolerable cut off for the pressure difference to be initial pressure for exponential integral in Matlab and conservative equation is 1 psia.
3. For different flowrate, the pressure difference needs to be modified based on the increment or decrement from the initial flowrate and initial pressure difference.
4. In simulation model, the pressure difference cutoff is slightly higher than exponential integral in Matlab and conservative equation. The pressure difference used is 4 psia to get tolerable radius of investigation to compare to exponential integral and conservative equation while for different flowrate, the concept is the same.
5. The radius of investigation is quite tricky where we cannot fully trust the equation or the calculated radius of investigation. We need to be careful to use the radius of investigation estimation. We might get deceived when we fully trust the estimated radius of investigation where actually the radius of investigation is way nearer or further in reality.


# Acknowledgement

The authors wish to thank A.P. Jang Hyun Lee for his work on the early stages of this work. Discussions with Naqiuddin Mahyuddin has been invaluable. The authors would also like to thank to our parents who give endless support. Further thanks goes to Universiti Teknologi PETRONAS for giving the opportunity to write this paper.

## References

1. L. Thompson and A. Reynolds, "Well Testing for Radially Heterogeneous Reservoirs under Single and Multiphase Flow Conditions," *SPEFE,* p. 57–64, 1997.
2. V. Sova, "Estimation of Wildcat Well Drainage Area From Well Test Data," *Oil and Gas Business Web Journal,* pp. 1-10, 2009.
3. C. S. Matthews and D. G. Russell, "Solutions for Radial Flow of Fluids of Small and Constant Compressibility," in *Pressure Buildup and Flow Tests in Wells*, New York, Society of Petroleum Engineers of AIME, 1967, pp. 130-133.
4. F. J. Kuchuk, "Radius of Investigation for Reserve Estimation from Pressure Transient Well," *SPE120515,* pp. 1-22, 2009.
5. B.-Z. Hsieh, G. V. Chilingar and Z.-S. Lin, "Propagation of Radius of Investigation from," *UESO #200271 (EXP),* pp. 1-15, 2006.
6. R. N. Horne, in *Modern Well Test Analysis A Computer-Aided Approach*, California, Petroway, Inc., 1990.
7. A. U. Chaudhry, in *Oil Well Testing Handbook*, Houston, Elsevier Science B.V., 2004, p. 135.
8. D. Bourdet, "Radial Flow Regime, Skin (Homogeneous Behaviour)," in *Well Test Analysis: The Use of Advanced Interpretation Models*, Paris, Elsevier Science B.V., 2002, pp. 11-12.
9. "Radius of Investigation," [Online]. Available: https://www.testwells.com/radius-of-investigation/.
10. "Radius of Investigation Definition," [Online]. Available: http://www.glossary.oilfield.slb.com/Terms/r/radius_of_investigation.aspx.