

Abstract and Concrete Decision Graphs for Choosing Extensions of Argumentation Frameworks

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Abstract. Most argumentation semantics allow for multiple extensions, which raises the question of how to choose among extensions. We propose to study this question as a decision problem. Inspired by decision trees commonly used in economics, we introduce the notion of a *decision graph* for deciding between the multiple extensions of a given AF in a given semantics. We distinguish between *abstract decision graphs* and concrete instantiations thereof. Inspired by the principle-based approach to argumentation, we formulate two principles that mappings from argumentation frameworks to decision graphs should satisfy, the principles of decision-graph directionality and that of directional decision-making. We then propose a concrete instantiation of decision graphs, which satisfies one of these principles. Finally, we discuss the potential for further research based on this novel methodology.

Keywords. abstract argumentation, argumentation semantics, extension selection

1. Introduction

Given that many argumentation semantics have been proposed in the literature [3] and that most argumentation semantics allow for multiple extensions [16], applications of abstract argumentation theory are faced with two decision problems: First, how to choose among the various argumentation semantics? Second, given an argumentation semantics, how to choose an extension?

An important methodology to support rational decisions concerning the first problem is the principle-based approach [5,11,16]. In this paper, we propose a novel methodology to support decision-making concerning the second problem, i.e. concerning the selection of one among many extensions of a given AF in a given semantics.

Sometimes the need to choose an extension is circumvented by merging all extensions into a single justification status for each argument [17,3]. For example, an argument is said to be *strongly accepted* iff it is in all extensions. However, this

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approach gives up the desirable properties of extensions that have been built into the chosen semantics. For example, the set of strongly accepted arguments in preferred semantics may not be admissible. This problem can be avoided by choosing one extension rather than merging all extensions into a single justification status. But this makes the question of how to choose among multiple extensions a very pressing question.

In this paper, we do not favor one particular method for choosing an extension, but instead propose a methodology for studying this problem as a decision problem. Inspired by decision trees commonly used in economics, we introduce the notion of a *decision graph* for deciding among the multiple extensions of a given AF in a given semantics. The edges of a decision graph represent specific decision steps that bring one closer to the final choice of a single extension. We distinguish between *abstract decision graphs*, where the only content present in the nodes is extension-labels on the leaves, and instantiations of these with *concrete decision graphs* that give a particular meaning to every node of the decision graph. Just like the distinction that Dung [12] introduced between abstract argumentation frameworks and structured instantiations thereof, this distinction helps to distill the features of decision graphs that come from the decision graph structure alone and study these separately.

Note that in this paper we do not propose to extend Dung's notion of argumentation frameworks. Dung has been criticized for its abstract nature and therefore Dung's formalism has been generalized in many ways, for example with structured frameworks [15], ADFs [6], etc. Such extensions are outside the scope of this paper, but some interesting possibilities suggested by our work are discussed in the future work section of this paper. Instead, in this paper we give a new perspective on existing abstract argumentation semantics in terms of decision graphs.

Furthermore, note that our choice to base decision graphs on the traditional extension-based approach to abstract argumentation semantics rather than on the labeling-based approach [2,3] is merely due to the fact that this simplifies the exposition of our ideas. All the ideas developed in this paper could also be developed with respect to the labelling-approach, and actually this would give rise to more fine-grained decision graphs, which allow for more flexibility in the decision-making process. We leave the exploration of this adaptation of our ideas to future work.

The layout of this paper is as follows. In Section 2 we introduce *abstract decision graphs* as well as the notion of *decision mappings* that map each AF to an abstract decision graph. Inspired by the principle-based approach to argumentation theory, we additionally define in this section two principles of decision mappings that seem desirable, the principles of decision-graph directionality and that of directional decision-making. In Section 3 we define a first concrete instantiation of decision graphs, namely *most fine-grained decision graphs*, whose corresponding decision mapping satisfies one of the two principles from Section 2 and does not satisfy the other. In Section 4 we discuss related work, and in Section 5 we conclude and discuss topics for further research.

In this paper, we briefly sketch and discuss our ideas without delving into formal details. For the interested readers, we refer to our online technical report[8].

2. Abstract decision graphs

In this section, we introduce *abstract decision graphs*, where the only content present in the nodes is extension-labels on the leaves. We will only informally describe the notions; for a full formal account, we refer the readers to our online technical report [8].

We do have a few requirements on the graph: It should be a directed acyclic graph, with a single root from which all other nodes are reachable, to represent our starting point in the decision-making process. Also, we require that each node connect to a distinct set of reachable endpoints, since we are interested in the processes where some extensions are discarded at every step as we traverse the graph. We refer to the nodes in a decision graph as the *decision points*.

Example 2.1. Consider the argumentation framework depicted in Fig. 1.(a). A possible abstract decision graph with respect to preferred semantics is the one depicted in Fig. 1.(b).

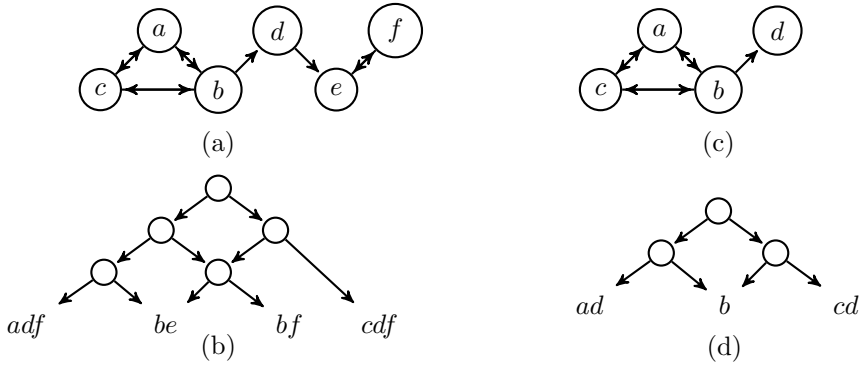


Figure 1. (a) Example argumentation framework $F_1 = \langle A_1, \rightarrow_1 \rangle$. (b) A possible abstract decision graph DG of F_1 with respect to preferred semantics. (c) An initial sub-framework $F'_1 = \langle A'_1, \rightarrow'_1 \rangle$ of F_1 , where $A'_1 = \{a, b, c, d\}$. (d) The restriction $DG \downarrow_{A'_1}$.

We now wish to examine some properties of functions which return abstract decision graphs for given pairs of AF and semantics, which we call *decision mappings*.

One important principle studied in the principle-based approach to argumentation theory is the Principle of Directionality, which was introduced by Baroni and Giacomin [5], and which has been extensively studied for abstract argumentation semantics [16]. We now propose a way to translate this principle to a similar principle for decision mappings.

We start with the notion of *initial sub-framework*, which is a sub-framework such that no argument outside of it attacks an argument inside of it. In terms of directionality, these are sub-frameworks that one should be able to evaluate locally, i.e. without having to take into account the rest of the framework.

We then wish to be able to contract a decision graph so that it only represents decisions made on a sub-framework of the original one, but while ensuring the conditions defined for an abstract decision graph are still respected.

We call such a contraction a *restriction* of the original decision graph with respect to a given subset of arguments.

Lemma 2.1. *Given an abstract decision graph DG of an $AF \langle A, \rightarrow \rangle$ and a set $A' \subseteq A$, the restriction $DG \downarrow_{A'}$ is also an abstract decision graph.*

Example 2.2. Fig. 1.(c) depicts an initial sub-framework $F' = \langle A', \rightarrow' \rangle$ of the framework F depicted in Fig. 1.(a). The restriction of the decision graph in 1.(b) to A' is depicted in Fig. 1.(d).

We then say that a decision mapping satisfies *decision-graph directionality* iff for any of its initial sub-frameworks, the restriction of the full decision graph is equal to the decision mapping of the sub-framework.

An interesting principle can be derived from the notions defined above, also on the topic of directionality, but this time while also incorporating the ideas of decision-making. The idea here is that the decision should follow the directionality of the graph, so that if an argument a can reach another argument b but not vice-versa, then decisions about the status of a should come no later than decisions about b .

We thus say that a decision mapping satisfies *directional decision-making* iff in all decision graphs it produces, disregarding the status of arguments which have strict ancestors with statuses yet to be decided does not affect the reachable extensions.

3. Most fine-grained decision graphs

In this section, we see an example of a decision mapping producing *concrete decision graphs*, where we now have labels on the intermediary points too. We will once again only informally describe the notions; for a full formal account, we refer the readers to our online technical report [8].

We first introduce the notion of a *partial extension*, which allows to represent intermediate steps in the decision process about which arguments to include in the extension and which arguments to exclude. Given an argument a , we denote the information that a has been chosen to be in the extension by $+a$, and the information that a has been chosen to not be in the extension by $-a$.

When neither $+a$ nor $-a$ is in a given partial extension, this means that the status of argument a has not been determined yet (not to be confused with the *undecided* label from the labeling-based approach). We refer to the set of arguments of which the status has already been determined by a partial extension as its *coverage*. When the status of all arguments has been determined, i.e. the coverage is the whole set of arguments, a *total extension* is reached.

Note that there is a direct correspondence between the classical notion of an extension as a subset of the sets of arguments, and the notion of a total extension defined here: Having $+a$ in the total extension corresponds to a being in the corresponding extension, and having $-a$ in the total extension corresponds to a not being in the corresponding extension.

We are now interested in fixing an argumentation framework and a semantics, and focusing on the decision structure of how the initial partial extension, from

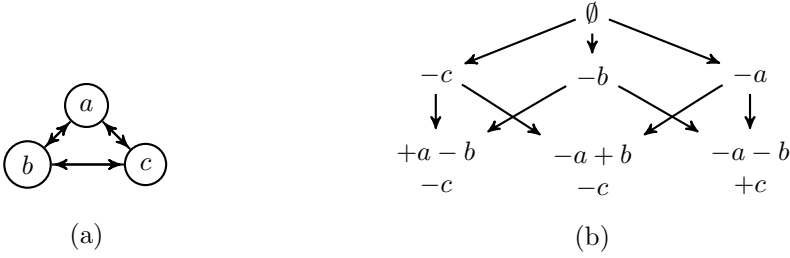


Figure 2. (a) Example argumentation framework F_2 . (b) Most fine-grained decision graph of F_2 with respect to complete semantics, $mfg(F_2, complete)$.

which all total extensions are reachable, leads to each one of these total extensions. For this purpose, we introduce the notion of *concrete decision graphs*. These are abstract decision graphs where the nodes are partial extensions such that the leaves are exactly the total extensions for the semantics of interest, and two partial extensions are related only if the first is a strict subset of the second.

We wish to define a decision mapping which constructs a concrete decision graph with as much granularity as possible. For this purpose, we start by observing the graph resulting from the subset relation on the partial extensions. This graph, to which we refer to as the *most exhaustive update*, consists of all partial extensions as nodes, together with the strict subset relation.

We distinguish between two kinds of edges in the most exhaustive update: the edges that relate two partial extensions that both lead to the same final total extensions, which we call *reasoning steps*, and the ones where the set of reachable total extensions becomes smaller, which we call *decision steps*. This corresponds to the idea that in some steps, no new information is gained, no decisions are made, and thus only reasoning is performed, while in other cases, the range of possible extensions is reduced and thus decisions are made.

We define the most fine-grained decision graphs by focusing on the decision steps in the most exhaustive update. For this, we need to condense the most exhaustive update such that reasoning is made automatically. This is akin to approaches in epistemic logic in which knowledge is assumed to be logically closed, i.e. in which reasoning is assumed to be instantaneously completed. We also identify the *decision points* in the most exhaustive update, which are the nodes where no more reasoning steps can be taken and decision steps are unavoidable. We call the *decision contraction* of the most exhaustive update *meu* the graph consisting of the decision points in *meu* together with the relation of *meu* restricted to those nodes.

We want to define a maximally fine-grained decision graph, so our intention is that no decisions are skipped, however small they may be. Thus we filter out the edges which relate two nodes already connected with more fine-grained paths. We call this process the *fine-grained filtering* of a graph.

We now define the *most fine-grained decision graph* of an AF with respect to a semantics as the decision contraction of the most exhaustive update of the AF in question with respect to that semantics, to which we apply the fine-grained filtering operation.

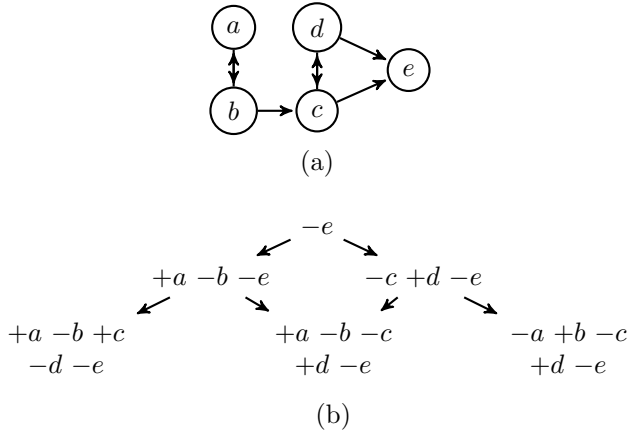


Figure 3. (a) Example argumentation framework F_3 . (b) Most fine-grained decision graph of F_3 with respect to preferred semantics, $mfg(F_3, \text{preferred})$.

Example 3.1. The concept of most fine-grained decision graph is illustrated in Fig. 2 and Fig. 3. In Fig. 2, one can see that if we did not differentiate partial extensions such as $-a$ from $\emptyset$, we would get a decision graph with one less layer of granularity. In Fig. 3, observe that the root is $-e$ instead of $\emptyset$, since $-e$ is an element of all total extensions. Also, one can see that the decision mapping mfg does not satisfy the principle of directional decision-making with respect to the preferred semantics, since it is also possible to initially make a decision about the status of the arguments c, d, e , even though there is a $\rightarrow$ -path from b to all of these arguments, but not vice-versa.

We now state some properties of most fine-grained decision graphs:

Theorem 3.1. *For any AF F and semantics sem , $mfg(F, sem)$ is a concrete decision graph.*

Theorem 3.2. *mfg satisfies the principle of decision-graph directionality with respect to any semantics that satisfies the principle of directionality defined in [5].*

Theorem 3.3. *mfg does not satisfy the principle of directional decision-making with respect to any of complete, preferred, semi-stable, naive, stage, CF2 or stage2 semantics.*

4. Related research

There is substantial work of applying formal argumentation theory to support decision-making [14,10,1]. In these papers, argumentation is used to support making decisions about other things than argumentation. That is quite different from our approach, in which we study how to theoretically study the making of decisions about which extension to choose among multiple extensions of an AF. It remains an open problem whether our decision graphs can be extended such that they can be applied to support decision-making as well.

Moreover, there is work on decision procedures. For example, Dvorak et al. [13] study the complexity of evaluations of AFs by exploiting decision procedures for problems of lower complexity whenever possible. Whether and how our general update semantics methodology can be applicable to the systematic study of algorithms for computing extensions, also has to be left to future research.

5. Conclusion and further research

In this paper, we have proposed a methodologically novel approach to choosing extensions of argumentation frameworks by studying abstract and concrete decision graphs that correspond to step-wise decision-making processes about the choice of extension. Inspired by the principle-based approach to abstract argumentation, we have studied two principles that mappings from AFs to decision graphs should satisfy.

We believe that there are many potential applications of our decision graph methodology. We briefly sketch some of them.

Apart from the type of concrete decision graphs defined in this paper, there are many other types of concrete decision graphs that could be studied. For example, any algorithm for computing all extensions of a given AF with respect to a fixed semantics gives rise to a concrete decision graph with respect to that semantics, namely by reducing the search tree of the algorithm to its decision points, similarly as we have reduced the most exhaustive update to the fine-grained decision graph. Studying the properties of these decision trees could give novel insights into the study of algorithms for computing extensions.

Further principles of decision graphs can be defined and studied. This will help to differentiate better between different semantics as well as the different decision graphs that they give rise to.

Furthermore, one can study properties of the different decision-paths (paths through the decision graph) that a given decision graph gives rise to. Here as well a principle-based approach can make sense: These principles could help to choose an extension in a systematic way, and could thus be very relevant to application of abstract argumentation in which a unique extension has to be chosen from the set of all extensions.

Though this is not our intention in this paper, decision graphs can also be used to generalize Dung's semantic framework, in the following sense. Instead of associating a set of extensions with a framework, we can associate a set of decision graphs with a framework. These decision graphs can be our most fine-grained decision graphs, or some other kind of decision graphs. We have made a contribution in this direction in a paper accepted for the second Chinese Conference on Logic and Argumentation [9].

In this paper we have only studied the decision graph methodology with respect to Dung's AFs, but the methodology could also be applied to extensions of Dung's formalisms such as bipolar AFs [7], ADFs [6], higher-order AFs [4] etc.

A further interesting line of future research is to study whether and how our methodology could be applied outside abstract argumentation, e.g. to structured argumentation, logic programming, answer set programming, Reiter's default logic, causal theories, social choice theory etc.

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