Computational Models of Argument
S. Modgil et al. (Eds.)

doi:10.3233/978-1-61499-906-5-181

Credulous and Skeptical Acceptance in Incomplete Argumentation Frameworks

Dorothea BAUMEISTER, Daniel NEUGEBAUER and Jörg ROTHE
Institut für Informatik, Heinrich-Heine-Universität Düsseldorf, Düsseldorf, Germany

Abstract. We propose natural generalizations of the credulous and skeptical acceptance problems in abstract argumentation for incomplete argumentation frameworks [3]. This continues earlier work on a similar generalization of the verification problem. We provide a full analysis of the computational complexity of the generalized problems for all original semantics, showing that, in almost all cases, acceptance problems for incomplete argumentation frameworks are significantly harder than the respective problems for argumentation frameworks without uncertainty. All our hardness results for the classes NP, coNP, Π_2^p, and Σ_2^p are derived from one generic reduction.

Keywords. Incomplete argumentation framework, credulous acceptance, skeptical acceptance, computational complexity

1. Introduction

Abstract argumentation frameworks [11] are a formal model that represents an argumentation by a set of atomic arguments and an attack relation between arguments. Unquantified uncertainty about the existence of particular attacks or arguments in abstract argumentation frameworks was first introduced by Coste-Marquis et al. [6] for the set of attacks and by Baumeister et al. [4] for the set of arguments. Baumeister et al. [3] subsequently generalized both models to *incomplete argumentation frameworks*, which allow uncertainty about both arguments and attacks. An incomplete argumentation framework can be seen as a representation of a set of possible worlds, called *completions*, each of which is a standard argumentation framework that shares all definite elements of the incomplete framework and where each of its uncertain elements is either included or excluded. Existing problems for argumentation frameworks can then be generalized to incomplete argumentation frameworks by either asking whether they are satisfied *possibly* (in at least one completion) or *necessarily* (in all completions), i.e., whether the uncertainty either can or must be resolved in a way that satisfies the conditions of the given problem. In applications, that answer may help with decisions in strategic scenarios, where the uncertainty represents possible moves. In scenarios where uncertainty represents missing information, the preliminary answer may be sufficient for the task at hand, removing the need to actually resolve the uncertainty.

In this paper, we continue that line of research and turn to the well-understood problems of *credulous* and *skeptical acceptance*, which are parameterized by a semantics and, for a given argumentation framework and an argument in that framework, either ask whether that argument is in *at least one* extension (for credulous acceptance) or *all* extensions (for skeptical acceptance) of the framework with respect to the semantics. For incomplete argumentation frameworks, we study the following four problem combinations, each of which covers interesting questions that arise in different application scenarios.

- **Possible Credulous Acceptance**: Is there *any* way to accept the given argument?
- **Necessary Credulous Acceptance**: Is the given argument in at least one extension, *regardless* of how the uncertainty is resolved?
- **Possible Skeptical Acceptance**: Can the uncertainty be resolved in such a way that the given argument is in *all* extensions?
- **Necessary Skeptical Acceptance**: Is the given argument absolutely *guaranteed* to be accepted?

We continue with formal definitions of the required notions in Section 2, followed by a full analysis of the complexity of possible and necessary acceptance problems in incomplete argumentation frameworks in Section 3, and a conclusion in Section 4.

2. Model

We decribe the standard model of (abstract) argumentation framework in Section 2.1, including the skeptical and credulous acceptance problems, and we introduce the more general model of incomplete argumentation framework in Section 2.2.

2.1. Argumentation Frameworks

An *argumentation framework* $AF = \langle \mathscr{A}, \mathscr{R} \rangle$ consists of a finite set $\mathscr{A}$ of *arguments* and a binary *attack relation* $\mathscr{R} \subseteq \mathscr{A} \times \mathscr{A}$ on the arguments, where $(a,b) \in \mathscr{R}$ indicates that *a attacks b*. An argument $a \in \mathscr{A}$ is *defended by a set* $A \subseteq \mathscr{A}$ *of arguments in AF* if, for each attacker $b \in \mathscr{A}$ of a with $(b,a) \in \mathscr{R}$, there is a defender $d \in A$ with $(d,b) \in \mathscr{R}$. The *characteristic function of AF*, $F_{AF} : 2^{\mathscr{A}} \to 2^{\mathscr{A}}$, outputs all arguments defended by a given set, i.e., $F_{AF}(A) = \{a \in \mathscr{A} \mid a \text{ is defended by } A \text{ in } AF\}$. F^k_{AF} denotes the k-fold composition of F_{AF}, and F^*_{AF} denotes its infinite composition. A set $A \subseteq \mathscr{A}$ is *conflict-free* (CF) if $(a,b) \notin \mathscr{R}$ for all $a,b \in A$. A conflict-free set $A \subseteq \mathscr{A}$ is further *admissible* (AD) if $A \subseteq F_{AF}(A)$, *complete* (CP) if $A = F_{AF}(A)$, *grounded* (GR) if $A = F^*_{AF}(\emptyset)$, *preferred* (PR) if A is admissible and has no admissible superset, and *stable* (ST) if for every $b \in \mathscr{A} \setminus A$ there is an $a \in A$ with $(a,b) \in \mathscr{R}$. A set of arguments that satisfies one of these *semantics* is called an *extension* of the argumentation framework with respect to that semantics. Every stable extension is preferred, every preferred extension is complete, every complete extension is admissible, and every admissible set is conflict-free. Further, the unique grounded extension is complete. There are argumentation frameworks that have no stable extension, all other extensions are guaranteed to exist.

We study the *credulous acceptance* and *skeptical acceptance* problems in argumentation frameworks, which were first defined by Dunne and Bench-Capon [12] for the preferred semantics alone, but that have since been adopted for various other semantics. The problems are defined as follows, where $\mathbf{s} \in \{\text{CF}, \text{AD}, \text{CP}, \text{GR}, \text{PR}, \text{ST}\}$ is a placeholder for any of the above semantics.

s-CREDULOUS-ACCEPTANCE (s-CA)	
Given:	An argumentation framework $\langle \mathscr{A}, \mathscr{R} \rangle$ and an argument $a \in \mathscr{A}$.
Question:	Is there an **s** extension $\mathscr{E}$ of $\langle \mathscr{A}, \mathscr{R} \rangle$ with $a \in \mathscr{E}$?

s-SKEPTICAL-ACCEPTANCE (s-SA)	
Given:	An argumentation framework $\langle \mathscr{A}, \mathscr{R} \rangle$ and an argument $a \in \mathscr{A}$.
Question:	For all **s** extensions $\mathscr{E}$ of $\langle \mathscr{A}, \mathscr{R} \rangle$, does $a \in \mathscr{E}$ hold?

The stable semantics is a special case, since there may be no stable extension in an argumentation framework. We use the standard formalization of ST-SA which has a "yes" answer for all instances where there is no stable extension, following the convention that a universal quantifier over an empty space defaults to *true*. This problem was shown to be coNP-complete by Dimopoulos and Torres [10]. However, this means that it is possible for an argument to be skeptically accepted but at the same time not credulously accepted, which may be undesired. An alternative formalization incorporates an exception for instances without stable extensions, treating them as "no"-instances. In this case, the skeptical acceptance problem is even DP-complete (see [13]), where DP (the second level of the boolean hierarchy over NP) is the class of differences of any two NP sets and contains both NP and coNP. We leave the analysis of this variant in the context of incomplete argumentation frameworks to future work.

We make a few observations about the CA and SA problems that will be used later.

Observation 1. Skeptical acceptance for CF and AD is trivial: The empty set is always conflict-free and admissible, so the answer is always "no" for all problem instances.

Observation 2. Since the grounded extension is unique, there is no difference between skeptical and credulous acceptance for the grounded semantics, i.e., GR-CA = GR-SA.

Observation 3. Since the grounded extension is exactly the intersection of all complete extensions, skeptical acceptance of an argument is the same for the grounded and the complete semantics: GR-SA = CP-SA.

Note that Observations 2 and 3 together yield that GR-CA = GR-SA = CP-SA.

Observation 4. The credulous acceptance problem is the same for the admissible and the preferred semantics (AD-CA = PR-CA): An argument is a member of (at least) one preferred extension if and only if it is in (at least) one admissible extension, since every preferred set is admissible and every admissible set is a subset of some preferred set.

The computational complexity of CA and SA for the six semantics considered in this paper was studied by Dimopoulos and Torres [10], Coste-Marquis et al. [7], and Dunne and Bench-Capon [12]. We present their results together with our new findings in Table 2 in Section 4.

2.2. Incomplete Argumentation Frameworks

An incomplete argumentation framework $\langle \mathscr{A}, \mathscr{A}^?, \mathscr{R}, \mathscr{R}^? \rangle$ splits both the set of arguments and the set of attacks into two disjoint parts, a *definite* part ($\mathscr{A}$ and $\mathscr{R}$) and an *uncertain* part ($\mathscr{A}^?$ and $\mathscr{R}^?$), where both attack types are subsets of $(\mathscr{A} \cup \mathscr{A}^?) \times (\mathscr{A} \cup \mathscr{A}^?)$. For uncertain elements (members of $\mathscr{A}^?$ or $\mathscr{R}^?$), it is not known whether they are part of the argumentation—they might be added or removed in the future, or the uncertainty may just represent the limited knowledge of some agent about those elements. Definite arguments (elements of $\mathscr{A}$) are known to exist, while definite attacks (elements of $\mathscr{R}$) exist if and only if both incident arguments exist, too. To account for this, we call attacks in $\mathscr{R}$ that are incident to at least one uncertain argument *conditionally definite*, since these attacks may vanish alongside an incident uncertain argument, while attacks in $\mathscr{R}$ that are only incident to definite arguments are called *definite*. If $\mathscr{A}^? = \emptyset$, we have a purely *attack-incomplete* argumentation framework; for $\mathscr{R}^? = \emptyset$, a purely *argument-incomplete* argumentation framework; and $\mathscr{A}^? = \mathscr{R}^? = \emptyset$ yields standard argumentation frameworks without uncertainty. An attack-incomplete argumentation framework may be abbreviated as $\langle \mathscr{A}, \mathscr{R}, \mathscr{R}^? \rangle$ and an argument-incomplete argumentation framework as $\langle \mathscr{A}, \mathscr{A}^?, \mathscr{R} \rangle$.

Example 5. An argumentation framework can be identified with a directed graph by representing arguments as nodes and attacks as directed edges. Figure 1 is a graph representation of the incomplete argumentation framework $\langle \mathscr{A}, \mathscr{A}^?, \mathscr{R}, \mathscr{R}^? \rangle$ with $\mathscr{A} = \{b, c\}$, $\mathscr{A}^? = \{a, d\}$, $\mathscr{R} = \{(a,b), (b,b), (d,c)\}$, and $\mathscr{R}^? = \{(b,c), (c,d)\}$, where definite elements are solid (circles for arguments or arrows for attacks), uncertain elements are dashed, and conditionally definite attacks are dash-dotted.

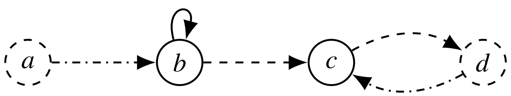

Figure 1. Graph representation of the incomplete argumentation framework in Example 5

A *completion* of an incomplete argumentation framework $AF = \langle \mathscr{A}, \mathscr{A}^?, \mathscr{R}, \mathscr{R}^? \rangle$ is any argumentation framework $AF^* = \langle \mathscr{A}^*, \mathscr{R}^* \rangle$ that satisfies $\mathscr{A} \subseteq \mathscr{A}^* \subseteq \mathscr{A} \cup \mathscr{A}^?$ and $\mathscr{R}|_{\mathscr{A}^*} \subseteq \mathscr{R}^* \subseteq \left(\mathscr{R} \cup \mathscr{R}^?\right)|_{\mathscr{A}^*}$. Here, the *restriction* $\mathscr{R}|_{\mathscr{A}^*}$ of an attack relation $\mathscr{R}$ to $\mathscr{A}^*$ is defined as $\mathscr{R}|_{\mathscr{A}^*} = \{(a,b) \in \mathscr{R} \mid a, b \in \mathscr{A}^*\}$. It represents the fact that conditionally definite attacks can only be part of a completion which includes that argument. However, a conditionally definite attack must be present in all completions containing both incident arguments, while an uncertain attack may vanish in a completion that contains both of its incident arguments. If at least one completion of an incomplete argumentation framework AF satisfies some property, this property is said to hold *possibly* for AF. On the other hand, if all completions of AF satisfy a property, it is said to hold *necessarily* for AF. Accordingly, we define both a possible and a necessary variant of the **s**-CA and **s**-SA problems for incomplete argumentation frameworks, for each semantics **s** considered here:

s-NECESSARY-CREDULOUS-ACCEPTANCE (**s**-NCA)	
Given:	An incomplete argumentation framework $\langle \mathscr{A}, \mathscr{A}^?, \mathscr{R}, \mathscr{R}^? \rangle$ and an argument $a \in \mathscr{A}$.
Question:	Is it true that for each completion $AF^* = \langle \mathscr{A}^*, \mathscr{R}^* \rangle$ of $\langle \mathscr{A}, \mathscr{A}^?, \mathscr{R}, \mathscr{R}^? \rangle$, there exists an **s** extension $\mathscr{E}$ of AF^* with $a \in \mathscr{E}$?

s-POSSIBLE-SKEPTICAL-ACCEPTANCE (s-PSA)	
Given:	An incomplete argumentation framework $\langle \mathscr{A}, \mathscr{A}^?, \mathscr{R}, \mathscr{R}^? \rangle$ and an argument $a \in \mathscr{A}$.
Question:	Does there exist a completion $AF^* = \langle \mathscr{A}^*, \mathscr{R}^* \rangle$ of $\langle \mathscr{A}, \mathscr{A}^?, \mathscr{R}, \mathscr{R}^? \rangle$ such that for each **s** extension $\mathscr{E}$ of AF^*, we have $a \in \mathscr{E}$?

We define **s**-POSSIBLE-CREDULOUS-ACCEPTANCE (**s**-PCA) analogously to **s**-NCA, except that we now quantify *existentially* over all completions AF^*, and we define **s**-NECESSARY-SKEPTICAL-ACCEPTANCE (**s**-NSA) analogously to **s**-PSA, except that we now quantify *universally* over all completions AF^*. Note that for the skeptical acceptance problems it is equivalent to ask whether each set of arguments that does not include a is not **s** in AF^*. This alternative quantifier formulation allows us to directly derive upper bounds. Due to Observations 2, 3, and 4, we have the following equalities: GR-PCA = GR-PSA = CP-PSA, GR-NCA = GR-NSA = CP-NSA, AD-PCA = PR-PCA, and AD-NCA = PR-NCA.

Note that we define the semantics of an incomplete argumentation framework through the completions. However, there are other ways for defining semantics of an incomplete argumentation framework as well. Cayrol et al. [5], for instance, restate the basic requirements of conflict-freeness and acceptability in the context of their "partial argumentation frameworks" (PAFs), consider related complexity issues, and establish links between semantics of PAFs and semantics of the completions.

3. Complexity Results

The number of completions for a given incomplete argumentation framework is exponential in the number of its uncertain elements. Therefore, possible and necessary problem generalizations are potentially harder than the respective baseline problem. In this section, we provide a full analysis of whether and how the computational complexity of the PCA, NCA, PSA, and NSA problem variants differs from that of CA and SA for the six semantics CF, AD, ST, CP, GR, and PR. For information about the relevant complexity classes of the polynomial hierarchy—in particular, P, NP, coNP, $\Pi_2^p = \mathrm{coNP}^{\mathrm{NP}}$, $\Sigma_2^p = \mathrm{NP}^{\mathrm{NP}}$, and $\Sigma_3^p = \mathrm{NP}^{\Sigma_2^p}$—as well as the concepts of hardness and completeness, we refer the reader to, e.g., Papadimitriou [14], Stockmeyer [16], and Rothe [15].

3.1. Upper Bounds

We start with some simple P membership results. Since the answer to CF-SA and AD-SA is trivially "no" for all completions of an incomplete argumentation framework due to Observation 1, so is the answer to their possible and necessary generalizations **s**-PSA and **s**-NSA for $\mathbf{s} \in \{\text{CF}, \text{AD}\}$, which are therefore in P, too. Further, both the possible and necessary generalizations of CF-CA are in P, as stated in Proposition 6.

Proposition 6. CF-PCA *and* CF-NCA *are in* P.

The proof of Proposition 6 is omitted due to space limitations. For all remaining problems, from their quantifier representation we can derive upper bounds potentially higher than P. Matching lower bounds in Section 3.2 will prove these bounds to be tight.

Table 1. Overview of quantified SAT problems used for hardness reductions

	instance	question	complexity
3-SAT	(φ, X)	$\exists\tau_X : \varphi[\tau_X] = \mathit{true}$	NP-complete
Σ_2SAT	(φ, X, Y)	$\exists\tau_X : \forall\tau_Y : \varphi[\tau_X, \tau_Y] = \mathit{false}$	Σ_2^p-complete
Σ_3SAT	(φ, X, Y, Z)	$\exists\tau_X : \forall\tau_Y : \exists\tau_Z : \varphi[\tau_X, \tau_Y, \tau_Z] = \mathit{true}$	Σ_3^p-complete
3-UNSAT	(φ, X)	$\forall\tau_X : \varphi[\tau_X] = \mathit{false}$	coNP-complete
Π_2SAT	(φ, X, Y)	$\forall\tau_X : \exists\tau_Y : \varphi[\tau_X, \tau_Y] = \mathit{true}$	Π_2^p-complete

Proposition 7. *1. For* $\mathbf{s} \in \{\text{AD}, \text{ST}, \text{CP}, \text{GR}, \text{PR}\}$ *and for* $\mathbf{s}' \in \{\text{CP}, \text{GR}\}$*,* $\mathbf{s}$-PCA *and* $\mathbf{s}'$-PSA *are in* NP.

2. *For* $\mathbf{s} \in \{\text{ST}, \text{CP}, \text{GR}\}$*,* $\mathbf{s}$-NSA *and* GR-NCA *are in* coNP.
3. *For* $\mathbf{s} \in \{\text{AD}, \text{ST}, \text{CP}, \text{PR}\}$*,* $\mathbf{s}$-NCA *is in* Π_2^p*.*
4. ST-PSA *is in* Σ_2^p*.*
5. PR-NSA *is in* Π_2^p *and* PR-PSA *is in* Σ_3^p*.*

Proof. Due to space limitations, we only prove the last item of this proposition. In the following, whenever we speak of "existential quantifiers" or "universal quantifiers," we mean polynomially length-bounded existential or universal quantifiers. A quantifier representation of skeptical acceptance for the preferred semantics is "$\forall \mathscr{E} \subseteq (\mathscr{A} \setminus \{a\}) : \exists \mathscr{E}' \supset \mathscr{E} : (\mathscr{E}$ is not AD in $AF)$ or $(\mathscr{E}'$ is AD in $AF)$", where $AF = \langle \mathscr{A}, \mathscr{R} \rangle$ and $a \in \mathscr{A}$. For PR-NSA, this is preceded by a universal quantifier over completions which collapses with the leading universal quantifier and provides Π_2^p membership. For PR-PSA, it is preceded by an existential quantifier over completions and provides Σ_3^p membership. ❑

3.2. Lower Bounds

Any hardness of the problems CA and SA is directly inherited by their possible and necessary generalizations. For several of these generalizations, the upper bound from Section 3.1 coincides with the lower bound inherited from CA and SA (cf. Table 2, columns 2 and 5), which is stated in Corollary 8.

Corollary 8. *1. For* $\mathbf{s} \in \{\text{AD}, \text{ST}, \text{CP}, \text{PR}\}$*,* $\mathbf{s}$-PCA *is* NP*-hard.*

2. ST-NSA *is* coNP*-hard.*
3. PR-NSA *is* Π_2^p*-hard.*

For our proofs of the remaining hardness results, we reduce from different versions of the satisfiability problem for quantified boolean formulas (QSAT), which are known to be hard for different classes in the polynomial hierarchy. Table 1 gives a short definition of all used problems along with their complexity, where X, Y, and Z are disjoint sets of propositional variables, φ denotes a formula in 3-CNF (conjunctive normal form with at most three literals per clause) over the respective variables, τ_S is a truth assignment on a set of literals S with $\tau_S : S \to \{\mathit{true}, \mathit{false}\}$, and $\varphi[\tau_S]$ is the truth value that φ evaluates to under τ_S.

Definition 9 describes a generic translation of 3-SAT, 3-UNSAT, Σ_2SAT, and Π_2SAT instances to incomplete argumentation frameworks that will be used in most of the remaining proofs. It allows to construct either purely attack-incomplete or purely argument-incomplete argumentation frameworks, so all hardness results obtained hold even in these special cases. The translation is loosely based on a similar construction originally used by Dimopoulos and Torres [10], which has since been frequently adopted and modified.

Definition 9. Given a QSAT instance (φ, X, Y) (where Y is considered empty if not part of an instance) with $\varphi = \bigwedge_i c_i$ and $c_i = \bigvee_j \alpha_j$ for each clause c_i, where the α_j are the literals in clause c_i, create either an attack-incomplete argumentation framework $\langle \mathscr{A}, \mathscr{R}, \mathscr{R}^? \rangle$ (left) or an argument-incomplete argumentation framework $\langle \mathscr{A}, \mathscr{A}^?, \mathscr{R} \rangle$ (right):

$$\mathscr{A} = \left\{ \begin{array}{ll} x_i, \bar{x}_i, & \text{for } x_i \in X \\ y_i, \bar{y}_i, & \text{for } y_i \in Y \\ \bar{c}_i, & \text{for } c_i \text{ in } \varphi \\ \varphi, \bar{\varphi}, g & \end{array} \right\}, \qquad \mathscr{A} = \left\{ \begin{array}{ll} \bar{x}_i, & \text{for } x_i \in X \\ y_i, \bar{y}_i, & \text{for } y_i \in Y \\ \bar{c}_i, & \text{for } c_i \text{ in } \varphi \\ \varphi, \bar{\varphi}, g & \end{array} \right\},$$

$$\mathscr{R} = \left\{ \begin{array}{ll} (\bar{x}_i, x_i), & \text{for } x_i \in X \\ (y_i, \bar{y}_i), (\bar{y}_i, y_i), & \text{for } y_i \in Y \\ (x_k, \bar{c}_i), & \text{if } x_k \text{ in } c_i \\ (\bar{x}_k, \bar{c}_i), & \text{if } \neg x_k \text{ in } c_i \\ (y_k, \bar{c}_i), & \text{if } y_k \text{ in } c_i \\ (\bar{y}_k, \bar{c}_i), & \text{if } \neg y_k \text{ in } c_i \\ (\bar{c}_i, \varphi), & \text{for } c_i \in \varphi \\ (\varphi, \bar{\varphi}), & \end{array} \right\}, \qquad \mathscr{R} = \left\{ \begin{array}{ll} (x_i, \bar{x}_i), & \text{for } x_i \in X \\ (y_i, \bar{y}_i), (\bar{y}_i, y_i), & \text{for } y_i \in Y \\ (x_k, \bar{c}_i), & \text{if } x_k \text{ in } c_i \\ (\bar{x}_k, \bar{c}_i), & \text{if } \neg x_k \text{ in } c_i \\ (y_k, \bar{c}_i), & \text{if } y_k \text{ in } c_i \\ (\bar{y}_k, \bar{c}_i), & \text{if } \neg y_k \text{ in } c_i \\ (\bar{c}_i, \varphi), & \text{for } c_i \in \varphi \\ (\varphi, \bar{\varphi}), & \end{array} \right\},$$

$$\mathscr{R}^? = \left\{ (g, \bar{x}_i), \text{ for } x_i \in X \right\}. \qquad \mathscr{A}^? = \left\{ x_i, \text{ for } x_i \in X \right\}.$$

All arguments x_i, y_i and $\bar{x}_i$, $\bar{y}_i$ are called *literal arguments* and all arguments $\bar{c}_i$ *clause arguments*. All arguments in pairs $\bar{a}$, a are called *counterparts* of each other. Argument g is without effect in the argument-incomplete version and only included there for uniformity.

For an incomplete argumentation framework AF created according to Definition 9, we associate a given truth assignment τ_X on X with a completion $AF^{\tau_X} = \langle \mathscr{A}^{\tau_X}, \mathscr{R}^{\tau_X} \rangle$ of AF. For an attack-incomplete argumentation framework $\langle \mathscr{A}, \mathscr{R}, \mathscr{R}^? \rangle$, that completion has $\mathscr{A}^{\tau_X} = \mathscr{A}$ and $(g, \bar{x}_i) \in \mathscr{R}^{\tau_X} \iff \tau_X(x_i) = \mathit{true}$. For an argument-incomplete argumentation framework $\langle \mathscr{A}, \mathscr{A}^?, \mathscr{R} \rangle$, that completion has $x_i \in \mathscr{A}^{\tau_X} \iff \tau_X(x_i) = \mathit{true}$ and $\mathscr{R}^{\tau_X} = \mathscr{R}|_{\mathscr{A}^{\tau_X}}$. Further, we identify an assignment τ_S on a set $S = \{s_1, \ldots, s_{|S|}\} \subseteq (X \cup Y)$ of variables with a set $\mathscr{A}^{\tau_X}[\tau_S]$ of arguments in the completion, namely, $\mathscr{A}^{\tau_X}[\tau_S] = \{s_i \mid \tau_S(s_i) = \mathit{true}\} \cup \{\bar{s}_i \mid \tau_S(s_i) = \mathit{false}\}$.

In Lemma 10, we prove that both constructions behave similarly and can, in effect, be used interchangeably.

Lemma 10. *Let (φ, X, Y) be a* QSAT *instance, let $\langle \mathscr{A}, \mathscr{R}, \mathscr{R}^? \rangle$ or $\langle \mathscr{A}, \mathscr{A}^?, \mathscr{R} \rangle$ be an incomplete argumentation framework created for it according to Definition 9, and let τ_X be an assignment on X. In the completion AF^{τ_X}, $\mathscr{A}^{\tau_X}[\tau_X] \cup \{g\}$ is a subset of the grounded extension and therefore contained in all complete extensions.*

Proof. g is always unattacked and therefore clearly in the grounded extension. Consider the attack-incomplete argumentation framework $AF = \langle \mathscr{A}, \mathscr{R}, \mathscr{R}^? \rangle$ and the completion AF^{τ_X} of AF according to τ_X. In AF^{τ_X}, g attacks each argument $\bar{x}_i$ for which

$\tau_X(x_i) = true$, thus defending its counterpart x_i, so these x_i are included in the grounded extension. All $\bar{x}_j$ for which $\tau_X(x_j) = false$ remain unattacked and are themselves included in the grounded extension, while they attack their counterparts x_j, which thus are not included. Consider now the argument-incomplete argumentation framework $AF = \langle \mathscr{A}, \mathscr{A}^?, \mathscr{R} \rangle$ and its completion AF^{τ_X}. Each argument x_i is always unattacked and therefore in the grounded extension if and only if it is included in the completion, which is the case for $\tau_X(x_i) = true$. Each $\bar{x}_j$ is only attacked by its counterpart x_j and therefore in the grounded extension if and only if that x_j is excluded from the completion, which is the case for $\tau_X(x_i) = false$. ❑

We now show a crucial correspondence between assignments in a QSAT instance and sets of arguments in the respective incomplete argumentation framework.

Lemma 11. *Given a* QSAT *instance* (φ, X, Y) *and full assignments* τ_X *and* τ_Y *(*τ_Y *only if applicable). Let AF be an incomplete AF created for* (φ, X, Y) *following Definition 9, let* AF^{τ_X} *be its completion corresponding to* τ_X *and let* $\mathscr{A}^{\tau_X}[\tau_X, \tau_Y]$ *be the set of literal arguments corresponding to the total assignment.*

- *If* $\varphi[\tau_X, \tau_Y] = true$*, then* $\mathscr{A}^{\tau_X}[\tau_X, \tau_Y] \cup \{g, \varphi\}$ *is admissible, complete, preferred, and stable in* AF^{τ_X}*, and for* $Y = \emptyset$ *also grounded.*
- *If* $\varphi[\tau_X, \tau_Y] = false$*, then* $\mathscr{A}^{\tau_X}[\tau_X, \tau_Y] \cup \{g, \bar{\varphi}\} \cup \{\bar{c}_i \mid \nexists d \in \mathscr{A}^{\tau_X}[\tau_X, \tau_Y] : (d, \bar{c}_i) \in \mathscr{R}^{\tau_X}\}$ *is admissible, complete, preferred, and stable in* AF^{τ_X}*, and for* $Y = \emptyset$ *also grounded.*

Proof. Assume that $\varphi[\tau_X, \tau_Y] = true$. We know that $\mathscr{A}^{\tau_X}[\tau_X] \cup \{g\}$ is a subset of the grounded extension of AF^{τ_X}. We show that $\mathscr{E} = \mathscr{A}^{\tau_X}[\tau_X, \tau_Y] \cup \{g, \varphi\}$ is stable in AF^{τ_X}. It is easy to see from Definition 9 that $\mathscr{E}$ is conflict-free, since there are no attacks between literal arguments for distinct literals, φ, or g. Further, $\mathscr{E}$ attacks each argument in $\mathscr{A}^{\tau_X} \setminus \mathscr{E}$. Argument $\bar{\varphi}$ is attacked by $\varphi \in \mathscr{E}$. Each literal argument from X that does not occur in $\mathscr{E}$ is either excluded from the completion, attacked by g, or attacked by its counterpart in $\mathscr{E}$. Each literal argument from Y that is not in $\mathscr{E}$ is attacked by its counterpart in $\mathscr{E}$. For each clause argument $\bar{c}_i$, we know by assumption that the corresponding clause c_i in φ is satisfied by the total assignment, since $\varphi[\tau_X, \tau_Y] = true$. Since c_i is satisfied, at least one literal in c_i must be satisfied. By construction of $\mathscr{E}$ we know that at least one literal argument corresponding to a literal in c_i is in $\mathscr{E}$, and by construction of AF, this argument attacks the clause argument $\bar{c}_i$. In total, this means that all clause arguments are attacked by $\mathscr{E}$, and we proved that $\mathscr{E}$ is stable in AF^{τ_X}. Since $\mathscr{E}$ is stable, it is also preferred, complete, and admissible. For $Y = \emptyset$, the set $\mathscr{A}^{\tau_X}[\tau_X] \cup \{g\}$, which is a subset of the grounded extension by Lemma 10, already attacks all clause arguments and thus defends φ, so $\mathscr{A}^{\tau_X}[\tau_X] \cup \{g, \varphi\}$ is the grounded extension of AF^{τ_X}.

Now assume that $\varphi[\tau_X, \tau_Y] = false$. Let $\mathscr{E}' = \mathscr{A}^{\tau_X}[\tau_X, \tau_Y] \cup \{g, \bar{\varphi}\} \cup \{\bar{c}_i \mid \nexists d \in \mathscr{A}^{\tau_X}[\tau_X, \tau_Y] : (d, \bar{c}_i) \in \mathscr{R}^{\tau_X}\}$. First, let us show that the subset $C = \{\bar{c}_i \mid \nexists d \in \mathscr{A}^{\tau_X}[\tau_X, \tau_Y] : (d, \bar{c}_i) \in \mathscr{R}^{\tau_X}\}$ of $\mathscr{E}'$ is non-empty. Since $\varphi[\tau_X, \tau_Y] = false$, there is at least one clause c'_i in φ that is not satisfied by the total assignment, so none of the literals in c'_i is satisfied. These literals correspond to literal arguments in AF, which are the only arguments in AF that attack the clause argument $\bar{c}'_i$. By construction of $\mathscr{E}'$, we know that none of these arguments are in $\mathscr{E}'$, so $\mathscr{E}'$ does not attack $\bar{c}'_i$ and thus $\bar{c}'_i \in C$. We now show that $\mathscr{E}'$ is stable in AF^{τ_X}. Again, $\mathscr{E}'$ is clearly conflict-free. All literal arguments from X that do not occur in $\mathscr{E}'$ are again either excluded from the completion, attacked by g, or attacked by their counterpart in $\mathscr{E}'$. Each literal argument from Y that is not in $\mathscr{E}'$ is attacked by its counterpart in $\mathscr{E}'$. Each clause argument that is not in C is attacked by some $d \in \mathscr{A}^{\tau_X}[\tau_X, \tau_Y]$ due to the definition of C. Finally, argument φ is attacked by all arguments in $C \subseteq \mathscr{E}'$, of which there is at least one since $C \neq \emptyset$. Since $\mathscr{E}'$ is stable, it is also preferred, complete, and admissible. For $Y = \emptyset$, the set $\mathscr{A}^{\tau_X}[\tau_X] \cup \{g\}$, which is a subset of the grounded extension due to Lemma 10, already attacks all clause arguments in $\mathscr{A}^{\tau_X} \setminus C$ and thus defends all arguments in C, which in turn defend $\bar{\varphi}$, so $\mathscr{E}'$ is the grounded extension of AF^{τ_X}. ❑

Theorem 12. GR-PCA *is* NP-*hard.*

Proof. We reduce from 3-SAT. Let (φ, X) be a 3-SAT instance. If $(\varphi, X) \in$ 3-SAT, we have $\exists \tau_X : \varphi[\tau_X] = \mathit{true}$, so by Lemma 11 there exists a completion of the corresponding argumentation framework AF where φ is in the grounded extension, and we have $(AF, \varphi) \in$ GR-PCA. If $(\varphi, X) \notin$ 3-SAT, we have $\forall \tau_X : \varphi[\tau_X] = \mathit{false}$, so $\bar{\varphi}$ is in the grounded extension of all completions of the corresponding argumentation framework AF, so φ cannot be in the grounded extension of any completion, and we have $(AF, \varphi) \notin$ GR-PCA. ❑

Together with Observations 2 and 3, the following corollary follows immediately.

Corollary 13. GR-PSA *and* CP-PSA *are* NP-*hard.*

Theorem 14. GR-NCA *is* coNP-*hard.*

Proof. We reduce from 3-UNSAT. Let (φ, X) be a 3-UNSAT instance. If $(\varphi, X) \in$ 3-UNSAT, we have $\forall \tau_X : \varphi[\tau_X] = \mathit{false}$, so by Lemma 11, $\bar{\varphi}$ is in the grounded extension of all completions of the corresponding argumentation framework AF and we have $(AF, \bar{\varphi}) \in$ GR-NCA. If $(\varphi, X) \notin$ 3-UNSAT, we have $\exists \tau_X : \varphi[\tau_X] = \mathit{true}$, so there exists a completion of the corresponding argumentation framework AF where φ is in the grounded extension, so $\bar{\varphi}$ cannot be in the grounded extensions of all completions, and we have $(AF, \bar{\varphi}) \notin$ GR-NCA. ❑

Again, Observations 2 and 3 immediately give the following corollary.

Corollary 15. GR-NSA *and* CP-NSA *are* coNP-*hard.*

Theorem 16. *For* $\mathbf{s} \in \{\text{AD}, \text{CP}, \text{ST}, \text{PR}\}$, $\mathbf{s}$-NCA *is* Π_2^p-*hard.*

Proof. We reduce from Π_2SAT. Let (φ, X, Y) be a Π_2SAT instance. If $(\varphi, X, Y) \in \Pi_2$SAT, we have $\forall \tau_X : \exists \tau_Y : \varphi[\tau_X, \tau_Y] = \mathit{true}$, so by Lemma 11, for all completions of the corresponding argumentation framework AF, there is a τ_Y such that the set $\mathscr{A}^{\tau_X}[\tau_X, \tau_Y] \cup \{g, \varphi\}$ is admissible, complete, preferred, and stable, so $(AF, \varphi) \in \mathbf{s}$-NCA for $\mathbf{s} \in \{\text{AD}, \text{CP}, \text{ST}, \text{PR}\}$. If $(\varphi, X, Y) \notin \Pi_2$SAT, we have $\exists \tau_X : \forall \tau_Y : \varphi[\tau_X, \tau_Y] = \mathit{false}$, so there is a completion AF^{τ_X} of the corresponding argumentation framework AF where $\mathscr{A}^{\tau_X}[\tau_X, \tau_Y] \cup \{g, \bar{\varphi}\} \cup \{c_i \mid \nexists d \in \mathscr{A}^{\tau_X}[\tau_X, \tau_Y] : (d, c_i) \in \mathscr{R}^{\tau_X}\}$ is stable for any choice of τ_Y. This means that φ cannot be a member of any admissible set in that completion—and therefore neither in a complete, stable, or preferred set—so $(AF, \varphi) \notin \mathbf{s}$-NCA for $\mathbf{s} \in \{\text{AD}, \text{CP}, \text{ST}, \text{PR}\}$. ❑

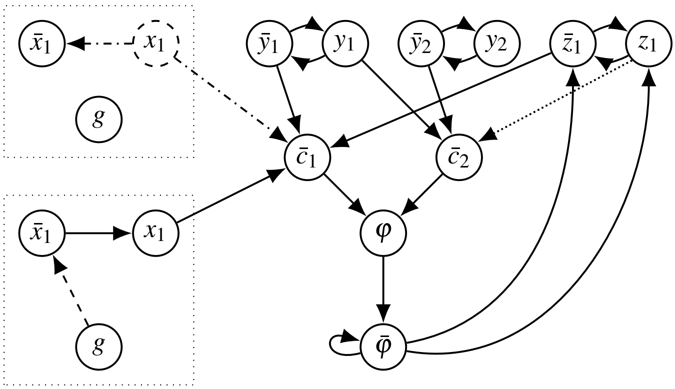

Figure 2. PR-PSA instance created from clauses $c_1 = x_1 \vee \neg y_1 \vee \neg z_1$ and $c_2 = y_1 \vee \neg y_2 \vee z_1$ following a construction of Dunne and Bench-Capon [12]. Add either the framed part at the top to create an argument-incomplete argumentation framework or the one at the bottom for an attack-incomplete argumentation framework. A slight modification that uses $c_2' = y_1 \vee \neg y_2$ instead of c_2 can be obtained by excluding the dotted attack $(z_1, \bar{c}_2)$.

Theorem 17. ST-PSA *is* Σ_2^p*-hard.*

Proof. We reduce from Σ_2SAT. Let (φ, X, Y) be a Σ_2SAT instance. If $(\varphi, X, Y) \in \Sigma_2$SAT, we have $\exists \tau_X : \forall \tau_Y : \varphi[\tau_X, \tau_Y] = \mathit{false}$, so by Lemma 11, there is a completion AF^{τ_X} of the corresponding argumentation framework AF where $\mathscr{A}^{\tau_X}[\tau_X, \tau_Y] \cup \{g, \bar{\varphi}\} \cup \{c_i \mid \nexists d \in \mathscr{A}^{\tau_X}[\tau_X, \tau_Y] : (d, c_i) \in \mathscr{R}^{\tau_X}\}$ is stable for any choice of τ_Y. There clearly can be no stable extension other than these, so $(AF, \bar{\varphi}) \in$ ST-PSA. If $(\varphi, X, Y) \notin \Sigma_2$SAT, we have $\forall \tau_X : \exists \tau_Y : \varphi[\tau_X, \tau_Y] = \mathit{true}$, so for all completions of the corresponding argumentation framework AF, there is some τ_Y such that the set $\mathscr{A}^{\tau_X}[\tau_X, \tau_Y] \cup \{g, \varphi\}$ is stable. Therefore, $(AF, \bar{\varphi}) \notin$ ST-PSA. ❑

Theorem 18. PR-PSA *is* Σ_3^p*-hard.*

Proof. Due to space constraints, we only sketch the proof of Theorem 18. To prove Σ_3^p-hardness, we can extend a reduction that Dunne and Bench-Capon [12, Def. 13] used to prove Π_2^p-hardness of PR-SA (see also the related work of Atkinson et al. [1]). Given an instance (φ, X, Y, Z) of Σ_3SAT, create an incomplete argumentation framework AF according to their construction using Y for their x-arguments, Z for their y-arguments, and clause arguments and φ for their *gate* arguments. In addition, create arguments from literals X along with argument g the same way as in our Definition 9.

If $(\varphi, X, Y, Z) \in \Sigma_3$SAT, there is a completion of AF in which, by their result, argument φ is skeptically preferred, so $(AF, \varphi) \in$ PR-PSA. If $(\varphi, X, Y, Z) \notin \Sigma_3$SAT, for all completions of AF, there is a preferred extension that does not include φ, so $(AF, \varphi) \notin$ PR-PSA. ❑

Example 19. Consider a Σ_3SAT instance $(\varphi, \{x_1\}, \{y_1, y_2\}, \{z_1\})$, where $\varphi = c_1 \wedge c_2$ with $c_1 = x_1 \vee \neg y_1 \vee \neg z_1$ and $c_2 = y_1 \vee \neg y_2 \vee z_1$. Figure 2 displays a graph representation of the incomplete argumentation framework created for this instance of Σ_3SAT: For $\tau_X(x_1) = \mathit{true}$, any assignment τ_Y on $\{y_1, y_2\}$, and $\tau_Z(z_1) = \mathit{true}$, we have $\varphi[\tau_X, \tau_Y, \tau_Z] = \mathit{true}$. Accordingly, in the completion AF^{τ_X} all preferred extensions are of the form $\{g, x_1, z_1, \varphi\} \cup \mathscr{A}[\tau_Y]$ for some τ_Y, so φ is skeptically preferred.

Table 2. Overview of existing and new complexity results for credulous and skeptical acceptance problems, where existing results are ascribed to the respective source (references are numbers in brackets). Results marked with an asterisk (*) are straight-forward and and results due to this paper are marked by their theorem numbers.

	s-CA		**s**-PCA		**s**-NCA		**s**-SA		**s**-PSA		**s**-NSA	
CF	$\in$ P	*	$\in$ P	6	$\in$ P	6	trivial	*	trivial	*	trivial	*
AD	NP-c.	[10]	NP-c.	8	Π_2^p-c.	16	trivial	*	trivial	*	trivial	*
ST	NP-c.	[10]	NP-c.	8	Π_2^p-c.	16	coNP-c.	[10]	Σ_2^p-c.	17	coNP-c.	8
CP	NP-c.	[7]	NP-c.	8	Π_2^p-c.	16	$\in$ P	[7]	NP-c.	13	coNP-c.	15
GR	$\in$ P	*	NP-c.	12	coNP-c.	14	$\in$ P	*	NP-c.	13	coNP-c.	15
PR	NP-c.	[10]	NP-c.	8	Π_2^p-c.	16	Π_2^p-c.	[12]	Σ_3^p-c.	18	Π_2^p-c.	8

When changing c_2 to $c_2' = y_1 \vee \neg y_2$ and $\varphi' = c_1 \wedge c_2'$, we obtain a "no" instance. For τ_Y with $\tau_Y(y_1) = false$ and $\tau_Y(y_2) = true$, along with any assignments τ_X and τ_Z, we have $\varphi'[\tau_X, \tau_Y, \tau_Z] = false$. In the corresponding argumentation framework, in both completions either $\{g, x_1, \bar{y}_1, y_2, \bar{c}_2'\}$ or $\{g, \bar{x}_1, \bar{y}_1, y_2, \bar{c}_2'\}$ is a preferred extension that does not include φ, so φ is not skeptically preferred.

4. Conclusions and Relations to Other Models

Table 2 summarizes all complexity results of this paper and compares them to the existing results for CA and SA.

Compared to the possible and necessary variants of the verification problem for incomplete argumentation frameworks [3], which are not harder to solve than the respective baseline problem for many semantics, in this paper we observe a jump in complexity of necessary credulous acceptance and possible skeptical acceptance in almost all cases. This indicates that the presence of uncertainty, as described by attack incompleteness or argument incompleteness or both, is very likely to make acceptance problems harder.

Possible-credulous acceptance problems in incomplete argumentation frameworks are related to extension enforcement problems [2,8], where the question is, given an argumentation framework and a subset of its arguments, how the attacks and/or the arguments of the argumentation framework can be modified most efficiently such that the given set becomes part of an extension. Instances for acceptance problems in incomplete argumentation frameworks and for enforcement problems coincide if the incomplete argumentation framework has only uncertain attacks and no uncertain arguments, and if the enforcement instance allows only changes to the attack relation and its given subset is a singleton. However, enforcement problems aim at finding a minimal number of changes to the argumentation framework, which is not an aim in incomplete argumentation frameworks. On the other hand, the question of whether acceptance of the target argument can *at all* be achieved is trivially true in most variants of enforcement, while this is the key question for possible-credulous acceptance problems in incomplete argumentation frameworks.

The model of incomplete argumentation frameworks is further closely related to the recently proposed *control argumentation frameworks* (CAF) [9], which use a similar, yet much more specific formalism of uncertain elements in argumentation frameworks. Though technically similar, neither model can be fully expressed by the other: CAFs have no feature to represent uncertain attacks in "possible" problem variants, while incomplete argumentation frameworks cannot express uncertain attacks where the attack itself is known, but its direction is not. However, there are various special cases where both models coincide. For example, "possible" problem variants in purely argument-incomplete argumentation frameworks can be represented by CAFs using their *control*-part, while "necessary" problem variants in incomplete argumentation frameworks can be represented by CAFs using their *uncertain*-part. The results of this paper may therefore be useful for the complexity analysis of similar problems in CAFs.

Acknowledgments: We thank the reviewers for many helpful suggestions. This work has been supported in part by the project "Online Participation," funded by the NRW Ministry for Innovation, Science, and Research.

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