Computational Models of Argument S. Modgil et al. (Eds.) © 2018 The authors and IOS Press. This article is published online with Open Access by IOS Press and distributed under the terms of the Creative Commons Attribution Non-Commercial License 4.0 (CC BY-NC 4.0). doi:10.3233/978-1-61499-906-5-129

Equipping Sequent-Based Argumentation with Defeasible Assumptions

AnneMarie Borg¹

Institute for Philosophy II, Ruhr-University Bochum, Germany

Abstract. In many expert and everyday reasoning contexts it is very useful to reason on the basis of defeasible assumptions. For instance, if the information at hand is incomplete we often use plausible assumptions, or if the information is conflicting we interpret it as consistent as possible. In this paper sequent-based argumentation, a form of logical argumentation in which arguments are represented by a sequent, is extended to incorporate assumptions. The resulting assumption framework is general, in that some other approaches to reasoning with assumptions can adequately be represented in it. To exemplify this, we show that assumption-based argumentation.

Keywords. nonmonotonic reasoning, structured argumentation, sequent-based argumentation, assumption-based argumentation, defeasible assumptions

1. Introduction

Assumptions are an important concept in defeasible reasoning. Often, in both expert and everyday reasoning, the information provided is not complete or it is inconsistent. By assuming additional information or considering consistent subsets of information, a conclusion can be reached in such cases. A well-known formal method for modeling defeasible reasoning is abstract argumentation theory, introduced by Dung [6]. In logical argumentation, the arguments have a specific structure on which the attacks depend [4,12]. One such logical argumentation framework is *sequent-based argumentation* [2], in which arguments are represented by sequents, as introduced by Gentzen [8] and well-known in proof theory. Attacks between arguments are formulated by *sequent elimination rules*, which are special inference rules. The resulting framework is generic and modular, in that any logic with a corresponding sound and complete sequent calculus can be taken as the deductive base (the so-called *core logic*).

In this paper we extend sequent-based argumentation. To each sequent a component for assumptions is added. This way, a distinction can be made between strict and defeasible premises, to reach further conclusions. As an instance of the obtained framework, assumption-based argumentation (ABA) [5,7] is studied and the relation to reasoning with maximally consistent subsets is investigated. ABA is a structural argumentation framework which is also abstract, in that there are only limited assumptions on the

¹The author is supported by by the Alexander von Humboldt Foundation and the German Ministry for Education and Research, and the Israel Science Foundation (grant 817/15)

underlying deductive system. It was introduced to determine a set of assumptions that can be accepted as a conclusion from the given information.

Arguments in ABA are constructed by applying modus ponens to simple clauses of an inferential database. Only recently logic-based instantiations of ABA have been studied, mostly with classical logic as the core logic. Sequent-based argumentation, and the here introduced assumptive generalization, are more general and modular, in that these are based on a (Tarskian) core logic and the arguments are constructed via the inference rules of the corresponding sequent calculus. Logics that can be equipped with defeasible assumptions by means of assumptive sequent-based argumentation include, in addition to classical logic, intuitionistic logic, many of the well-known modal logics and several relevance logics. Hence, the results of this paper generalize to many deductive core systems.

2. Sequent-based argumentation

Throughout the paper only propositional languages are considered, denoted by \mathscr{L} . Atomic formulas are denoted by p,q, formulas are denoted by ϕ, ψ , sets of formulas are denoted by S, T, and finite sets of formulas are denoted by Γ, Δ , later on we will denote sets of assumptions by A and finite sets of assumptions by A, all of which can be primed or indexed.

Definition 1. A logic for a language \mathcal{L} is a pair $L = \langle \mathcal{L}, \vdash \rangle$, where \vdash is a consequence relation for \mathcal{L} , having the following properties: reflexivity: if $\phi \in S$, then $S \vdash \phi$; and transitivity: if $S \vdash \phi$ and $S', \phi \vdash \psi$, then $S, S' \vdash \psi$.

As usual in logical argumentation (see, e.g., [4,12]), arguments have a specific structure based on the underlying formal language, the *core logic*. In the current setting arguments are represented by the well-known proof theoretical notion of a *sequent*.

Definition 2. Let $L = \langle \mathscr{L}, \vdash \rangle$ be a logic and S a set of \mathscr{L} -formulas.

- An \mathscr{L} -sequent (sequent for short) is an expression of the form $\Gamma \Rightarrow \Delta$, where Γ and Δ are finite sets of formulas in \mathscr{L} and \Rightarrow is a symbol that does not appear in \mathscr{L} .
- An L-argument (argument for short) is an \mathscr{L} -sequent $\Gamma \Rightarrow \psi$,² where $\Gamma \vdash \psi$. Γ is called the support set of the argument and ψ its conclusion.
- An L-argument based on S is an L-argument $\Gamma \Rightarrow \psi$, where $\Gamma \subseteq S$. We denote by $Arg_{1}(S)$ the set of all the L-arguments based on S.

Given an argument $a = \Gamma \Rightarrow \psi$, we denote $\text{Supp}(a) = \Gamma$ and $\text{Conc}(a) = \psi$.

The formal systems used for the construction of sequents (and so of arguments) for a logic $L = \langle \mathcal{L}, \vdash \rangle$, are *sequent calculi* [8], denoted here by C. In what follows we shall assume that C is sound and complete for $L = \langle \mathcal{L}, \vdash \rangle$, i.e., $\Gamma \Rightarrow \psi$ is provable in C iff $\Gamma \vdash \psi$. One of the advantages of sequent-based argumentation is that any logic with a corresponding sound and complete sequent calculus can be used as the core logic.³

Argumentation systems contain also attacks between arguments. In our case, attacks are represented by *sequent elimination rules*. Such a rule consists of an attacking argu-

²Set signs in arguments are omitted.

³See [2] for further advantages of this approach.

ment (the first condition of the rule), an attacked argument (the last condition of the rule), conditions for the attack (the conditions in between) and a conclusion (the eliminated attacked sequent). The outcome of an application of such a rule is that the attacked sequent is 'eliminated'. The elimination of a sequent $a = \Gamma \Rightarrow \Delta$ is denoted by \overline{a} or $\Gamma \neq \Delta$.

Definition 3. A sequent elimination rule (or attack rule) is a rule \mathcal{R} of the form:

$$\frac{\Gamma_1 \Rightarrow \Delta_1 \quad \dots \quad \Gamma_n \Rightarrow \Delta_n}{\Gamma_n \Rightarrow \Delta_n} \quad \mathscr{R} \tag{1}$$

It is said that $\Gamma_1 \Rightarrow \Delta_1 \mathscr{R}$ -attacks $\Gamma_n \Rightarrow \Delta_n$.

Example 1. Suppose \mathscr{L} contains $a \vdash$ -negation \neg (where $p \nvDash \neg p$ and $\neg p \nvDash p$ for every atom p) and $a \vdash$ -conjunction \land (where $S \vdash \phi \land \psi$ iff $S \vdash \phi$ and $S \vdash \psi$). We refer to [2] for a definition of a variety of attack rules. Assuming that $\Gamma_2 \neq \emptyset$, two such rules are:

$$\begin{array}{l} \begin{array}{ccc} & \displaystyle \frac{\Gamma_1 \Rightarrow \psi_1 & \Rightarrow \psi_1 \leftrightarrow \neg \wedge \Gamma_2 & \Gamma_2, \Gamma_2' \Rightarrow \psi_2 \\ \\ & \displaystyle \textit{Undercut} (\textit{Ucut}): & & \displaystyle \frac{\Gamma_1 \Rightarrow \psi_1 & \Rightarrow \psi_1 \leftrightarrow \neg \gamma & \gamma, \Gamma_2' \Rightarrow \psi_2 \\ \\ & \displaystyle \textit{Direct Ucut} (\textit{DUcut}): & & \displaystyle \frac{\Gamma_1 \Rightarrow \psi_1 & \Rightarrow \psi_1 \leftrightarrow \neg \gamma & \gamma, \Gamma_2' \Rightarrow \psi_2 \\ \end{array} \end{array}$$

A sequent-based framework is now defined as follows:

Definition 4. A sequent-based argumentation framework for a set of formulas S based on the logic $L = \langle \mathscr{L}, \vdash \rangle$ and a set AR of sequent elimination rules, is a pair AF_{L,AR}(S) = $\langle Arg_L(S), AT \rangle$, where AT $\subseteq Arg_L(S) \times Arg_L(S)$ and $(a_1, a_2) \in AT$ iff there is an $\mathscr{R} \in AR$ such that $a_1 \mathscr{R}$ -attacks a_2 .

In what follows, to simplify notation, we will omit the subscripts L and/or AR when these are clear from the context or arbitrary.

Given a (sequent-based) framework, Dung-style semantics [6] can be applied to it:

Definition 5. Let $AF_L(S) = \langle Arg_L(S), AT \rangle$ be an argumentation framework and $\mathscr{S} \subseteq Arg_L(S)$ a set of arguments. \mathscr{S} attacks an argument a if there is an $a' \in \mathscr{S}$ such that $(a', a) \in AT$; \mathscr{S} defends an argument a if \mathscr{S} attacks every attacker of a; \mathscr{S} is conflict-free if there are no arguments $a_1, a_2 \in \mathscr{S}$ such that $(a_1, a_2) \in AT$; \mathscr{S} is admissible if it is conflict-free and it defends all of its elements. An admissible set that contains all the arguments that it defends is a complete extension of $AF_L(S)$.

Some particular complete extensions of $AF_L(S)$ are: a preferred extension of $AF_L(S)$ is a maximal (with respect to \subseteq) complete extension of $Arg_L(S)$; a stable extension of $Arg_L(S)$ is a complete extension that attacks every argument not in it; the grounded extension of $AF_L(S)$ is the minimal (with respect to \subseteq) complete extension of $Arg_L(S)$.

We denote by $Ext_{sem}(AF_L(S))$ the set of all the extensions of $AF_L(S)$ under the semantics sem $\in \{cmp, grd, prf, stb\}$.

Definition 6. Given a sequent-based argumentation framework AF_L(S), the semantics as defined in Definition 5 induces corresponding (nonmonotonic) entailment relations:⁴

⁴Since the grounded extension is unique, $\succ_{L,grd}^{\cap}$, $\succ_{L,grd}^{\cup}$ and $\succ_{L,grd}^{\cap}$, are the same, and will be denoted by $\succ_{L,grd}$.

132 A.M. Borg / Equipping Sequent-Based Argumentation with Defeasible Assumptions

- $S \models_{L,sem}^{\cap} \phi$ ($S \models_{L,sem}^{\cup} \phi$) iff for every (some) extension $\mathscr{E} \in Ext_{sem}(AF_L(S))$, there is an argument $\Gamma \Rightarrow \phi \in \mathscr{E}$ for $\Gamma \subseteq S$,
- $S \stackrel{\text{\tiny Constraint}}{\vdash} for every & \in Ext_{sem}(AF_L(S)) \text{ there is an } a \in & and Conc(a) = \phi.$

Example 2. Let $AF_{CL,{Ucut}}(S)$ be an argumentation framework, with classical logic as its core logic, Ucut the only attack rule and the set $S = \{p, p \supset q, \neg q\}$. Some of the arguments are: $a = p, p \supset q \Rightarrow q$, $b = \neg q \Rightarrow \neg q$, $c = p \Rightarrow p$, $d = \Rightarrow q \lor \neg q$ and $e = p \supset q, \neg q \Rightarrow \neg p$.

The argument $d \Rightarrow q \lor \neg q$ *is not attacked and hence* $S \models_{\mathsf{CL},\mathsf{grd}} q \lor \neg q$ *. For the other formulas in* $\phi \in S$ *we have that* $S \models_{\mathsf{CL},\mathsf{sem}}^{\cap} \phi$ *and* $S \models_{\mathsf{CL},\mathsf{sem}}^{\cup} \phi$ *, for* $\mathsf{sem} \in \{\mathsf{cmp}, \mathsf{prf}, \mathsf{stb}\}$ *.*

3. Assumptive sequent-based argumentation

There are many ways in which assumptions are handled in the literature, e.g., default logic [13], assumption-based argumentation [5], default assumptions [11] and adaptive logics [3]. In this section we extend the sequent-based argumentation framework from the previous section, to incorporate assumptions. This generalization is formulated in a general way: independent of the core logic, the nature of the assumptions, or the way that the system allows for deriving conclusions based on these assumptions.

In what follows we assume that, instead of one set of formulas, the input contains two sets of \mathscr{L} -formulas: A, a set of, possibly conflicting, assumptions or defeasible premises, the form of which depends on the application and the logic; and S, a consistent set, the formulas which can intuitively be understood as facts or strict premises. We assume again that a logic L = $\langle \mathscr{L}, \vdash \rangle$ has a corresponding sequent calculus C. This calculus will, depending on the application, be extended to C', in order to allow for assumptions.

Definition 7. Let $L = \langle \mathcal{L}, \vdash \rangle$ be a logic, with a corresponding sound and complete sequent calculus C and sequent calculus extension C', let S be a consistent set of \mathcal{L} -formulas and A a set of assumptions.

- An assumptive \mathscr{L} -sequent ((assumptive) sequent for short) is a sequent $A \subseteq \Gamma \Rightarrow \Delta$.
- An assumptive L-argument ((assumptive) argument for short) is an assumptive sequent $A \ \Gamma \Rightarrow \Delta$, that is provable in C'.⁵
- An assumptive L-argument based on S and A is an assumptive argument $A \ \Gamma \Rightarrow \Delta$ such that $\Gamma \subseteq S$ and $A \subseteq A$. As before, we denote by $Arg_{L}(S, A)$ the set of all the assumptive L-arguments based on S and A.

Notation 1. Let $a = A \ rightharpoondown \cap \Delta B$ be an assumptive argument. Then Ass(a) = A denotes the assumptions of the argument a. Furthermore, for \mathscr{S} a set of arguments, $Concs(\mathscr{S}) = \{Conc(a) \mid a \in \mathscr{S}\}$, $Supps(\mathscr{S}) = \bigcup \{Supp(a) \mid a \in \mathscr{S}\}$ and $Ass(\mathscr{S}) = \bigcup \{Ass(a) \mid a \in \mathscr{S}\}$. In case that $A = \emptyset$, a will sometimes be written as $\Gamma \Rightarrow \Delta$.

An important rule in sequent calculi is [Cut]. In assumptive notation there are two:

$$\frac{A_1 \land \Gamma_1 \Rightarrow \Delta_1, \phi \quad A_2 \land \Gamma_2, \phi \Rightarrow \Delta_2}{A_1, A_2 \land \Gamma_1, \Gamma_2 \Rightarrow \Delta_1, \Delta_2} \quad [Cut] \qquad \frac{A_1 \land \Gamma_1 \Rightarrow \Delta_1, \phi \quad A_2, \phi \land \Gamma_2 \Rightarrow \Delta_2}{A_1, A_2 \land \Gamma_1, \Gamma_2 \Rightarrow \Delta_1, \Delta_2} \quad [Cut]$$

⁵Often, C' will be the result of adding rules, to divide the support set of each argument into the set of defeasible premises on the left-hand-side and the set of strict premises on the right-hand-side of ζ , to C.

Let $a = A \ \Gamma \Rightarrow \Delta$ be an argument. We continue using \overline{a} and $A \ \Gamma \Rightarrow \Delta$ to denote that *a* has been eliminated. Arguments are attacked in the set of assumptions, we give an example in the next section.

Definition 8. An assumptive sequent-based argumentation framework for a set of formulas S, set of assumptions A, based on a logic $L = \langle \mathcal{L}, \vdash \rangle$ and a set AR of sequent elimination rules, is a pair AF_{L,AR}(S,A) = $\langle Arg_L(S,A), AT \rangle$, where AT $\subseteq Arg_L(S,A) \times$ $Arg_L(S,A)$ and $(a_1,a_2) \in$ AT iff there is an $\mathcal{R} \in$ AR such that $a_1 \mathcal{R}$ -attacks a_2 .

Like before, when these are clear from the context or arbitrary, we will omit the subscripts L, AR and/or A. The semantics, as defined in Definition 5 can be applied to assumptive sequent-based argumentation frameworks. The corresponding entailment relations (from Definition 6) are denoted by $\sim_{A.sem}^{\pi}$ for $\pi \in \{\cap, \cup, \mathbb{n}\}$.

4. Incorporating ABA

Assumption-based argumentation (ABA) was introduced in [5]. It takes as input a formal deductive system, a set of assumptions and a contrariness mapping for each assumption. There are only few requirements placed on each of these, keeping the framework abstract on the one hand, while the arguments have a formal structure and the attacks are based on the latter. First some of the most important definitions for the ABA-framework, from [5]:

Definition 9. A deductive system is a pair $\langle \mathcal{L}, \mathcal{R} \rangle$, where \mathcal{L} is a formal language and \mathcal{R} is a set of rules of the form $\phi_1, \ldots, \phi_n \to \phi$, for $\phi_1, \ldots, \phi_n, \phi \in \mathcal{L}$ and $n \ge 0$.

Definition 10. A deduction from a theory Γ is a sequence ψ_1, \ldots, ψ_m , where m > 0, such that for all $i = 1, \ldots, m$, $\psi_i \in \Gamma$, or there is a rule $\phi_1, \ldots, \phi_n \to \psi_i \in \mathscr{R}$ with $\phi_1, \ldots, \phi_n \in \{\psi_1, \ldots, \psi_{i-1}\}$. We denote by $\Gamma \vdash^{\mathscr{R}} \psi$ a deduction for ψ from Γ using rules in \mathscr{R} . It is assumed that Γ is \subseteq -minimal.

From this ABA argumentation frameworks can be defined:

Definition 11. An ABA-framework is a tuple $AF_{(\mathscr{L},\mathscr{R})}(S,A) = \langle \mathscr{L},\mathscr{R}, S, A, \overline{\cdot} \rangle$ where:

- $\langle \mathcal{L}, \mathcal{R} \rangle$ is a deductive system;
- $S \subseteq \mathscr{L}$ a set of formulas, that satisfies non-triviality ($S \nvDash^{\mathscr{R}} \phi$ for all ϕ that do not share an atom with any of the formulas in S);⁶
- $A \subseteq \mathcal{L}$ a non-empty set of assumptions for which $S \cap A = \emptyset$; and
- $\overline{\cdot}$ a mapping from A into \mathcal{L} , where $\overline{\phi}$ is said to be the contrary of ϕ .

A simple way of defining contrariness in the context of classical logic is by $\overline{\phi} = \neg \phi$.

Definition 12. Given an ABA-framework $AF_{\langle \mathscr{L}, \mathscr{R} \rangle}(S, A)$, a set $A \subseteq A$ is: consistent iff there is no $\phi \in A$ such that $A', \Gamma \vdash^{\mathscr{R}} \overline{\phi}$ for some $A' \subseteq A$ and some $\Gamma \subseteq S$. A is maximally consistent iff there is no A' such that $A \subset A' \subseteq A$ and A' is consistent, then $A \in MCS(S, A)$. Thus MCS(S, A) contains all subsets of A that are maximally consistent with S. The closure of $T \subseteq \mathscr{L}$ is defined as $CN(T) = \{\phi \mid \Gamma \vdash^{\mathscr{R}} \phi \text{ for } \Gamma \subseteq T\}$.

⁶In the remainder, if a set of formulas S satisfies non-triviality, it is said that S is non-trivializing.

ABA-arguments are defined in terms of deductions and an attack is on the assumptions of the attacked argument. As in [7], arguments are not required to be consistent.

Definition 13. Let $AF_{\langle \mathscr{L}, \mathscr{R} \rangle}(S, A) = \langle \mathscr{L}, \mathscr{R}, S, A, \overline{\cdot} \rangle$. An ABA-argument for $\phi \in \mathscr{L}$ is a deduction $A \cup \Gamma \vdash^{\mathscr{R}} \phi$, where $A \subseteq A$ and $\Gamma \subseteq S$. The set $Arg^{ABA}_{\langle \mathscr{L}, \mathscr{R} \rangle}(S, A)$ denotes the set of all ABA-arguments for S and A.

Definition 14. Let $AF_{\langle \mathcal{L}, \mathcal{R} \rangle}(S, A) = \langle \mathcal{L}, \mathcal{R}, S, A, \overline{\cdot} \rangle$. An argument $A \cup S \vdash^{\mathcal{R}} \phi$ attacks an argument $A' \cup S \vdash^{\mathcal{R}} \phi'$ iff $\phi = \overline{\psi}$ for some $\psi \in A'$.

Semantics are defined as usual, see Definition 5. From this we can define the corresponding entailment relation:

Definition 15. Let $AF_{\langle \mathscr{L}, \mathscr{R} \rangle}(S, A) = \langle \mathscr{L}, \mathscr{R}, S, A, \overline{\cdot} \rangle$ and sem $\in \{ grd, cmp, prf, stb \}.$

- $A \cup S \models_{ABA,sem}^{\cup} \phi$ $(A \cup S \models_{ABA,sem}^{\cap} \phi)$ if and only if for some (every) extension $\mathscr{E} \in Ext_{sem}(AF_{\langle \mathscr{L},\mathscr{R} \rangle}(S,A))$ there is an argument $A \cup \Gamma \models^{\mathscr{R}} \phi \in \mathscr{E}$ for $A \subseteq A$ and $\Gamma \subseteq S$.
- $A \cup S \sim_{ABA,sem}^{m} \phi$ if and only if for every $\mathscr{E} \in Ext_{sem}(AF_{\langle \mathscr{L},\mathscr{R} \rangle}(S,A))$ there is an $a \in \mathscr{E}$ and $Conc(a) = \phi$.

Based on the above notions from assumption-based argumentation, a corresponding sequent-based ABA-framework can be defined:

Definition 16. Let $AF_{\langle \mathscr{L}, \mathscr{R} \rangle}(S, A) = \langle \mathscr{L}, \mathscr{R}, S, A, \overline{\cdot} \rangle$ be an ABA-framework as defined above. The corresponding sequent-based ABA-framework is then $AF_{\langle \mathscr{L}, \mathscr{R} \Rightarrow \rangle}^{ABA_{\Rightarrow}}(S, A) = \langle Arg_{\langle \mathscr{L}, \mathscr{R} \Rightarrow \rangle}^{ABA_{\Rightarrow}}(S, A), AT \rangle$, where:

- $\mathscr{R}_{\Rightarrow}$ is defined as:
 - * if $\langle \mathscr{L}, \mathscr{R} \rangle$ is a logic with corresponding sequent calculus C, $\mathscr{R}_{\Rightarrow} = C \cup \{AS_{ABA}\}$ such that:

$$\frac{A}{A} \underbrace{ \Gamma, \phi \Rightarrow \psi}_{A, \phi} AS_{ABA} \qquad \frac{A, \phi \underbrace{ \Gamma \Rightarrow \psi}_{A \underbrace{ \Gamma, \phi \Rightarrow \psi}} AS_{ABA} \qquad where \ \phi \in A$$

- * otherwise $\mathscr{R}_{\Rightarrow} = \{\mu(r) \mid r \in \mathscr{R}\} \cup \{AS_{\mathsf{ABA}}, [Cut], [\mathsf{id}]\}\$ where, for each $r = \phi_1, \ldots, \phi_n \rightarrow \phi \in \mathscr{R}, \ \mu(r) = \phi_1, \ldots, \phi_n \Rightarrow \phi \ and: \frac{1}{\phi \Rightarrow \phi} [id]$
- $a = A \ (\Gamma \Rightarrow \phi \in Arg_{\langle \mathscr{L}, \mathscr{R} \Rightarrow \rangle}^{\mathsf{ABA} \Rightarrow}(\mathsf{S}, \mathsf{A}) \text{ for } \mathsf{A} \subseteq \mathsf{A}, \Gamma \subseteq \mathsf{S} \text{ iff there is a derivation of a using rules in } \mathscr{R}_{\Rightarrow}.$
- $(a_1, a_2) \in AT$ iff $a_1 \mathscr{R}$ -attacks a_2 as defined in Definition 4, for $AR = \{AT_{ABA}\}$ and:

$$\frac{A_1 \left\{ \Gamma_1 \Rightarrow \overline{\phi} \quad A_2, \phi \right\} \Gamma_2 \Rightarrow \psi}{A_2, \overline{\phi} \left\{ \Gamma_2 \Rightarrow \psi \right\}} AT_{\mathsf{ABA}}$$
(2)

Remark 1. $A \cup \Gamma \Rightarrow \phi$ is derivable iff $A \ \Gamma \Rightarrow \phi$ is derivable.

Let $\langle \mathscr{L}, \mathscr{R} \rangle$ be a deductive system, $S \subseteq \mathscr{L}$ a non-trivializing set of formulas and $A \subseteq \mathscr{L}$ a set of assumptions, such that $\Gamma \subseteq S$ and $A \subseteq A$ are finite. Moreover, let $\mathsf{AF}^{\mathsf{ABA}_{\Rightarrow}}_{\langle \mathscr{L}, \mathscr{R}_{\Rightarrow} \rangle}(S, A) = \left\langle \mathsf{Arg}^{\mathsf{ABA}_{\Rightarrow}}_{\langle \mathscr{L}, \mathscr{R}_{\Rightarrow} \rangle}(S, A), \mathsf{AT} \right\rangle$ be a sequent-based ABA-framework and $\mathsf{AF}_{\langle \mathscr{L}, \mathscr{R} \rangle}(S, A) = \langle \mathscr{L}, \mathscr{R}, \mathsf{S}, \mathsf{A}, \overline{\cdot} \rangle$. Then:

Proposition 1. $A \cup S \mathrel{\sim}^{\pi}_{ABA.sem} \phi$ *iff* $A \cup S \mathrel{\sim}^{\pi}_{A.sem} \phi$ *for* sem $\in \{\text{grd}, \text{cmp}, \text{prf}, \text{stb}\}$ and $\pi \in \{\cup, \cap, \mathbb{m}\}$.

In the next example we show how classical logic, with corresponding sequent calculus LK can be taken as underlying deductive system.

Example 3. Let $\mathsf{CL} = \langle \mathscr{L}, \vdash \rangle$, where $\overline{\phi} = \neg \phi$ and $\mathscr{R}_{\Rightarrow} = \mathsf{LK} \cup \{AS_{\mathsf{ABA}}\}$. According to Definition 7 A $\Gamma \Rightarrow \phi \in Arg_{L}(S,A)$ iff $\Gamma \cup A \Rightarrow \phi$ is derivable in LK, for some finite $A \subseteq A$ and $\Gamma \subseteq S$. It follows immediately that $A \cup \Gamma \Rightarrow \phi$ is derivable in $\mathscr{R}_{\Rightarrow}$ iff it is derivable in LK.

Now consider $AF^{ABA_{\Rightarrow}}_{\langle \mathscr{L}, \mathscr{R}_{\Rightarrow} \rangle}(S, A) = \left\langle Arg^{ABA_{\Rightarrow}}_{\langle \mathscr{L}, \mathscr{R}_{\Rightarrow} \rangle}(S, A), AT \right\rangle$ with $S = \{s\}$ and $A = \{p, q, \neg p \lor \neg q, \neg p \lor r, \neg q \lor r\}$. Some of the arguments in $Arg^{ABA_{\Rightarrow}}_{\langle \mathscr{L}, \mathscr{R}_{\Rightarrow} \rangle}(S, A)$ are: $a = s \Rightarrow s$, $b = p, \neg p \lor \neg q \ \Rightarrow \neg q, \ c = q, \neg p \lor \neg q \ \Rightarrow \neg p \text{ and } d = p, q, \neg p \lor r, \neg q \lor r \ \Rightarrow r.$ Note that a cannot be attacked, since $Ass(a) = \emptyset$. We thus have $A \cup S \mid \sim_{A,sem}^{\pi} s$.

for sem $\in \{ \text{grd}, \text{cmp}, \text{prf}, \text{stb} \}$ and $\pi \in \{ \cup, \cap, \cap \}$. However, the argument d is attacked by both b and c. Moreover b attacks c and c attacks b. It can be shown that, for $\phi \in \{p, q, \neg p \lor \neg q\}, A \cup S \not\models_{A,sem}^{\pi} \phi \text{ for sem} \in \{grd, cmp, prf, stb\} \text{ and } \pi \in \{\cap, \cap\} \text{ but}$ also $A \cup S \vdash_{A \text{ sem}'}^{\cup} \phi$ for $\text{sem}' \in \{\text{cmp}, \text{prf}, \text{stb}\}$.

Remark 2. The examples and explanations in this paper are based on classical logic. However, Proposition 1 holds for a large range of deductive systems. The only requirements are that the consequence or deducability relation is reflexive and transitive, and that a contrariness function can be defined. Thus, not only classical logic can be taken as the underlying deductive system, but for example non-classical logics and nonmonotonic deductive systems as well. The construction of an assumptive sequent-based argumentation framework for such a deductive system follows the steps in Definition 16.

Reasoning with maximally consistent subsets is a well-known way to maintain consistency when provided with inconsistent information. The relations between ABA and reasoning with maximally consistent subsets and between sequent-based argumentation and maximally consistent subsets have been studied [1,9]. In our setting we have:

Definition 17. Let $L = \langle \mathscr{L}, \vdash \rangle$, S a consistent set of formulas and A a set of assumptions. *Recall that* MCS(S,A) *denotes all* $A \subset A$ *which are maximally complete w.r.t.* S. Define:

- $S \models_{mcs}^{\cap,A} \phi \ iff \phi \in CN(\bigcap MCS(S,A) \cup S);$ $S \models_{mcs}^{\cup,A} \phi \ iff \phi \in \bigcup_{T \in MCS(S,A)} CN(S \cup T);$ $S \models_{mcs}^{\oplus,A} \phi \ iff \phi \in \bigcap_{T \in MCS(S,A)} CN(S \cup T).$

Proposition 2. Let $AF_{\langle \mathscr{L}, \mathscr{R}_{\Rightarrow} \rangle}^{ABA_{\Rightarrow}}(S,A)$, for a deductive system $\langle \mathscr{L}, \mathscr{R} \rangle$, $S \subseteq \mathscr{L}$ a nontrivializing set of formulas and A a set of assumptions. When $\vdash^{\mathscr{R}}$ is contrapositive for assumptions (for $\phi, \psi \in A, A \cup \Gamma \vdash^{\mathscr{R}} \overline{\psi}$ iff $(A \setminus \{\phi\}) \cup \{\psi\} \cup \Gamma \vdash^{\mathscr{R}} \overline{\phi}$), then: $A \cup S \vdash^{\pi}_{A, prf} \phi$ *iff* $A \cup S \mid_{A,stb}^{\pi} \phi$ *iff* $S \mid_{mcs}^{\pi,A} \phi$, *for* $\pi \in \{\cap, \cup, \mathbb{m}\}$.

⁷Due to space restrictions, some details and proofs are omitted. A version of the paper with proofs is available on https://arxiv.org/pdf/1804.08674.pdf.

5. Conclusion

In order to allow for reasoning with assumptions, sequent-based argumentation was extended by adding a component for assumptions to each argument, resulting in assumptive sequent-based argumentation. As in sequent-based argumentation, any logic, with a corresponding sound and complete sequent calculus, can be taken as the core logic. Due to its generic and modular setting, assumptive sequent-based argumentation is more general than other approaches to reasoning with assumptions, such as assumption-based argumentation (where arguments are constructed by applying modus ponens to an inferential database and for which it was shown that it can be embedded in the here introduced framework), default assumptions [11] (defined in terms of classical logic) and adaptive logics [3,14] (based on a supra-classical Tarskian logic). Moreover, the proofs of the results in this paper do not rely on the concrete nature of the underlying core logic. It therefore paves the way to equip many well-known logics (e.g., intuitionist logic and many modal logics) with defeasible assumptions.

Recently, the relation between different nonmonotonic reasoning systems have been studied, see for an overview [10]. There translations from $ASPIC^+$ [12] and adaptive logics into ABA are provided as well. Though it remains an open question to see how sequent-based argumentation fits within this group of nonmonotonic reasoning systems, these translations suggest that, in the flat case (i.e., without priorities), assumptive sequent-based argumentation is expressive enough to capture $ASPIC^+$ and adaptive logics.

References

- Ofer Arieli, AnneMarie Borg, and Christian Straßer. Argumentative approaches to reasoning with consistent subsets of premises. In *Proc. IEA/AIE'17*, pages 455–465. Springer, 2017.
- [2] Ofer Arieli and Christian Straßer. Sequent-based logical argumentation. *Argument & Computation*, 6(1):73–99, 2015.
- [3] Diderik Batens. A universal logic approach to adaptive logics. *Logica Universalis*, 1(1):221–242, 2007.
- Philippe Besnard and Anthony Hunter. A logic-based theory of deductive arguments. Artificial Intelligence, 128(1-2):203 – 235, 2001.
- [5] Andrei Bondarenko, Phan Minh Dung, Robert A. Kowalski, and Francesca Toni. An abstract, argumentation-theoretic approach to default reasoning. *Artificial Intelligence*, 93(1):63 – 101, 1997.
- [6] Phan Minh Dung. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial Intelligence*, 77(2):321–357, 1995.
- [7] Phan Minh Dung, Robert A. Kowalski, and Francesca Toni. Assumption-Based Argumentation, pages 199–218. Springer, 2009.
- [8] Gerhard Gentzen. Untersuchungen über das logische Schließen I, II. Mathematische Zeitschrift, 39:176– 210, 405–431, 1934.
- [9] Jesse Heyninck and Ofer Arieli. On the semantics of simple contrapositive assumption-based argumentation frameworks. In *Proc. COMMA'18*, 2018. This volume.
- [10] Jesse Heyninck and Christian Straßer. Relations between assumption-based approaches in nonmonotonic logic and formal argumentation. In Proc. NMR'16, 2016.
- [11] David Makinson. Bridges between classical and nonmonotonic logic. *Logic Journal of the IGPL*, 11(1):69, 2003.
- [12] Henry Prakken. An abstract framework for argumentation with structured arguments. Argument & Computation, 1(2):93–124, 2010.
- [13] Raymond Reiter. A logic for default reasoning. Artificial Intelligence, 13(1-2):81–132, 1980.
- [14] Christian Straßer. Adaptive Logics for Defeasible Reasoning. Applications in Argumentation, Normative Reasoning and Default Reasoning, volume 38 of Trends in Logic. Springer, 2014.