Transdisciplinary Engineering: Crossing Boundaries
M. Borsato et al. (Eds.)

doi:10.3233/978-1-61499-703-0-461

Active Vibration Control Applying H-Infinity Norm in a Composite Laminated Beam

Camila Albertin Xavier da SILVA[1], Daniel Almeida COLOMBO[2], Renan Franco CORRÊA[3], Fabian Andres LARA-MOLINA[4] and Edson Hideki KOROISHI[5]
Department of Mechanical Engineering, Federal Technological University of Parana, Brazil

Abstract. The use of active vibration control techniques applying piezoelectric actuators can show satisfactory results. This paper proposes an active vibration control technique based on H-infinity Norm, which is applied to a piezoelectric actuator bonded to a composite structure forming a so-called smart composite structure. The structural model is stablished for a rectangular shape beam by Serendipity type finite element based on first-order shear deformation theory with eight nodes, five degrees of freedom (DOF) per node and eight electrical DOF per piezoelectric. Furthermore, a mixed theory that uses a single equivalent layer for the discretization of the mechanical displacement field and a layerwise representation of the electrical field is adopted. Temperature effects are neglected. Simulation results show the effectiveness of the proposed control methodology for composite structures.

Keywords. Active Vibration Control, H-Infinity, Composite Materials.

Introduction

In the last few years, there is a great interest in researches about composite materials that possess innovative layouts, characterized by low weight, mechanical resistance and the possibility of optimization to specific work conditions. Such materials are made of fiber layers that can be oriented in specified directions, which enables them for singulars applications.

The addition of piezoelectric layers to composite materials offers a wide application area, such as precision positioning and active vibration control. The association of composite materials with piezoelectric layers is known as Smart Composite Structure. The representation of the behavior for the Smart Composite Structures is achieved through Mixed Theory, based in one equivalent layer to discretize the mechanical displacement field and another to represent the electric field [1]. In the current work, both electrical and mechanical fields are represented by Finite Element Model, by means of Hamilton's Variational Principle, which takes into account all the energy that is present in the structure.

The number of publications related to the development of new active vibration control techniques increases in recent years. Many of them show successful results.

[1]camilaalbertin@alunos.utfpr.edu.br, [2]danielcolombo@alunos.utfpr.edu.br,
[3] renancorrea@alunos.utfpr.edu.br, [4] fabianmolina@utfpr.edu.br, [5] edsonh@utfpr.edu.br

The present work models a Smart Composite structure, which is a fixed beam in one end, and uses modal active vibration control to optimize the performance of this dynamical system. The aim is to improve the dynamical response behavior, considering uncertainties in the model. For the control it was used the minimization by H-infinity Norm.

1. Modeling of a Smart Composite Structure

1.1. Mechanical Deflection Field according to the Mixed Theory

The mechanical behavior of a structure, according to the Mixed Theory is represented by the First Shear Displacement Theory (FSDT), given by Equation 1:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + (z\psi_x(x,y,t)) \\ u(x,y,z,t) &= v_0(x,y,t) + (z\psi_y(x,y,t)) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{1}$$

where u_0, v_0 and w_0 are the displacements in directions x, y and z, respectively. The plane x-y is the material plane and ψ_x ψ_y are the rotation regarding x and y, respectively.

The mechanical variables in Equation 1 are achieved by shape functions and nodal variables using Finite Element Method. The element considered in this formulation is known as Serendipity, which is a planar element with three nodes per side, resulting in eight nodes per element [2].

From the FSDT, the mechanical displacement as function of nodal coordinates is obtained:

$$\{U(\xi,\eta,z,t)\} = [A_u(z)][N_u(\xi,\eta)]\{u_e(t)\} \tag{2}$$

where $\{U(\xi,\eta,z,t)\} = \{u(\xi,\eta,z,t)\}\ \{v(\xi,\eta,z,t)\}\ \{w(\xi,\eta,z,t)\}^T$, $[A_u(z)]$ is the matrix of the thickness variables in z direction as functions of the five displacements $(u_0, v_0, w_0, \psi_x\ e\ \psi_y)$, $\{u_e(t)\}$ is the vector that contain all the nodal variables and $[N_u(\xi,\eta)]$ is the dynamical matrix of the mechanical function for the beam.

The elongation is expressed as function of shape functions and nodal variables, shown in Equation 3:

$$\{\varepsilon(\xi,\eta,z,t)\} = [B_u(\xi,\eta,z)]\{u_e(t)\} \tag{3}$$

where $[B_u(\xi,\eta,z)] = [D(z)][N_u(\xi,\eta)]$, $[D(z)]$ is the matrix that contains differential operators present in the strain-displacement relation, which is detailed in [3].

1.2. Linear Electrical Potential Distributed in the Structure Layers

After reaching the mechanical displacements, it is necessary to deal with the electrical field calculation. Through the Mixed Theory, the electrical potential is expressed by Equation 4, in which the direction along the thickness z is uncoupled from the reference plane x-y.

$$\varphi(x,y,z,t) = \sum_{j=1}^{nc+1} L_j(z)\varphi_j(x,y,t) \tag{4}$$

Where $L_j(z)$ is related to the function of equivalent layers, and $\varphi_j(x,y,t)$ is the interface function of the j-th interface of the composite constituted by *nc* layers.

The electrical potential described in the local coordinates for the k-th layer and e-th layer element is expressed by Equation 5, also originated from finite element formulation.

$$\varphi(\xi,\eta,z,t)_e^k = \left[N_\varphi(\xi,\eta,z)\right]\{\varphi_e(t)\} \tag{5}$$

Where the matrix of the electric shape functions $N_\varphi(\xi,\eta,z)$ is associated to the serendipity shape functions and Lagrangian interpolation functions $\{\varphi_e(t)\}$ that contain the nodal values for the electrical potential.

Applying the definition of electric field as negative gradient of the electrical potential, the expansion of the electric field for the k-th layer is expressed by Equation 6:

$$\left\{E(\xi,\eta,z,t)_e^k\right\} = -\nabla\left[\mathrm{N}_\varphi(\xi,\eta,z)\right]\{\varphi_e(t)\} \tag{6}$$

where $\vec{\nabla}\left[N_\varphi(\xi,\eta,Z)\right] = \left[B_\varphi(\xi,\eta,z)_k\right]$[3].

1.3. Elementary Matrix

When modeling a smart structure, the coupling between the composite structure and the piezoelectric element is achieved by means of Hamilton's Variational Principle, which incorporates all energy distribution present in the structure. According to [4], the elementary coupling matrices are shown from Equation 7 to Equation 10:

$$\left[M^e\right] = \int_{Ve} \rho\left[N_u\right]^T\left[A_u\right]^T\left[N_u\right]dV_e \tag{7}$$

$$\left[K_{uu}^e\right] = \sum_{k=1}^{nc}\int_{\xi=-1}^{+1}\int_{\eta}^{+1}\int_{z=z_k}^{z_k+1}\left(\left[B_u\right]^T\left[c\right]\left[B_u\right]\right)Jdzd\eta d\xi \tag{8}$$

$$\left[K_{u\varphi}^{e}\right] = \sum_{k=1}^{nc} \int_{\xi=-1}^{+1} \int_{\eta}^{+1} \int_{z=z_k}^{z_k+1} \left(\left[B_u\right]^T \left[e\right]^T \left[B_\varphi\right] \right) Jdzd\eta d\xi \tag{9}$$

$$\left[K_{\varphi\varphi}^{e}\right] = \sum_{k=1}^{nc} \int_{\xi=-1}^{+1} \int_{\eta}^{+1} \int_{z=z_k}^{z_k+1} - \left(\left[B_\varphi\right]^T \left[\chi\right]^T \left[B_\varphi\right] \right) Jdzd\eta d\xi \tag{10}$$

where ρ is the material density, $\left[M^e\right]$ is the mass elementary matrix, $\left[K_{uu}^e\right]$ is the stiffness elementary matrix and both $\left[K_{u\varphi}^e\right]$ and $\left[K_{\varphi u}^e\right]$ are elementary matrices of the electric-mechanical coupling. $\left[K_{\varphi\varphi}^e\right]$ is the dielectric elementary matrix. $\left[c\right]$, $\left[e\right]$ and $\left[\chi\right]$ are respectively the matrices of, elastic stiffness, piezoelectric stress and permittivity matrices, all formed by constant values. $\left[B_u\right]$ and $\left[B_\varphi\right]$ are input matrices and V_e is the elementary volume. J ís the Jacobian of transformation [2].

Based in the previous equations, the global model matrix, presented in Equation 11 is built through a standard procedure in which g indicates global magnitude.

$$\begin{bmatrix} \left[M_g\right] & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u}_g \\ \ddot{\phi}_g \end{Bmatrix} + \begin{bmatrix} \left[K_{uu}\right] & \left[K_{u\phi}\right] \\ \left[K_{\phi u}\right] & \left[K_{\phi\phi}\right] \end{bmatrix} \begin{Bmatrix} u_g \\ \phi_g \end{Bmatrix} = \begin{Bmatrix} \left\{F_g\right\} \\ \left\{Q_g\right\} \end{Bmatrix} \tag{11}$$

Where $\left\{F_g\right\}$ and $\left\{Q_g\right\}$ are respectively, the generalized force and the electric charge.

2. Control Approach

The modal active control is an advantageous technique, which is very effective when applied in flexible structures, because these structures require a reduced number of sensors and actuators.

In the present work, modal active control is employed to control the smart composite structure. In Figure 1, there is a scheme of the arrangement for the modal control, where δ corresponds to the displacement, X to the modal state, F_{exc} to the external force and u represents the control effort.

The estimator acts to determine the modal states demanded by the controller, however, in order to apply the Kalman Estimator, it is necessary that the model be represented in the state space form. Therefore, in the control scheme in Figure 1, the smart composite structure, expressed by Equation 11, is denoted in the space state form as in Equations 12 and 13, in order to enable them for numerical simulations.

The Kalman Estimator is used when it is desired to reduce the effects of noises in both input and output signals, to estimate the states and the outputs of the system [6]. Equation 12 represent the Kalman Estimator:

$$\{\dot{x}_r(t)\} = [A_r]\{x_r(t)\} + [B_r]\{u(t)\} + [L]\{\delta(t) - \bar{\delta}(t)\} \tag{12}$$

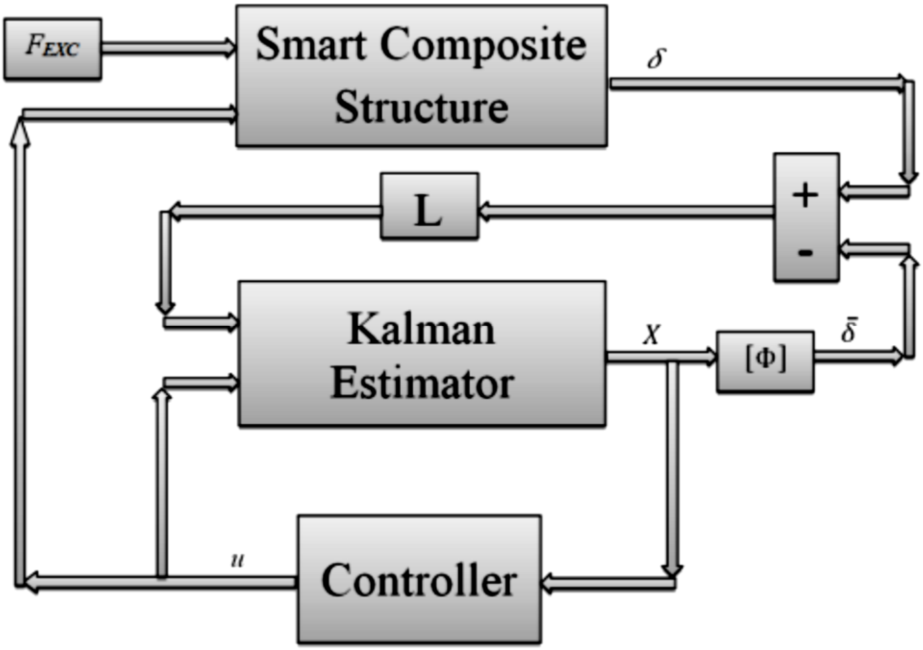

Figure 1. Active modal control based on modal state feedback control (adapted from [5]).

where $[A_r] = \begin{bmatrix} 0 & I \\ -M^{-1}K & 0 \end{bmatrix}$ and $[B_r]=[0\ \ M^{-1}]^T$, $[L]$ is the observer gain matrix, determined by lqe.m in Matlab® *software*, $\delta(t)$ is the displacement vector and $\bar{\delta}(t)$ is the estimated displacement vector.

The output equation is as follow:

$$\{\delta(t)\} = [C_r]\{x_r(t)\} \tag{13}$$

In Figure 1, it can be observed that in the modal state, the feedback require modal displacements and velocities to determine the control effort. In the current work, it was used the H-infinity Norm as controller.

3. Norm H-infinity

In [7] it is shown how to calculate de norm H-infinity by LMIs. The H-infinity Norm can be solved by using the following optimization convex problem:

$$\|G\|_\infty = \min \mu$$
$$\begin{bmatrix} A^T P + PA + C^T CPB \\ B^T P - \mu \end{bmatrix} < 0 \tag{14}$$
$$P > 0, \ \mu > 0$$

where μ is a scalar. Therefore the controller gain is achieved when μ is minimized.

4. Numerical Simulation

The smart laminated beam structure analyzed in this work is illustrated in Figure 2. The beam has a 306*mm* length, 25.5*mm* width and 1*mm* thickness, and it is composed by five layers of graphite/epoxy 0.2*mm* thickness each layer. The layers are oriented as [45º /0º /45º /0º /45º], where 0º refers to a direction parallel to *x* axis.

The piezoelectric ceramic actuator is attached to the upper side of the beam, 1*mm* far from the fixed end, with 1*mm* thickness and dimensions of 45.9 x 25.5*mm*.

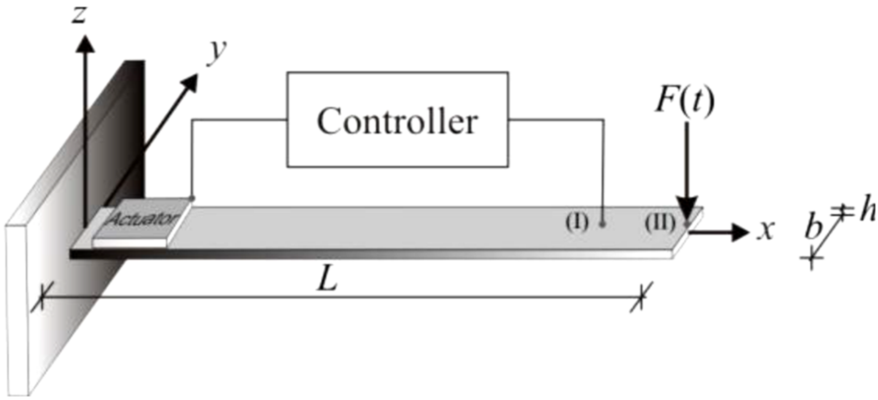

Figure 2. Laminated beam one fixed end with active vibration control (From [8]).

The constants of elastic stiffness of the beam, made of graphite/epoxy (AS4/3501) given in [*GPa*], are respectively:

$c_{11} = 173.6$; $c_{22} = c_{33} = 7.61$; $c_{12} = c_{13} = 2.48$; $c_{23} = 2.31$; $c_{44} = 1.38$; $c_{55} = c_{66} = 3.45$.

The piezoelectric stress constants:

$c_{11} = c_{22} = c_{33} = 102.23$; $c_{12} = c_{13} = c_{23} = 5.035$; $c_{11} = c_{22} = c_{33} = 102.23$; $c_{44} = c_{55} = c_{66} = 2.594$.

The piezoelectric electric constants in [*C/m²*] are: $e_{31} = 18.300$, $e_{32} = e_{33} = -9.013$.

The electric permittivity, given in [*F/m*], are: $\chi_{11} = \chi_{22} = \chi_{33} = 1800\varepsilon_0$. The density of the composite structure and of the piezoelectric material, given in [*kg/m³*], are respectively 1578 e 7700.

The excitation force of *1 N* was applied at point (II), as shown in Figure 2 and the responses in the time domain were measured at point (I). The piezoelectric actuator is connected to an active control system, and the amplitude of vibration is minimized. In order to have more similarity possible with an experimental condition, white noise were added in the displacement calculation.

Robustness analyzes were performed considering uncertainties of (±10%) in the dynamical matrix of the state space and also in the Kalman Estimator model. The balanced realization was used to reduce the model to the first two vibration modes of the structure, given that the system is both controllable and observable when considering these modes.

Regarding control, three cases were analyzed, the first was the deterministic, the second considered non-parametric uncertainties in the model of the smart composite structure, the uncertainties were inserted directly in the dynamical matrix of the system model, and in the third case, the uncertainties were input in the Kalman Estimator model.

5. Results

The first important results is that for the first two modes considered, the system was indeed observable and controllable. Then, the methodology discussed was applied to the system. For this condition the response of the system was analyzed in both time and frequency domain, presenting the voltage used in each case

The first case was the deterministic case in order to verify the efficiency of the proposed methodology. Figures 3, 4 e 5 show the control results.

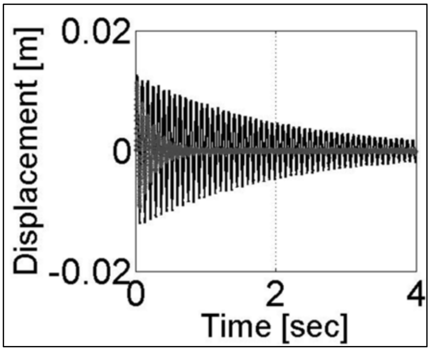

Figure 3. Impact Response control off (black line) and control on (gray line).

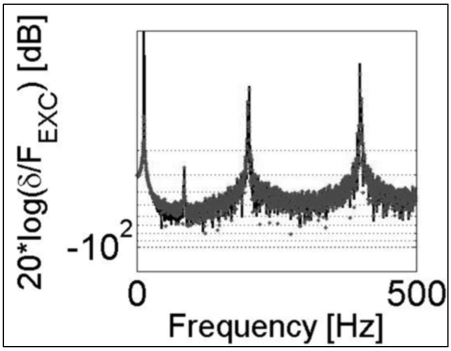

Figure 4. Frequency Response Function: control off (black line) and control on (gray line).

In Figure 3, it can be observed that the system was controlled, and the response was completely attenuated after *1.0 s*. Figure 4 shows that the control strategy was capable of expressively reduce the first mode of the system. Figure 5 shows the voltage due to the control effort of the piezoelectric actuator.

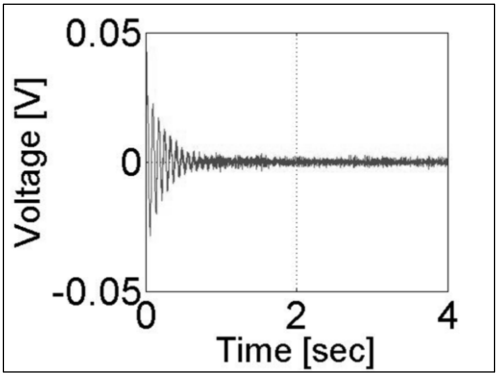

Figure 5. Control effort.

Figures 6, 7 and 8 show the results for the second case.

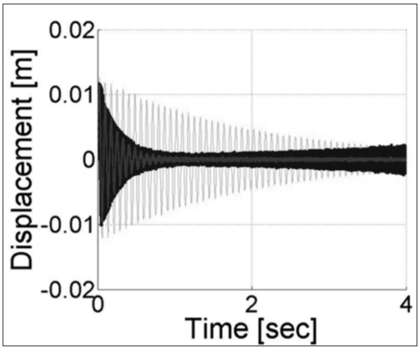

Figure 6. Impact Response: control on (gray line), envelope (black) and control off (light gray line).

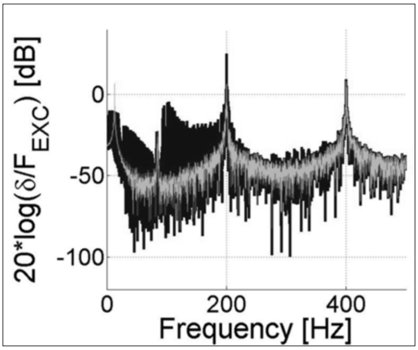

Figure 7. Frequency Response Function: deterministic (light gray line) and envelope (black).

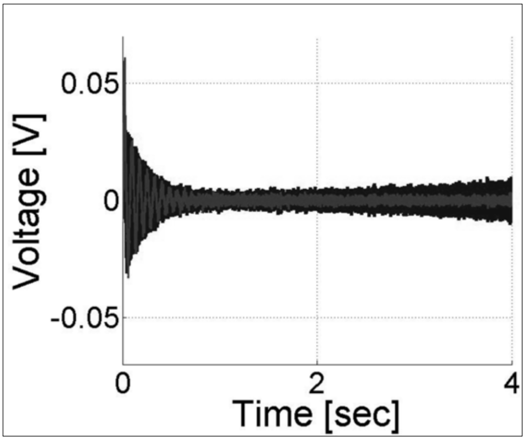

Figure 8. Control effort: deterministic value (gray line) and envelope (black).

In terms of impact response, it is noticeable that the system response was attenuated, as can be seen in Figure 6, where the light gray line represents the system without control. It can be observed robustness of the controller until a certain period shown by the small envelope (in black), however in a posterior moment it can be noticed that this envelope increases, what characterize instability of the system in steady state. The same tendency was observed in terms of control effort in Figure 8. The FRF in Figure 7 shows that the two modes considered for controlling were reduced, however there is an increase of the third mode, demonstrating the Spillover effect, which causes increase in the steady state envelope.

In the third case, it was considered uncertainties in the Kalman Estimator model. Figures 9, 10 and11 show the results for this case.

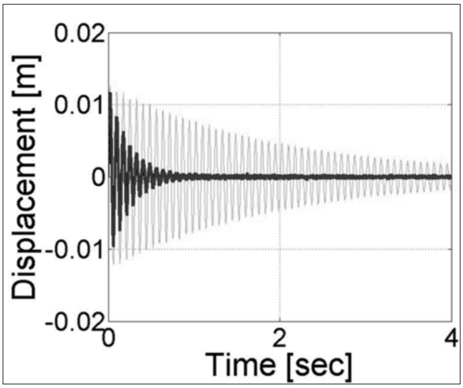

Figure 9. Impact Response: contron on (gray line), envelope (black) and control off (light gray line).

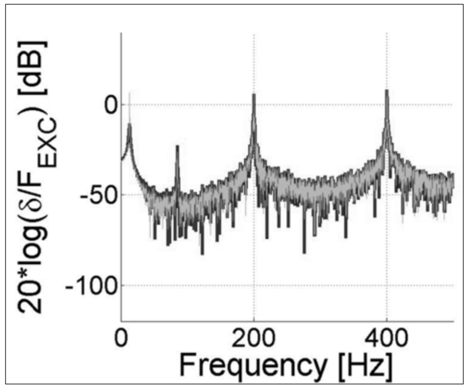

Figura 10. Frequency Response Function: deterministic (light gray) and envelope (black).

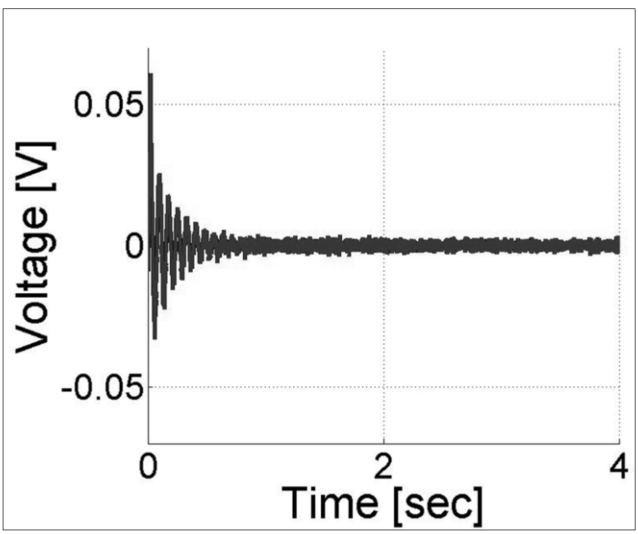

Figure 11. Impact Response: control on (gray line) and envelope (black).

From the graphs presented in Figure 9, 10 and 11 it is noticeable that the condition considering uncertainties in the Estimator model, considerably approaches the real model for all measured values, showing more robustness in this case than in the previous cases.

6. Conclusion

The current work deal with the study of active vibration control in smart composite structures through the control by H-infinity Norm. The numerical results had shown that the control was effective to attenuate the beam vibration using a piezoelectric actuator. They also reveal that the number of modes considered, which in this case were two, had been sufficient to achieve satisfactory control results. It is worth comment the importance of the balanced realization at this stage, given that this technique classifies the modes in order of relevance, allowing the consideration of the most important modes to the system response. The analysis had shown the robustness of the system, mainly in the third condition analyzed, which occurred even with the uncertainties added to the estimator model. The results obtained matched the values of the deterministic methodology.

Acknowledgement

The authors are thankful to the Brazilian Research Agency CAPES and Foundation Araucária for the financial support to this work.

References

[1] D. A. Savaranos, P. R. Heyliger and D. A. Hopkins, Layerwise Mechanics and Finite Element for the Dynamic Analysis of Piezoelectric Composite Plates, *International Journal of Solids and Structures*, Vol. 34, 1997, No. 3, pp. 359-378.

[2] J. N. Reddy, *Mechanics of Laminated Composite Plates: Theory and Analysis*, Second Edition, CRC Press, London, UK, 1997.

[3] C. Y. K. Chee, *Static Shape Control of Laminated Composite Plate Smart Structure Using Piezoelectric Actuators*, Ph.D. Thesis, University of Sydney, Sydney, Australia, 2000.

[4] C. Y. K. Chee, L. Tong and G. Steven, A Mixed Model for Adaptive Composite Plates with Piezoelectric for Anisotropic Actuation, *Computers & Structures*, Vol. 77, 2000, No. 3, pp. 253-268.

[5] E. H. Koroishi, A. S. Borges, A. A. Cavalini Jr and V. Steffen Jr, Numerical and Experimental Modal Control of Flexible Rotor Using Electromagnetic Actuator, *Mathematical Problems in Engineering*, Article ID 361418, 14 pages, 2014.

[6] G. Welch and G. Bishop, *An Introduction to Kalman Filter*, Technical Report, University of North Carolina at Chapel Hill, 1995.

[7] S. Boyd, V. Balakrishnan, E. Feron and L. El Ghaoui, Linear Matrix Inequalities in Systems and Control Theory, *Siam Studies in Applied Mathematics*, USA, 1994.

[8] E. H. Koroishi, F. A. Lara-Molina, A.W. Faria and V. Steffen Jr, Fuzzy Modal Control Applied to Smart Composite Structure. In: *11th International Conference on Damage Assessment of Structures DAMAS 2015*, Ghent – Belgium, 2015