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A Specialized Set Theoretic Semantics for Acceptability Dynamics of Arguments[1]

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Abstract
Inspired by the standard set theoretic Tarskian semantics, we propose a novel interpretation structure for studying the acceptability dynamics of arguments (i.e., the eventual changes on their acceptability condition) for logic-based argumentation. Interpretation structures identify possible scenarios in which a given argument would be accepted, or not, according to some standard extension-based argumentation semantics. These scenarios are configured in accordance to the consideration or inconsideration of other arguments from the given argumentation framework. Thereafter, it would be possible to ensure the acceptability of an argument by handling the evolution of the argumentation framework throughout the use of argumentative change operations. Hence, an interpretation structure which is a model of a given argument specifies a possible epistemic state to which the argumentation framework could evolve towards the argument's positive acceptability. Moreover, the analysis of several models of a given argument brings the opportunity of satisfying additional restrictions towards the evolution of a framework. Finally, we propose a revision operator whose rationality is ensured through postulates and a corresponding representation theorem.

Keywords. Argumentation, Belief Revision, Argumentation Dynamics

1. Introduction

We propose a new perspective for studying acceptability dynamics of arguments upon logic-based argumentation. Although an argumentation change operator is presented at the end, the main objective of this article is not precisely its proposal, but the introduction of a new way of reasoning about dynamics in argumentation by considering the changes on the acceptability status of arguments. We intend to deal with the question of which arguments provide, or interfere with, the acceptability of others, studying the interaction between acceptance and rejection through the consideration of sub-frameworks, and facilitating a theoretical analysis of the implications of change on a framework in advance, before the formal application of some argumentation change operation. We get inspiration from the

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standard set theoretic Tarskian semantics and the idea of constructing interpretation structures for reasoning about dynamics in argumentation. The cornerstone for such structures relies on the notions of core and remainder sets [11], two different constructions for recognizing acceptance and rejection of arguments. An interpretation structure proposes a "further" epistemic state in which the acceptability of a formula is analyzed in contrast with its acceptability status on the current epistemic state, *i.e.*, the current framework. Since that formula is supported by the claim of certain arguments, the interpretation structure ends up analyzing their acceptability as well. Afterwards, an interpretation ensuring the acceptability on the further epistemic state is referred as a model. Since several different models may appear, we obtain alternatives of change for analyzing and deciding which should be the most appropriate according to rationality conditions. On its basis, we propose thereafter an acceptance revision operator which deals with the matter of incorporating a new argument while ensuring its acceptance. Finally, its rationality is guaranteed through the axiomatic characterization and corresponding representation theorem according to classic belief revision [2] literature and an argument-based belief revision model like Argument Theory Change (ATC) [12].

2. Fundamentals for Reasoning on Logic-based Frameworks

We refer to the *argument domain set* $\mathbb{A}_{\mathcal{L}}$ for identifying *(logic-based) arguments* $a \in \mathbb{A}_{\mathcal{L}}$ built with formulae $\vartheta \in \mathcal{L}$, where $\mathcal{L}$ is some underlying logic. Arguments are expressed through a pair $\langle S, \vartheta \rangle$ where $S \subseteq \mathcal{L}$ is the argument's *support*, and $\vartheta \in \mathcal{L}$ its *claim*. The functions $\mathfrak{cl} : \mathbb{A}_{\mathcal{L}} \longrightarrow \mathcal{L}$ and $\mathfrak{sp} : \mathbb{A}_{\mathcal{L}} \longrightarrow \wp(\mathcal{L})$ are used for identifying the claim $\mathfrak{cl}(a) \in \mathcal{L}$ and support set $\mathfrak{sp}(a) \subseteq \mathcal{L}$, of an argument $a \in \mathbb{A}_{\mathcal{L}}$. The function $\mathfrak{sp}$ will be overloaded to apply over sets of arguments, $\mathfrak{sp} : \wp(\mathbb{A}_{\mathcal{L}}) \longrightarrow \wp(\mathcal{L})$, such that $\mathfrak{sp}(\Theta) = \bigcup_{a \in \Theta} \mathfrak{sp}(a)$ will identify the *base* determined by the set of supports of arguments in $\Theta \subseteq \mathbb{A}_{\mathcal{L}}$. The logic $\mathcal{L}$ will be considered along with its corresponding inference operator $\models$. Thus, we can say that an argument $a \in \mathbb{A}_{\mathcal{L}}$ *supports*, or *is a supporter of* ϑ, to specify that $\mathfrak{cl}(a) \models \vartheta$ holds. In order to avoid multiple representation of arguments with a same support set, we will restrict their construction to the *canonical form* [4], in which for any argument a, its claim is $\mathfrak{cl}(a) = \bigwedge \mathfrak{sp}(a)$. Hence, we will assume $\mathbb{A}_{\mathcal{L}}$ as the domain of canonical arguments. In consequence, for any pair $a, b \in \mathbb{A}_{\mathcal{L}}$, $a = b$ *iff* if $\mathfrak{sp}(a) = \mathfrak{sp}(b)$ then $\mathfrak{cl}(a) = \mathfrak{cl}(b)$. We write $a \sqsubseteq b$ for expressing that an argument $a \in \mathbb{A}_{\mathcal{L}}$ is a *sub-argument* of argument $b \in \mathbb{A}_{\mathcal{L}}$ (and also that b is a *super-argument* of a), implying that $\mathfrak{sp}(a) \subseteq \mathfrak{sp}(b)$ holds. When $\mathfrak{sp}(a) \subset \mathfrak{sp}(b)$, we say that a is a *strict sub-argument* of b, by writing $a \sqsubset b$. Arguments with no strict sub-arguments inside are referred as *atomic arguments*, thus $a \in \mathbb{A}_{\mathcal{L}}$ is *atomic iff* $|\mathfrak{sp}(a)| = 1$. The *atoms function* $\mathfrak{at} : \mathbb{A}_{\mathcal{L}} \longrightarrow \wp(\mathbb{A}_{\mathcal{L}})$ identifies the set $\mathfrak{at}(a) \subseteq \mathbb{A}_{\mathcal{L}}$ of all the atomic arguments of $a \in \mathbb{A}_{\mathcal{L}}$. The atoms function will be overloaded as $\mathfrak{at} : \wp(\mathbb{A}_{\mathcal{L}}) \longrightarrow \wp(\mathbb{A}_{\mathcal{L}})$ to apply over sets $\Theta \subseteq \mathbb{A}_{\mathcal{L}}$ such that $\mathfrak{at}(\Theta) = \bigcup_{a \in \Theta} \mathfrak{at}(a)$. The set $\mathbf{R}_{\Theta} \subseteq \mathbb{A}_{\mathcal{L}} \times \mathbb{A}_{\mathcal{L}}$ identifies the *defeat relation* between pairs of arguments from $\Theta \subseteq \mathbb{A}_{\mathcal{L}}$. A pair $(a, b) \in \mathbf{R}_{\Theta}$ implies that $a \in \Theta$ *defeats* $b \in \Theta$, or equivalently, a is a *defeater* of b, meaning that $\mathfrak{sp}(a) \cup \mathfrak{sp}(b) \models \bot$ and $a \succcurlyeq b$, where $\succcurlyeq \subseteq \mathbb{A}_{\mathcal{L}} \times \mathbb{A}_{\mathcal{L}}$ is an abstract *preference relation* assumed to be *total*

–thus, for any pair of arguments $a, b \in \mathbb{A}_{\mathcal{L}}$ we know either $a \succcurlyeq b$ or $b \succcurlyeq a$ (or both). This is a necessary condition to ensure a functional construction of the defeat relation $\mathbf{R} : \wp(\mathbb{A}_{\mathcal{L}}) \longrightarrow \wp(\mathbb{A}_{\mathcal{L}} \times \mathbb{A}_{\mathcal{L}})$, verifying $\mathfrak{sp}(a) \cup \mathfrak{sp}(b) \models \perp$ *iff* $(a, b) \in \mathbf{R}_{\Theta}$ or $(b, a) \in \mathbf{R}_{\Theta}$, for any pair $a, b \in \Theta$. In addition, for guaranteeing $\mathfrak{sp}(\Theta) \models \perp$ *iff* $\mathbf{R}_{\Theta} \neq \emptyset$, we will rely upon *closed sets of arguments*: a set containing all the sub- and super-arguments that can be constructed from its arguments. We provide such implementation through an *argumentation closure operator* $\mathbb{C}$ such that for any $\Theta \subseteq \mathbb{A}_{\mathcal{L}}$, $\mathbb{C}(\Theta) = \{a \in \mathbb{A}_{\mathcal{L}} | \mathfrak{at}(a) \subseteq \Theta$ or $a \sqsubseteq b$, for any $b \in \Theta\}$. Thus, we will say $\mathbf{A} \subseteq \mathbb{A}_{\mathcal{L}}$ is *closed iff* $\mathbf{A} = \mathbb{C}(\mathbf{A})$, and will usually note as $\mathbf{A}$ any closed set.

Example 1 *Assuming a propositional logic $\mathcal{L}$ and a set $\Theta \subseteq \mathbb{A}_{\mathcal{L}}$ such that $\Theta = \{a, b, c\}$ where $a = \langle\{p\}, p\rangle$, $b = \langle\{q\}, q\rangle$, and $c = \langle\{\neg p \vee \neg q\}, \neg p \vee \neg q\rangle$; the functional construction of the defeat relation will trigger a set $\mathbf{R}_{\Theta} = \emptyset$, although $\mathfrak{sp}(\Theta) \models \perp$ holds. However, the argumentation closure renders a closed set $\mathbf{A} = \mathbb{C}(\Theta) = \{a, b, c, d, e, f\}$, where $d = \langle\{p, q\}, p \wedge q\rangle$, $e = \langle\{p, \neg p \vee \neg q\}, p \wedge (\neg p \vee \neg q)\rangle$, and $f = \langle\{q, \neg p \vee \neg q\}, q \wedge (\neg p \vee \neg q)\rangle$. Afterwards, the defeat relation ends up as $\mathbf{R}_{\mathbf{A}} = \{(a, f), (b, e), (c, d), (d, e), (d, f), (e, d), (f, d)\}$, for a preference relation prioritizing arguments in Θ over others, being symmetric otherwise.*

A *(canonical logic-based) argumentation framework* (AF) is identified through the structure $\langle\Theta, \mathbf{R}_{\Theta}\rangle$, where $\Theta \subseteq \mathbb{A}_{\mathcal{L}}$, and whenever $\mathbf{A} \subseteq \mathbb{A}_{\mathcal{L}}$ is known to be closed, the structure $\langle\mathbf{A}, \mathbf{R}_{\mathbf{A}}\rangle$ identifies a *closed* AF. Since the defeat relation is a function over $\mathbb{A}_{\mathcal{L}}$-arguments, we refer to an operator $\mathbb{F}_{\Theta}$ as the AF *generator from* Θ *iff* $\mathbb{F}_{\Theta} = \langle\Theta, \mathbf{R}_{\Theta}\rangle$. Note that $\mathbb{F}_{\Theta}$ is the AF constructed from Θ. Finally, we refer to an AF $\mathbb{F}_{\mathbf{A}}$, implying that $\mathbb{F}_{\mathbf{A}}$ is the closed AF $\langle\mathbf{A}, \mathbf{R}_{\mathbf{A}}\rangle$, and thus, $\mathbf{A} = \mathbb{C}(\mathbf{A})$. Given an AF $\mathbb{F}_{\mathbf{A}}$, for any not necessarily closed set $\Theta \subseteq \mathbf{A}$, it is possible to construct the *sub-framework* $\mathbb{F}_{\Theta}$. In such a case, we overload the sub-argument operator '$\sqsubseteq$' by also using it for identifying sub-frameworks, writing $\mathbb{F}_{\Theta} \sqsubseteq \mathbb{F}_{\mathbf{A}}$. Observe that, if $\mathbb{C}(\Theta) = \mathbf{A}'$ and $\mathbf{A}' \subset \mathbf{A}$, then $\mathbb{F}_{\mathbf{A}'}$ is a closed *strict sub-framework* of $\mathbb{F}_{\mathbf{A}}$, *i.e.*, $\mathbb{F}_{\mathbf{A}'} \sqsubset \mathbb{F}_{\mathbf{A}}$. Our intention is to simplify AF*s* for concentrating on acceptability dynamics of arguments. Consequently, for an AF τ, we refer to its set of arguments through the set $\mathbf{A}(\tau)$ and to its set of defeats through $\mathbf{R}(\tau)$.

Given an AF $\mathbb{F}_{\mathbf{A}}$, as usual in abstract argumentation [8], for any $\Theta \subseteq \mathbf{A}$ we say that Θ *defeats* an argument $a \in \mathbf{A}$ *iff* there is some $b \in \Theta$ such that b defeats a; Θ *defends* an argument $a \in \mathbf{A}$ *iff* Θ defeats every defeater of a; Θ is *conflict-free iff* $\mathbf{R}_{\Theta} = \emptyset$; and Θ is *admissible iff* it is conflict-free and defends all its members. However, as seen before, a logic-based framework should be closed to ensure that all sources of conflict are identified through the defeat relation. For instance, in Ex. 1, $\Theta \subseteq \mathbf{A}$ is admissible given that it is conflict-free and defends all its members, however $\mathfrak{sp}(\Theta) \models \perp$. This is undesirable since an admissible set could trigger an inconsistent set of supports. Thus, we reformulate the classic notion of admissibility for abstract argumentation into *logic-based admissibility* [11]:

Definition 1 (Logic-based Admissibility [11]) *For any $\Theta \subseteq \mathbf{A}$ we say that Θ is* **admissible** *iff Θ is closed, conflict-free, and defends all its members.*

We will just say admissibility to refer to logic-based admissibility. (In Ex. 1, Θ cannot be admissible since it is not closed, thus, the only admissible sets are

$\{a\}$, $\{b\}$, and $\{c\}$.) The *extension semantics*, which rely upon admissibility, will also be affected by the notion of logic-based admissibility without inconvenience. We will only refer to the *complete semantics* in some examples, however, any of the extension semantics could also be applied. Thus, given an AF $\tau = \mathbb{F}_{\mathbf{A}}$, a set $\mathbf{E} \subseteq \mathbf{A}$ is a *complete extension* if $\mathbf{E}$ is admissible and contains every argument it defends. Afterwards, the set $\mathbb{E}_{\mathfrak{s}}(\tau) \subseteq \wp(\mathbf{A})$ identifies the *set of* $\mathfrak{s}$*-extensions* $\mathbf{E}$ from τ, where an $\mathfrak{s}$*-extension* is an extension in τ according to some specific extension semantics $\mathfrak{s}$. Observe that any extension $\mathbf{E} \in \mathbb{E}_{\mathfrak{s}}(\tau)$ is admissible and thus, it contains a consistent support base, *i.e.*, $\mathfrak{sp}(\mathbf{E}) \not\models \bot$ holds.

We refer as *acceptance criterion* to the determination of acceptance of arguments in either a *sceptical* or *credulous* way. Several postures may appear. For instance, a *sceptical set* may be obtained by intersecting every $\mathfrak{s}$-extension $\bigcap \mathbb{E}_{\mathfrak{s}}(\tau)$, while a *credulous set* may arise from the selection of a single extension $\mathbf{E} \in \mathbb{E}_{\mathfrak{s}}(\tau)$ according to some specific preference. For instance, selecting "the best" extension among those of maximal cardinality. We will abstract the implementation of any acceptance criterion by referring to an *acceptance function* $\delta : \wp(\wp(\mathbf{A})) \longrightarrow \wp(\mathbf{A})$ where $\delta(\mathbb{E}_{\mathfrak{s}}(\tau))$ determines the outcome of the adopted criterion. In addition, we refer as *(argumentation) semantics specification* $\mathcal{S}$ to a tuple $\langle \mathfrak{s}, \delta \rangle$, where $\mathfrak{s}$ stands for identifying some extension semantics and δ for an acceptance function implementing some acceptance criterion. Afterwards, we refer to the set $\mathcal{A}_{\mathcal{S}}(\tau) \subseteq \mathbf{A}$ as the *acceptable set* of τ according to $\mathcal{S}$ *iff* $\mathcal{A}_{\mathcal{S}}(\tau) = \delta(\mathbb{E}_{\mathfrak{s}}(\tau))$. Finally, for any $a \in \mathbf{A}$, a is $\mathcal{S}$*-accepted* in τ (resp. of, $\mathcal{S}$*-rejected*) *iff* $a \in \mathcal{A}_{\mathcal{S}}(\tau)$ (resp. of, $a \notin \mathcal{A}_{\mathcal{S}}(\tau)$).

3. Acceptability Analysis through Core and Remainder Sets

We rely upon the notions of *admissible* and *core sets* ([11]) of an argument as the fundamentals for recognizing the sources of an argument's acceptability condition, and upon *rejecting sets* for the argument's rejecting condition.

Definition 2 (Admissible Sets of an Argument) *Given an* AF $\tau = \mathbb{F}_{\mathbf{A}}$ *and an argument* $a \in \mathbf{A}$*; for any* $\Theta \subseteq \mathbf{A}$*, we say that:* 1) Θ *is an* a**-admissible set** *in* τ *iff* Θ *is an admissible set*[3] *such that* $a \in \Theta$*, and* 2) Θ *is a* **minimal** a**-admissible set** *in* τ *iff* Θ *is a-admissible and for any* $\Theta' \subset \Theta$*, it follows that* Θ' *is not a-admissible.*

Definition 3 (Core Sets) *Given an* AF $\tau = \mathbb{F}_{\mathbf{A}}$ *and an argumentation semantics specification* $\mathcal{S}$*, for any* $\mathcal{C} \subseteq \mathbf{A}$*, we say that* $\mathcal{C}$ *is an* a**-core** *in* τ*, noted as* a-$\mathtt{core}_{\mathcal{S}}$ *iff* $\mathcal{C}$ *is a minimal a-admissible set and a is* $\mathcal{S}$*-accepted in* τ*.*

Definition 4 (Rejecting Sets of an Argument) *Given an* AF $\mathbb{F}_{\mathbf{A}}$*, a semantics specification* $\mathcal{S}$*, and an argument* $a \in \mathbf{A}$*; for any* $\Theta \subseteq \mathbf{A}$*, we say that* Θ *is a* $\mathcal{S}$**-a-rejecting set** *in* $\mathbb{F}_{\mathbf{A}}$ *iff a is* $\mathcal{S}$*-rejected in* $\mathbb{F}_{\mathbf{A}}$ *but it is* $\mathcal{S}$*-accepted in* $\mathbb{F}_{\mathbf{A} \setminus \Theta}$*.*

We have defined rejecting sets in an intuitive manner. For constructing rejecting sets of an argument a (see [11]) we need to identify those arguments that interpose to the construction of an a-$\mathtt{core}_{\mathcal{S}}$ set. This is the seed for further con-

[3] Recall that from now on by admissibility we refer only to its logic-based definition.

structing *remainder sets*. However, the acceptability analysis must apply upon closed frameworks for avoiding inconveniences as described in Ex. 1. We only can be sure that a is $\mathcal{S}$-accepted in $\mathbb{F}_{\mathbf{A}\setminus\Theta}$ if we can ensure that $\mathbb{F}_{\mathbf{A}\setminus\Theta}$ is a closed $\mathtt{AF}$. An *expansive closure* is a sort of "complementary closure operator" which ensures that removing an *expanded set* from a closed set delivers a closed set.

Definition 5 (Expansive Closure) *Given* $\Theta \subseteq \mathbf{A}$*,* $\mathbb{P}$ *is an* **expansive closure** *iff* $\mathbb{P}(\Theta) = \{a \in \mathbf{A} | b \sqsubseteq a, \textit{ for every } b \in \mathfrak{at}(\mathbb{P}_0(\Theta))\}$*, where* $\mathbb{P}_0(\Theta) = \{a \in \Theta |$ *there is no* $b \in \Theta$ *such that* $b \sqsubset a\}$*. We say that* Θ *is* **expanded** *iff it holds* $\Theta = \mathbb{P}(\Theta)$*.*

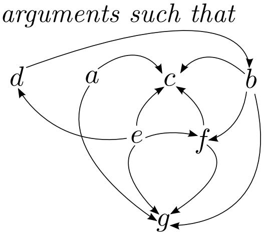
Example 2 *Suppose* $\{a_1, a_2, a, b_1, b, c\} \subseteq \mathbf{A}$*, where* $a \sqsubset b$ *and* $b \sqsubset c$*,* $\mathfrak{at}(a) = \{a_1, a_2\}$*, and* $\mathfrak{at}(b) = \{a_1, a_2, b_1\}$*; and* $\Theta = \{a, b\}$*. This means that* $\mathbb{P}_0(\Theta) = \{a\}$*. Removing* a *from* $\mathbf{A}$ *should prevent its construction, thus,* a_1 *and* a_2 *should not be simultaneously present (since* $a \in \mathbb{C}(\{a_1, a_2\})$*,* $\mathbf{A} = \mathbb{C}(\mathbf{A} \setminus \{a\})$*). The expanded set ends up being* $\mathbb{P}(\Theta) = \{a_1, a_2, a, b, c\}$*, which ensures that* $\mathbf{A} \setminus \mathbb{P}(\Theta)$ *is a closed set. Observe however that* $\mathbb{P}(\Theta)$ *is not a minimal expanded set for the removal of arguments* a *and* b*: for instance if* $\Theta' = \{a_1, a, b\}$ *then* $\mathbb{P}(\Theta') = \{a_1, a, b, c\}$*, which is a minimal alternative for such purpose.*

Proposition 1 *Given two sets* $\mathbf{A} \subseteq \mathbb{A}_\mathcal{L}$ *and* $\Theta \subseteq \mathbb{A}_\mathcal{L}$*, where* $\mathbf{A}$ *is closed; if* $\Theta \subseteq \mathbf{A}$ *then* $\mathbf{A}' = \mathbf{A} \setminus \mathbb{P}(\Theta)$ *is a closed set, i.e.,* $\mathbf{A}' = \mathbb{C}(\mathbf{A}')$*.*

Remainder sets identify "responsible" arguments for the non-acceptability of an argument. Intuitively, an a-remainder is a minimal expanded $\mathcal{S}$-a-rejecting set.

Definition 6 (Remainder Sets) *Given an* $\mathtt{AF}$ $\mathbb{F}_\mathbf{A}$ *and a semantics specification* $\mathcal{S}$*, for any* $\mathcal{R} \subseteq \mathbf{A}$*,* $\mathcal{R}$ *is an* a**-remainder** *in* $\mathbb{F}_\mathbf{A}$*, noted as* a-$\mathtt{remainder}_\mathcal{S}$ *iff* $\mathcal{R}$ *is a minimal expanded* $\mathcal{S}$*-*a*-rejecting set:* 1) $\mathcal{R}$ *is a* $\mathcal{S}$*-*a*-rejecting set,* 2) $\mathcal{R} = \mathbb{P}(\mathcal{R})$*, and* 3) *for any set* $\Theta \subset \mathcal{R}$ *such that* $\Theta = \mathbb{P}(\Theta)$*, it holds* a *is* $\mathcal{S}$*-rejected in* $\mathbb{F}_{\mathbf{A}\setminus\Theta}$*.*

Example 3 *Assume* $\mathcal{L}$ *as the propositional logic and* $\mathbb{A}_\mathcal{L}$ *as the domain of canonical arguments. Let* $\Theta = \{a, b, c, d\} \subseteq \mathbb{A}_\mathcal{L}$ *be a set of canonical arguments such that* $\Theta = \{a, b, c, d\}$*, where* $a = \langle\{p \wedge q_1\}, p \wedge q_1\rangle$*,* $b = \langle\{p \wedge q_2\}, p \wedge q_2\rangle$*,* $c = \langle\{\neg p\}, \neg p\rangle$*, and* $d = \langle\{\neg q_2\}, \neg q_2\rangle$*. The argumentation closure renders the closed set of arguments* $\mathbf{A} = \mathbb{C}(\Theta) = \{a, b, c, d, e, f, g\}$*, where:*
$e = \langle\{p \wedge q_1, p \wedge q_2\}, p \wedge q_1 \wedge q_2\rangle$ $(a \sqsubseteq e,\ b \sqsubseteq e)$
$f = \langle\{p \wedge q_1, \neg q_2\}, p \wedge q_1 \wedge \neg q_2\rangle$ $(a \sqsubseteq f,\ d \sqsubseteq f)$
$g = \langle\{\neg p, \neg q_2\}, \neg p \wedge \neg q_2\rangle$ $(c \sqsubseteq g,\ d \sqsubseteq g)$



$\mathbb{F}_\mathbf{A}$

Thus, $\mathbb{F}_\mathbf{A}$ *is closed and due to some preference relation:* $\mathbf{R_A} = \{(a, c), (b, c), (d, b), (e, c), (e, d), (b, f), (f, c), (a, g), (b, g), (e, f), (e, g), (f, g)\}$*. Assuming* $\mathcal{S} = \langle\mathfrak{s}, \delta\rangle$*, where* $\mathfrak{s}$ *is a complete semantics and* δ *selects "the best"* $\mathfrak{s}$*-extension of higher cardinality (credulous), a* b-$\mathtt{core}_\mathcal{S}$ $\mathcal{C}_b = \{a, b, e\}$ *is constructed by* $\mathbb{C}(\{b, e\})$*. Since* c *and* d *are* $\mathcal{S}$*-rejected, we have remainders for both of them: a* c-$\mathtt{remainder}_\mathcal{S}$ $\mathcal{R}_c = \{a, e, f\}$ *and two* d-$\mathtt{remainder}_\mathcal{S}$ *sets* $\mathcal{R}_d = \{a, e, f\}$ *and* $\mathcal{R}'_d = \{b, e\}$*. Note* $\{a, b, e, f\} = \mathbb{P}(\{e\})$ *is not a* d-$\mathtt{remainder}_\mathcal{S}$ *since it is not minimal given that it contains* $\mathbb{P}(\{a, e\}) = \mathcal{R}_d$ *and* $\mathbb{P}(\{b, e\}) = \mathcal{R}'_d$*.*

Proposition 2 *Given an* $\texttt{AF}$ $\mathbb{F}_{\mathbf{A}}$*, it holds:* 1) $a \in \mathcal{A}_{\mathcal{S}}(\mathbb{F}_{\mathbf{A}})$ *iff there is some* a-$\texttt{core}_{\mathcal{S}}$ *in* $\mathbf{A}$*,* 2) $a \notin \mathcal{A}_{\mathcal{S}}(\mathbb{F}_{\mathbf{A}})$ *iff there is some* a-$\texttt{remainder}_{\mathcal{S}}$ *in* $\mathbf{A}$*, and* 3) *there is some* a-$\texttt{core}_{\mathcal{S}}$ *in* $\mathbf{A}$ *iff there is no* a-$\texttt{remainder}_{\mathcal{S}}$ *in* $\mathbf{A}$*.*

4. Argumentation Dynamics Contra-Semantics

The idea behind the theory of *Argumentation Dynamics Contra-Semantics* is to analyze the current epistemic state determined by an $\texttt{AF}$ $\tau = \mathbb{F}_{\mathbf{A}}$ for answering whether a formula $\vartheta \in \mathcal{L}$ is $\mathcal{S}$-accepted in τ, and in the case ϑ is $\mathcal{S}$-rejected in τ, whether it is possible, and how, to provoke the evolution of τ to reach a *further epistemic state* in which ϑ would end up $\mathcal{S}$-accepted. The language $\mathcal{L}$ is interpreted in terms of a specialized set theoretic semantics *à la* Tarski, through an *argumentation dynamics interpretation structure* $\mathcal{I}_{\mathcal{S}} = \langle \Delta^{\mathcal{I}}, \Gamma^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$, which considers an *(acceptability dynamics) interpretation function* $\cdot^{\mathcal{I}}$ and two different domains containing the "constants" of the $\texttt{AF}$ which would be the lower-case letters naming arguments from $\mathbf{A}$. The positive domain $\Delta^{\mathcal{I}}$ referred as *interpretation domain*, describes which arguments are considered in the proposed interpretation, and the negative domain $\Gamma^{\mathcal{I}}$ referred as *interpretation contra-domain*, describes undesired arguments. Through the interpretation function, the acceptability dynamics of a formula $\vartheta \in \mathcal{L}$ are interpreted as $\vartheta^{\mathcal{I}} \subseteq \mathbb{W}_{\mathcal{L}}$, where $\mathbb{W}_{\mathcal{L}} \subseteq \mathbb{A}_{\mathcal{L}} \times \wp(\mathbb{A}_{\mathcal{L}}) \times \wp(\mathbb{A}_{\mathcal{L}})$.

Definition 7 (Dynamics Interpretation Structure) *Given an* $\texttt{AF}$ $\mathbb{F}_{\mathbf{A}}$ *and a semantics specification* $\mathcal{S}$*; a structure* $\mathcal{I}_{\mathcal{S}} = \langle \Delta^{\mathcal{I}}, \Gamma^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$*, where* $\Delta^{\mathcal{I}} \subseteq \mathbf{A}$*,* $\Gamma^{\mathcal{I}} \subseteq \mathbf{A}$*, and* $\cdot^{\mathcal{I}} : \mathcal{L} \longrightarrow \wp(\mathbb{W}_{\mathcal{L}})$*; is referred as* **(argumentation dynamics) interpretation structure** *iff* 1) $\Delta^{\mathcal{I}} \cup \Gamma^{\mathcal{I}} = \mathbf{A}$*,* 2) $\Delta^{\mathcal{I}} \cap \Gamma^{\mathcal{I}} = \emptyset$*,* 3) $\Delta^{\mathcal{I}}$ *is closed, and* 4) *for any* $\vartheta \in \mathcal{L}$*,* $\vartheta^{\mathcal{I}}$ *is the set of* **interpretation triples** $(a, X, Y) \in \mathbb{W}_{\mathcal{L}}$ *verifying:*

a) $a \in \mathbf{A}$*, is a support for* ϑ*, i.e.,* $\mathfrak{cl}(a) \models \vartheta$

b) $X \subseteq \Delta^{\mathcal{I}}$*, is an* a-$\texttt{core}_{\mathcal{S}}$ *in* $\mathbb{F}_{\Delta^{\mathcal{I}}}$ *or else the empty set*

c) $Y \subseteq \Gamma^{\mathcal{I}}$*, is an* a-$\texttt{remainder}_{\mathcal{S}}$ *in* $\mathbb{F}_{\mathbf{A}}$ *or else the empty set*

We refer to $\Delta^{\mathcal{I}}$ *as the* **interpretation domain***,* $\Gamma^{\mathcal{I}}$ *as the* **interpretation contra-domain***, and* $\cdot^{\mathcal{I}}$ *as the* **(acceptability dynamics) interpretation function***.*

The interpretation function related to an interpretation $\mathcal{I}_{\mathcal{S}}$ brings the possibility to understand the consequences of discarding arguments (those in the contra-domain) from the acceptability analysis. $\mathcal{I}_{\mathcal{S}}$ proposes a further epistemic state, *i.e.*, an evolutive step of the $\texttt{AF}$, in which we can see the concrete implications by observing the corresponding interpreted formula and its triples. As being specified above, the interpretation structure counts with two mutually exclusive domains and with an interpretation triple composed by an argument $a \in \mathbf{A}$ which is a support of the interpreted formula ϑ, and two sets of arguments, an a-$\texttt{core}_{\mathcal{S}}$ and an a-$\texttt{remainder}_{\mathcal{S}}$. The a-$\texttt{remainder}_{\mathcal{S}}$ indicates which are the arguments contained in the contra-domain that interfere with the acceptability of a and the a-$\texttt{core}_{\mathcal{S}}$ specifies which is the core set of arguments that will ensure a positive acceptability for a in the interpretation's proposed evolutive step, *i.e.*, the $\texttt{AF}$ determined by the interpretation domain: $\mathbb{F}_{\Delta^{\mathcal{I}}}$. Note that the evolved $\texttt{AF}$ is closed.

Example 4 (From Ex. 3) *Assume an interpretation* $\mathcal{I}_{\mathcal{S}} = \langle \Delta^{\mathcal{I}}, \Gamma^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ *for the* AF $\mathbb{F}_{\mathbf{A}}$*, where* $\Delta^{\mathcal{I}} = \mathbf{A} \setminus \Gamma^{\mathcal{I}}$ *and* $\Gamma^{\mathcal{I}} = \{a, b, e, f\}$*. The interpretation function* $\cdot^{\mathcal{I}}$ *applied over a formula* $\vartheta = \neg p \vee \neg q_2$ *ends up containing four triples,* $\vartheta^{\mathcal{I}} = \{(c, \{c, d\}, \{a, e, f\}), (d, \{d\}, \{a, e, f\}),\ (d, \{d\}, \{b, e\}), (g, \{g\}, \{a, b, e, f\})\}$*. For a formula like* $p \wedge q_2$*, the function* $\cdot^{\mathcal{I}}$ *brings some empty triples:* $(p \wedge q_2)^{\mathcal{I}} = \{(b, \emptyset, \emptyset), (e, \emptyset, \emptyset)\}$ *since both* b *and* e *are part of the contra-domain, and moreover, for a formula like* $\neg q_1$*, the function* $\cdot^{\mathcal{I}}$ *ends up empty, i.e.,* $(\neg q_1)^{\mathcal{I}} = \emptyset$*, since there is no argument in* $\mathbf{A}$ *supporting* $\neg q_1$*.*

A dynamics interpretation may be accurate for the positive acceptability of some formulae. This is captured by the notion of *interpretation model.*

Definition 8 (Interpretation Model) *Given an* AF τ*, an interpretation* $\mathcal{I}_{\mathcal{S}}$*, and a formula* $\vartheta \in \mathcal{L}$*; we say* $\mathcal{I}_{\mathcal{S}}$ *is a* **model** *of* ϑ *in* τ*, noted* $\mathcal{I}_{\mathcal{S}} \approx\!\!\!\!\!\mid\ \vartheta$ *iff there is some triple* $(a, X, Y) \in \vartheta^{\mathcal{I}}$ *such that* $X \neq \emptyset$ *holds. On the contrary, if there is no such triple, we say* $\mathcal{I}_{\mathcal{S}}$ *is not a model of* ϑ*, writing* $\mathcal{I}_{\mathcal{S}} \not\approx\!\!\!\!\!\mid\ \vartheta$*.*

Proposition 3 *if* $\mathcal{I}_{\mathcal{S}} \approx\!\!\!\!\!\mid\ \vartheta$ *then for any* $(a, X, Y) \in \vartheta^{\mathcal{I}}$*, it holds* 1) *a is* $\mathcal{S}$*-accepted in* $\mathbb{F}_{\Delta^{\mathcal{I}}}$ *and* 2) *a is* $\mathcal{S}$*-accepted in* $\mathbb{F}_{\mathbf{A} \setminus Y}$*.*

The previous proposition shows that when $Y \subset \Gamma^{\mathcal{I}}$, the argument's acceptability is unaffected by the additional contra-domain's arguments given that they do not belong to the associated remainder set Y. This brings about the need for restricting the contra-domain only to the exclusively necessary arguments to achieve a further positive acceptability. This can be understood as a pathway to minimal change (discussed later). We look for a construction which minimizes the contra-domain. This means that a *minimal model* will ensure that each argument in the contra-domain is necessarily there in order for ϑ to be accepted.

Definition 9 (Minimal Model of a Formula) *Given an* AF τ*, an interpretation* $\mathcal{I}_{\mathcal{S}}$*, and a formula* $\vartheta \in \mathcal{L}$ *such that* $\mathcal{I}_{\mathcal{S}} \approx\!\!\!\!\!\mid\ \vartheta$*, we say* $\mathcal{I}_{\mathcal{S}}$ *is a* **minimal model** *of* ϑ *in* τ *iff there is no interpretation* $\mathcal{I}'_{\mathcal{S}}$ *such that* $\mathcal{I}'_{\mathcal{S}} \approx\!\!\!\!\!\mid\ \vartheta$*, where* $\Gamma^{\mathcal{I}'} \subset \Gamma^{\mathcal{I}}$ *holds.*

Proposition 4 $\mathcal{I}_{\mathcal{S}}$ *is a minimal model of* ϑ *iff for every* $(a, X, Y) \in \vartheta^{\mathcal{I}}$*,* $Y = \Gamma^{\mathcal{I}}$*.*

Example 5 (From Ex. 4) *Although* $\mathcal{I}_{\mathcal{S}}$ *models* ϑ*, it is not a minimal model since its contra-domain* $\Gamma^{\mathcal{I}}$ *strictly contains* $\{a, e, f\} \subseteq \Gamma^{\mathcal{I}}$ *and* $\{b, e\} \subseteq \Gamma^{\mathcal{I}}$*, defining two minimal models* $\mathcal{I}^1_{\mathcal{S}} = \langle \{b, c, d, g\}, \{a, e, f\}, \cdot^{\mathcal{I}^1} \rangle$ *and* $\mathcal{I}^2_{\mathcal{S}} = \langle \{a, c, d, f, g\}, \{b, e\}, \cdot^{\mathcal{I}^2} \rangle$*. Observe that* ϑ *is supported by* c*,* d*, and* f*; several alternatives are available. Through the minimal model* $\mathcal{I}^1_{\mathcal{S}}$*,* ϑ *is* $\mathcal{S}$*-accepted through the acceptance of* c *and* d*, while in the case of the second minimal model* $\mathcal{I}^2_{\mathcal{S}}$*,* ϑ *is* $\mathcal{S}$*-accepted through the acceptance of* d *and* f*. Finally,* $\vartheta^{\mathcal{I}^1} = \{(c, \{c, d\}, \{a, e, f\}), (d, \{d\}, \{a, e, f\})\}$ *and* $\vartheta^{\mathcal{I}^2} = \{(d, \{d\}, \{b, e\}), (f, \{a, d, f\}, \{b, e\})\}$*.*

It is observable a fine distinction between what an interpretation says about the acceptability of a modeled formula in the current epistemic state and its acceptability dynamics on a further epistemic state. As we have already discussed, an interpretation $\mathcal{I}_{\mathcal{S}}$ proposes an alternative framework $\mathbb{F}_{\Delta^{\mathcal{I}}}$ in which a modeled formula ϑ would end up accepted. However, much more than this can be said.

We refer as *dynamic model* to that which proposes an alternative of change for modifying the current acceptability state of ϑ, from rejected to accepted. Such a situation can be observed when for every triple $(a, X, Y) \in \vartheta^{\mathcal{I}}$ both X and Y are always non empty sets, showing that in the current epistemic state there is always an a-$\mathtt{remainder}_{\mathcal{S}}$ set, *i.e.*, a set of arguments blocking the acceptability of the supporter of ϑ, *i.e.*, a. On the other hand, when there is some triple $(a, X, Y) \in \vartheta^{\mathcal{I}}$ where $X \neq \emptyset$ and $Y = \emptyset$, we have that there is no a-$\mathtt{remainder}_{\mathcal{S}}$ given the a-$\mathtt{core}_{\mathcal{S}}$ X, which means that ϑ is already accepted through argument a in the current epistemic state $\mathbb{F}_{\mathbf{A}}$, and also in the further epistemic state $\mathbb{F}_{\Delta^{\mathcal{I}}}$ where the a-$\mathtt{core}_{\mathcal{S}}$ X is built. We refer to such interpretation as *static model*. In summary, if $\mathcal{I}_{\mathcal{S}}$ statically models ϑ we can ensure ϑ is accepted in the current framework $\mathbb{F}_{\mathbf{A}}$ as well as in $\mathbb{F}_{\Delta^{\mathcal{I}}}$, on the other hand, if $\mathcal{I}_{\mathcal{S}}$ dynamically models ϑ we can ensure ϑ is rejected in the current framework $\mathbb{F}_{\mathbf{A}}$ whereas it is accepted in $\mathbb{F}_{\Delta^{\mathcal{I}}}$.

As we have seen before, when for every triple $(a, X, Y) \in \vartheta^{\mathcal{I}}$, X is an empty set, we would be considering an interpretation which does not model ϑ. However, this can be still meaningful. Whenever, Y is a non-empty set, we have an interpretation which ensures that ϑ is rejected in the current epistemic state, given that it is possible to identify a set of arguments blocking its acceptability (the a-$\mathtt{remainder}_{\mathcal{S}}$ set Y), but since no a-$\mathtt{core}_{\mathcal{S}}$ set can be built considering only arguments from the interpretation domain $\Delta^{\mathcal{I}}$, we can infer that the contra-domain contains arguments that are needed for constructing an a-$\mathtt{core}_{\mathcal{S}}$ set. We refer to such an interpretation as a *contra-model*. On the other hand, when both X and Y are empty, we know ϑ will not be accepted in $\mathbb{F}_{\Delta^{\mathcal{I}}}$, however the interpretation does not tell anything about ϑ's acceptability in $\mathbb{F}_{\mathbf{A}}$ –given that it may be the case (or not) that some a-$\mathtt{remainder}_{\mathcal{S}}$ set is constructible but not from the arguments in the contra-domain– and therefore, the acceptability dynamics of ϑ will be unknown. Such an interpretation will be referred as a *failure* for ϑ.

$X \neq \emptyset$	$Y \neq \emptyset$	$a \in \mathcal{A}_{\mathcal{S}}(\mathbb{F}_{\mathbf{A}})$	$a \in \mathcal{A}_{\mathcal{S}}(\mathbb{F}_{\Delta^{\mathcal{I}}})$	Referred as
✓	✓	×	✓	Dynamic Model
✓	×	✓	✓	Static Model
×	✓	×	×	Contra-Model
×	×	?	×	Failure

Example 6 (From Ex. 5) *Both $\mathcal{I}^1_{\mathcal{S}}$ and $\mathcal{I}^2_{\mathcal{S}}$ are dynamic models for ϑ. Let $\vartheta' = p \wedge q_2$, and two interpretations $\mathcal{I}^3_{\mathcal{S}} = \langle \{a, b, d, e, f\}, \{c, g\}, \cdot^{\mathcal{I}^3} \rangle$, where $(b, \{a, b, e\}, \{\}) \in \vartheta'^{\mathcal{I}^3}$, and $\mathcal{I}^4_{\mathcal{S}} = \langle \{a, b, c, e\}, \{d, f, g\}, \cdot^{\mathcal{I}^4} \rangle$, where $(b, \{b\}, \{\}) \in \vartheta'^{\mathcal{I}^4}$. Both interpretations are static models for ϑ' since b is $\mathcal{S}$-accepted in $\mathbb{F}_{\mathbf{A}}$ and its positive acceptability is maintained in each evolved* AF *since b-$\mathtt{core}_{\mathcal{S}}$ sets are identified in each case. Note that the "canonical interpretation" $\langle \mathbf{A}, \emptyset, \cdot^{\mathcal{I}^c} \rangle$ is always a static model for any $\mathcal{S}$-accepted formula, like ϑ'. A contra-model can be seen by considering $\mathcal{I}^5_{\mathcal{S}} = \langle \{b, c\}, \{a, d, e, f, g\}, \cdot^{\mathcal{I}^5} \rangle$ where the formula $(\neg p)$ is interpreted as $(c, \{\}, \{a, e, f\}) \in (\neg p)^{\mathcal{I}^5}$. Here we have no c-$\mathtt{core}_{\mathcal{S}}$ set since a part of it is in the contra-domain (d would be required), which implies $\mathcal{I}^5_{\mathcal{S}} \not\approx \neg p$ and thus $(\neg p)$ will be $\mathcal{S}$-rejected in $\mathbb{F}_{\Delta^{\mathcal{I}^5}}$. However, since we can build a c-$\mathtt{remainder}_{\mathcal{S}}$, we know that there is a set of arguments responsible for the non-acceptability of c and therefore, we can also ensure that $(\neg p)$ is also $\mathcal{S}$-rejected in $\mathbb{F}_{\mathbf{A}}$. Two cases of failure can be referred to Ex. 4 through the interpretation $\mathcal{I}_{\mathcal{S}}$ for $(p \wedge q_2)$ and $(\neg q_1)$.*

We will refer to the special notational convention $\mathcal{I}_{\mathcal{S}}, a \models\!\approx \vartheta$ for specifying that $a \in \mathbf{A}$ is an specific argument by which $\mathcal{I}_{\mathcal{S}} \models\!\approx \vartheta$ holds. In this sense, it will be also possible to restrict the construction of an interpretation model of a formula ϑ through the acceptability of a specific argument a by requiring $\mathcal{I}_{\mathcal{S}}, a \models\!\approx \vartheta$ to be satisfied. Notice that it will be possible to have an interpretation model for a formula ϑ, that is, $\mathcal{I}_{\mathcal{S}} \models\!\approx \vartheta$ which does not model ϑ through the acceptability of a particular argument a, *i.e.*, $\mathcal{I}_{\mathcal{S}}, a \not\models\!\approx \vartheta$. It is clear that, if $\mathcal{I}_{\mathcal{S}}, a \models\!\approx \vartheta$ then $a \in \mathcal{A}_{\mathcal{S}}(\mathbb{F}_{\Delta^{\mathcal{I}}})$ and $\mathfrak{cl}(a) \models \vartheta$, and moreover, if $\mathcal{I}_{\mathcal{S}}$ is a static model, we also know that $a \in \mathcal{A}_{\mathcal{S}}(\mathbb{F}_{\mathbf{A}})$. For the specific case in which $\vartheta = \mathfrak{cl}(a)$ we will make a slight abuse of notation (for simplicity), writing $\mathcal{I}_{\mathcal{S}} \models\!\approx a$ instead of $\mathcal{I}_{\mathcal{S}}, a \models\!\approx \mathfrak{cl}(a)$. In such a case, we may say that $\mathcal{I}_{\mathcal{S}}$ is a *model of argument* a, although, its formal meaning is more likely to correspond to: $\mathcal{I}_{\mathcal{S}}$ models $\mathfrak{cl}(a)$ through the acceptability of argument a. Afterwards, with a slight abuse of notation, it will also be possible to write $(X, Y) \in a^{\mathcal{I}}$ as a shortcut for $(a, X, Y) \in \mathfrak{cl}(a)^{\mathcal{I}}$. Finally, we say $\mathcal{I}_{\mathcal{S}}$ is a *minimal model of argument* a *iff* $\mathcal{I}_{\mathcal{S}} \models\!\approx a$ and for every $(X, Y) \in a^{\mathcal{I}}$, $Y = \Gamma^{\mathcal{I}}$ holds.

5. Argumentation Dynamics through Contra-Semantics

We say that an argument $a \in \mathbb{A}_{\mathcal{L}}$ is *external to the* `AF` $\mathbb{F}_{\mathbf{A}}$ (or just, *external*) *iff* $a \notin \mathbf{A}$. An *expansion operation* incorporates an external argument ensuring a new resulting closed `AF`. Thus, given an `AF` $\mathbb{F}_{\mathbf{A}}$ and an external argument $a \in \mathbb{A}_{\mathcal{L}}$, the operator $+$ stands for an *expansion iff* $\mathbb{F}_{\mathbf{A}} + a = \mathbb{F}_{\mathbb{C}(\mathbf{A} \cup \{a\})}$.

We identify the domain of all interpretation structures of a given `AF` τ through the set $\mathbb{I}^{\tau}_{\mathcal{S}} \subseteq \wp(\mathbb{A}_{\mathcal{L}}) \times \wp(\mathbb{A}_{\mathcal{L}}) \times \mathbb{W}_{\mathcal{L}}$, in addition, we identify the *set of all minimal models* of an argument $a \in \mathbb{A}_{\mathcal{L}}$ in τ through the operator $\mathcal{M}_{\mathcal{S}}(a, \tau) \subseteq \mathbb{I}^{\tau}_{\mathcal{S}}$. Next we define a *selection function* for identifying "the best" minimal model.

Definition 10 (Minimal Model Selection) *Given an* `AF` $\tau = \mathbb{F}_{\mathbf{A}}$*, a semantics specification* $\mathcal{S}$*, and an argument* $a \in \mathbf{A}$*; a* **minimal model selection** *is obtained by a* **selection function** $\gamma : \wp(\mathbb{I}^{\tau}_{\mathcal{S}}) \longrightarrow \mathbb{I}^{\tau}_{\mathcal{S}}$ *applied over the set* $\mathcal{M}_{\mathcal{S}}(a, \tau)$ *for selecting some minimal model of* a *in* τ*, where* $\gamma(\mathcal{M}_{\mathcal{S}}(a, \tau)) \in \mathcal{M}_{\mathcal{S}}(a, \tau)$ *is such that for every* $\mathcal{I}_{\mathcal{S}} \in \mathcal{M}_{\mathcal{S}}(a, \tau)$ *it holds* $\gamma(\mathcal{M}_{\mathcal{S}}(a, \tau)) \preccurlyeq_{\gamma} \mathcal{I}_{\mathcal{S}}$*, where* $\preccurlyeq_{\gamma}$ *is a* **selection criterion** *by which it is possible to select the best representative minimal model.*

The selection criterion can be any method for ordering sets of arguments which takes in consideration any possible *perspective of minimal change*. Probably, the simplest perspective is to prefer the models of smaller contra-domain in order to remove as less arguments as possible (for instance, in Ex. 5, $\mathcal{I}^2_{\mathcal{S}}$ should be preferred over $\mathcal{I}^1_{\mathcal{S}}$), however, the criterion should look deeper into the set for deciding among several models with contra-domains of identical cardinality. A different perspective of minimal change could be to prefer those minimal models whose proposed evolutive step removes as less as possible conflicts between pairs of arguments, thus looking for a minimal change regarding the morphology of the graph of arguments. But probably, the most powerful and distinctive advantage of relying upon contra-semantics, for analyzing and selecting minimal models, is that we can study the impact of change directly over the resulting acceptable set. Not only for deciding to reduce as less as possible the acceptable set, but also for making a selective change operation which could valuate the positive acceptability of certain arguments more than others, or even for analyzing the classification of models in order to keep as controlled as possible the number of dynamic models, and to avoid reducing the number of static models while keeping as low as possible the cardinality of contra-models. Such a discussion deserves to be deepened with more space, and is part of the ongoing work about this theory. An *acceptance revision* will incorporate an external argument a to the AF ensuring the positive acceptability of a by referring to a minimal model selection.

Definition 11 (Acceptance Revision) *Given an* AF τ*, a semantics specification* $\mathcal{S}$*, and an external argument* $a \in \mathbb{A}_{\mathcal{L}}$*; the operator* $\circledast$ *stands for an* **acceptance revision** *iff* $\tau \circledast a = \mathbb{F}_{\Delta^{\mathcal{I}}}$*, where* $\Delta^{\mathcal{I}}$ *is the interpretation domain of the selected minimal model* $\mathcal{I}_{\mathcal{S}} = \gamma(\mathcal{M}_{\mathcal{S}}(a, \tau + a))$*. When necessary we will write* $\tau \circledast_{\gamma} a$ *to identify the minimal model selection* γ *by which the revision* $\tau \circledast a$ *is obtained.*

The axiomatization of the acceptance revision is achieved by analyzing the different characters of revisions from classical belief revision [2,10] and from ATC revision [12], for adapting the classical postulates to argumentation. For space reasons, we will not discuss the intuitions motivating each postulate. For a detailed discussion on this matter, the interested reader may refer to [11,12].

(closure) if $\mathbf{A}(\tau) = \mathbb{C}(\mathbf{A}(\tau))$ then $\mathbf{A}(\tau \circledast a) = \mathbb{C}(\mathbf{A}(\tau \circledast a))$
(success) a is $\mathcal{S}$-accepted in $\tau \circledast a$
(consistency) $\mathcal{A}_{\mathcal{S}}(\tau \circledast a)$ is conflict-free
(inclusion) $\mathbf{A}(\tau \circledast a) \subseteq \mathbf{A}(\tau + a)$
(vacuity) If a is $\mathcal{S}$-accepted in $\tau + a$ then $\mathbf{A}(\tau + a) \subseteq \mathbf{A}(\tau \circledast a)$
(core-retainment) If $b \in \mathbf{A}(\tau) \setminus \mathbf{A}(\tau \circledast a)$ then exists an AF τ' such that $\mathbf{A}(\tau') \subseteq \mathbf{A}(\tau)$ and a is $\mathcal{S}$-accepted in $\tau' + a$ but $\mathcal{S}$-rejected in $(\tau' + b) + a$
(uniformity) if $a \equiv b$ then $\mathbf{A}(\tau) \cap \mathbf{A}(\tau \circledast a) = \mathbf{A}(\tau) \cap \mathbf{A}(\tau \circledast b)$

The uniformity postulate makes reference to an *equivalence relation* "$\equiv$" for arguments (see [12]) to ensure that the revisions $\tau \circledast a$ and $\tau \circledast b$ have equivalent outcomes when arguments a and b are equivalent. For any pair of arguments $a, b \in \mathbb{A}_{\mathcal{L}}$, we say that a and b are *equivalent arguments*, noted as $a \equiv b$ *iff* $\mathfrak{cl}(a) \models \mathfrak{cl}(b)$ and $\mathfrak{cl}(b) \models \mathfrak{cl}(a)$ and for any $a' \sqsubset a$ there is $b' \sqsubset b$ such that $a' \equiv b'$. Inspired by smooth incisions in Hansson's Kernel Contractions [10], we introduce an additional condition on minimal models selection functions for guaranteeing uniformity. Under the consideration of two equivalent arguments a and b, the idea is to ensure that the selection function will trigger one minimal model for each argument (a and b) whose interpretation domains are identical except for the presence of a or b in each corresponding case. Note that we refer to the expansion closure operator $\mathbb{P}$ for looking at the common base of each interpretation domain.

Definition 12 (Smooth Minimal Model Selection) *Given an* AF τ *and two external arguments* $a, b \in \mathbb{A}_{\mathcal{L}}$*. If* $a \equiv b$ *then* $\Delta^{\mathcal{I}^a} \setminus \mathbb{P}(\{a\}) = \Delta^{\mathcal{I}^b} \setminus \mathbb{P}(\{b\})$*, where* $\mathcal{I}^a_{\mathcal{S}} = \gamma(\mathcal{M}_{\mathcal{S}}(a, \tau + a))$ *and* $\mathcal{I}^b_{\mathcal{S}} = \gamma(\mathcal{M}_{\mathcal{S}}(b, \tau + b))$*.*

An operation $\tau \circledast_{\gamma} a$ is a *smooth acceptance revision iff* $\tau \circledast_{\gamma} a$ is an acceptance revision obtained through a smooth minimal model selection 'γ'.

Representation Theorem 1 *Given an* AF τ*, a semantics specification* $\mathcal{S}$*, and an external argument* $a \in \mathbb{A}_{\mathcal{L}}$*;* $\tau \circledast a$ *is a smooth acceptance revision iff '*$\circledast$*' satisfies closure, success, consistency, inclusion, vacuity, core-retainment, and uniformity.*

6. Related Work & Conclusions

A revision approach in an AGM spirit is presented in [6] through revision formulae that express how the acceptability of some arguments should be changed. As a result, they derive argumentation systems which satisfy the given revision formula, and are such that the corresponding extensions are as close as possible to the extensions of the input system. The revision presented is divided in two subsequent levels: firstly, revising the extensions produced by the standard semantics. This is done without considering the attack relation. Secondly, the generation of argumentation systems fulfilling the outcome delivered by the first level. Minimal change is pursued in two different levels, firstly, by ensuring as less change as possible regarding the arguments contained in each extension, and secondly, procuring as less change as possible on the argumentation graph. The methods they provide do not provoke change upon the set of arguments, but only upon the attack relations. Their operator is more related to a distance based-revision which measures the differences from the actual extensions with respect to the ones obtained for verifying the revision formula. They give a basic set of rationality postulates in the very spirit of AGM, but closer to the perspective given in [9]. They only show that the model presented satisfies the postulates without giving the complete representation theorem for which the way back of the proof, *i.e.*, from postulates to the construction, is missing. However, the very recent work [7], which is in general a refinement of [6] and [5], proposes a generic solution to the revision of argumentation frameworks by relying upon complete representation theorems. In addition, the revision from the perspective of argumentation frameworks is also considered. A different approach, but still in an AGM spirit was presented in [3], where authors propose expansion and revision operators for Dung's abstract argumentation frameworks (AFs) based on a novel proposal called *Dung logics* with the particularity that equivalence in such logics coincides with strong equivalence for the respective argumentation semantics. The approach presents a reformulation of the AGM postulates in terms of monotonic consequence relations for AF*s*. They finally state that standard approaches based on measuring distance between models are not appropriate for AF*s*.

The aforementioned works differ from ours in the perspective of dealing with the argumentation dynamics. This also renders different directions to follow for achieving rationality. To our knowledge, [12] was the first work to propose AGM postulates for rationalizing argumentation dynamics, providing also complete representation theorems for the proposed revision operations built upon logic-based argumentation. The rationalization done here is mainly inspired by such results.

The main objective of the *dynamics contra-semantics* is to bring a new theoretical structure conceived from scratch to deal with acceptability dynamics of arguments. The expected virtue of this theory is to ease the proposal and rationality analysis of new models of argumentative change. We believe that it could be simpler to show that the outcome of a "rational" change operator coincides with an interpretation model than showing the complete rationality through a representation theorem. If this hypothesis is true, the full rationality of new change operators could be achieved by means of the representation theorem here presented. In this sense, the intuitions behind the notions of core and remainder sets exceed the scope of the standard argumentation semantics. Their constructions can be redefined for being applied over other kind of frameworks like abstract and dialectical argumentation. For instance, the concept of remainders could match well as a generalization of the idea proposed in ATC [13,12] about selectable con-arguments from a set of attacking lines in a dialectical tree [14] (argumentation lines whose parity interfere with the possibility of acceptance of the root argument). The study of the dynamics contra-semantics upon dialectical argumentation seems to be possible also, given that the reference to standard argumentation semantics in this work has been parametrized, thus allowing the modeling of marking criteria for trees of arguments. The intention would be to bring a formal methodology for studying acceptability dynamics upon an argumentation which fits better for reasoning about a main issue in dispute, *i.e.*, a root argument, as done in dialogues and legal reasoning. This is part of the ongoing work.

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