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On the Acceptability Semantics of Argumentation Frameworks with Recursive Attack and Support

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Abstract. The Attack-Support Argumentation Framework (ASAF) is an abstract argumentation framework that provides a unified setting for representing attack and support for arguments, as well as attack and support for the attack and support relations at any level. Currently, the extensions of the ASAF are obtained by translating it into a Dung's Argumentation Framework (AF). In this work we provide the ASAF with the ability of determining its extensions without requiring such a translation. We follow an extension-based approach for characterizing the acceptability semantics directly on the ASAF, considering the complete, preferred, stable and grounded semantics. Finally, we show that the proposed characterization satisfies different results from Dung's argumentation theory.

Keywords. abstract argumentation, bipolar argumentation, recursive interactions, acceptability semantics.

1. Introduction

The study of Abstract Argumentation Frameworks (AFs) has proved to be of great interest within the argumentation community since they allow to explore different properties on arguments and their relationships, as well as providing various characterizations for their acceptability status [8,15]. Whereas Dung's AFs only account for the existence of an attack relation between arguments, in the last decade, several proposals have been developed in order to enrich such AFs with a positive interaction between arguments: a support relation. A first line of work on such AFs, called Bipolar Argumentation Frameworks (BAFs) in [3], introduced a general support relation between arguments and proposed a series of complex attacks [4] enforcing acceptability constraints derived from the coexistence of attacks and supports. Later, alternative interpretations for the notion of support were proposed, the most well-known being evidential support [12], deductive support [16] and necessary support [11].

Starting from [4] and [6], where different interpretations of support are compared and discussed, the interest in studying AFs with support relations has greatly increased. Furthermore, recent works have focused on a deeper study of the necessary support re-

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lation (see [10,14,13,5]). For instance, in [14] the author gives an instantiation of necessary support in ASPIC+ using sub-arguments; and in [5] an axiomatization of necessary support is proposed through different frameworks.

Another line of work extending AFs that has gained attention amongst the researchers regards the consideration of high-order interactions. Motivated by [9], where second-order attacks are used for representing preferences between arguments, in [1] the authors proposed an AF with recursive attacks (AFRA). Moreover, in [16], the authors allow the attack and support relations of an AF to be attacked in order to model their defeasible nature. Further research on this area combined the above results by characterizing the *Attack-Support Argumentation Framework* (ASAF) [7], an AF that allows for attacks and supports between arguments, as well as attacks and supports from an argument to the attack and support relations, at any level.

A key feature of any argumentation system consists in determining the conditions under which the arguments are accepted, after accounting for their interactions [8,2]. A criticism on [7] is that such conditions are not specified directly on the ASAF; instead, the collectively acceptable sets of arguments are obtained by translating the ASAF into a Dung's AF. In this work we will provide the means for characterizing the acceptability semantics of the ASAF, hence addressing the above mentioned criticism. Since attacks and supports in an ASAF may be affected by other interactions, we will have to account for the conditions under which these attacks and supports are considered as accepted. Moreover, we will show that the characterization of the semantics proposed here satisfies properties given in [8] for Dung's argumentation theory.

The rest of this paper is organized as follows. Section 2 introduces some background notions, including definitions from Dung's theory [8] and the definition of the ASAF [7]. Then, Section 3 identifies conflicts between the elements of the ASAF, leading to the characterization of different kinds of defeat. Given those defeats, Section 4 starts by adapting Dung's basic semantic notions to then characterize the acceptability semantics of the ASAF. Finally, Section 5 discusses related work, presents some conclusions and comments on future lines of research.

2. Background

In this section we include the background required for characterizing the acceptability semantics of the ASAF. We first present some basic notions related to Dung's AFs [8] and then, the definition of the ASAF provided in [7].

The Abstract Argumentation Framework defined in [8] consists of a set of arguments and a set of conflicts between them:

Definition 1 (AF)**.** *An* abstract argumentation framework (AF) *is a pair* $\langle \mathbb{A}, \mathbb{R} \rangle$*, where* $\mathbb{A}$ *is finite and non-empty set of arguments and* $\mathbb{R} \subseteq \mathbb{A} \times \mathbb{A}$ *is an attack relation.*

Given an AF, [8] defines a series of semantic notions, leading to the characterization of collectively acceptable sets of arguments.

Definition 2 (Conflict-freeness, acceptability, admissibility)**.** *Let* $AF = \langle \mathbb{A}, \mathbb{R} \rangle$ *and* $\mathbf{S} \subseteq \mathbb{A}$. $\mathbf{S}$ *is* conflict-free *if* $\nexists \mathscr{A}, \mathscr{B} \in \mathbf{S}$ *s.t.* $(\mathscr{A}, \mathscr{B}) \in \mathbb{R}$. $\mathscr{A} \in \mathbb{A}$ *is* acceptable w.r.t. $\mathbf{S}$ *if* $\forall \mathscr{B} \in \mathbb{A}$ *s.t.* $(\mathscr{B}, \mathscr{A}) \in \mathbb{R}$, $\exists \mathscr{C} \in \mathbf{S}$ *s.t.* $(\mathscr{C}, \mathscr{B}) \in \mathbb{R}$. $\mathbf{S}$ *is* admissible *if it is conflict-free and* $\forall \mathscr{A} \in \mathbf{S}$*:* $\mathscr{A}$ *is acceptable w.r.t.* $\mathbf{S}$.

Then, by adding restrictions to the notion of admissibility, the complete, preferred, stable and grounded extensions of an AF are defined as follows:

Definition 3 (AF Extensions)**.** *Let* $AF = \langle \mathbb{A}, \mathbb{R} \rangle$ *and* $\mathbf{S} \subseteq \mathbb{A}$. $\mathbf{S}$ *is a* complete extension *of AF iff it is admissible and* $\forall \mathscr{A} \in \mathbb{A}$*: if* $\mathscr{A}$ *is acceptable w.r.t.* $\mathbf{S}$*, then* $\mathscr{A} \in \mathbf{S}$. $\mathbf{S}$ *is a* preferred extension *of AF iff it is a maximal* $($*w.r.t.* $\subseteq)$ *admissible set of AF.* $\mathbf{S}$ *is a* stable extension *of AF iff it is conflict-free and* $\forall \mathscr{A} \in \mathbb{A} \backslash \mathbf{S}$*:* $\exists \mathscr{B} \in \mathbf{S}$ *s.t.* $(\mathscr{B}, \mathscr{A}) \in \mathbb{R}$. $\mathbf{S}$ *is the* grounded extension *of AF iff it is the smallest* $($*w.r.t.* $\subseteq)$ *complete extension of AF.*

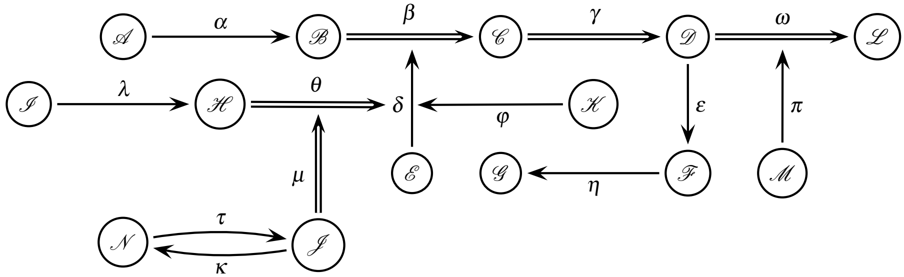
Next, we include the definition of the ASAF given in [7], corresponding to an AF with recursive attack and support relations.

Definition 4 (ASAF)**.** *An* Attack-Support Argumentation Framwork (ASAF) *is a tuple* $\langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ *where* $\mathbb{A}$ *is a set of arguments,* $\mathbb{R} \subseteq \mathbb{A} \times (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ *is an attack relation and* $\mathbb{S} \subseteq \mathbb{A} \times (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ *is a support relation. We assume that* $\mathbb{S}$ *is acyclic and* $\mathbb{R} \cap \mathbb{S} = \emptyset$*.*

The support relation of the ASAF follows the necessity interpretation of the Argumentation Framework with Necessities (AFN) [11], where if $\mathscr{A}$ supports $\mathscr{B}$ then it means that the acceptance of $\mathscr{A}$ is necessary to get the acceptance of $\mathscr{B}$; in other words, the acceptance of $\mathscr{B}$ implies the acceptance of $\mathscr{A}$ or, equivalently, the non-acceptance of $\mathscr{A}$ implies the non-acceptance of $\mathscr{B}$. As a result, the attack relation of the ASAF not only extends the attack relation of the AFRA [1] by allowing for attacks to the support relation, but it also extends the attack and support relations of the AFN [11] by allowing for recursive attacks and supports, as well as attacks to the support relation and vice-versa.

Given an attack or a support $\alpha = (\mathscr{A}, X) \in (\mathbb{R} \cup \mathbb{S})$, $\mathscr{A}$ is called the source of α and X is called the target of α, and they can be referred to as $\mathsf{src}(\alpha)$ and $\mathsf{trg}(\alpha)$, respectively. Moreover, an ASAF can be graphically represented using a graph-like notation: an argument $\mathscr{A} \in \mathbb{A}$ will be denoted as a node in the graph, an attack $\alpha = (\mathscr{A}, X) \in \mathbb{R}$ will be denoted as $\mathscr{A} \xrightarrow{\alpha} X$, and a support $\beta = (\mathscr{B}, Y) \in \mathbb{S}$ will be denoted as $\mathscr{B} \overset{\beta}{\Longrightarrow} Y$. To simplify the notation, the attack from an argument $\mathscr{C}$ to an attack or a support $\alpha = (\mathscr{A}, X)$ will be referred to as $(\mathscr{C}, \alpha)$. Similarly, the support from an argument $\mathscr{D}$ to an attack or a support $\beta = (\mathscr{B}, Y)$ will be referred to as $(\mathscr{D}, \beta)$. Since, as mentioned before, the attack and support relations of an ASAF are assumed to be disjoint, a pair $\gamma = (\mathscr{E}, Z)$ in the attack relation or the support relation will be unequivocally identified by γ. Thus, when referring to γ, it will be possible to identify the attack or support it represents. To illustrate this, let us consider the following example.

Example 1. *Let us consider the ASAF* Δ_1 *with the following graphical representation:*



We have the first-level attacks $\alpha = (\mathscr{A},\mathscr{B})$, $\varepsilon = (\mathscr{D},\mathscr{F})$, $\eta = (\mathscr{F},\mathscr{G})$, $\lambda = (\mathscr{I},\mathscr{H})$, $\tau = (\mathscr{N},\mathscr{J})$ and $\kappa = (\mathscr{J},\mathscr{N})$. The first-level supports are $\beta = (\mathscr{B},\mathscr{C})$, $\gamma = (\mathscr{C},\mathscr{D})$ and $\omega = (\mathscr{D},\mathscr{L})$. The second-level interactions are the attacks $\delta = (\mathscr{E},\beta)$ and $\pi = (\mathscr{M},\omega)$. Then, we have the third-level attack and support on δ: respectively, $\varphi = (\mathscr{K},\delta)$ and $\theta = (\mathscr{K},\delta)$. Finally, the only fourth-level interaction is the support $\mu = (\mathscr{J},\theta)$.

3. Defeats in the ASAF

Before characterizing the acceptability semantics of the ASAF we need to clearly identify all the conflicts between its elements, in addition to those already expressed in the attack relation. The set of all conflicts between the elements of the ASAF will be called the set of *defeats*, in order to distinguish them from the original attacks. In particular, similarly to [1], we consider a notion of defeat which regards attacks, rather than their source arguments, as the subjects able to defeat arguments, attacks or supports.

In the following we will distinguish between two types of defeats: those that can be inferred directly by looking at the attack relation of the ASAF, and those that are conditioned by the existence of supports. The former will be referred to as *unconditional defeats*, and are defined in Section 3.1, whereas the latter are the *conditional defeats*, defined in Section 3.2.

3.1. Unconditional Defeats

The first case of unconditional defeats corresponds to conflicts already captured by the attack relation of the ASAF, which we call *direct defeats*.

Definition 5 (Direct Defeat)**.** *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF, $\alpha \in \mathbb{R}$ and $X \in (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$. We say that α* directly defeats *X, noted α d-def X, iff $\mathsf{trg}(\alpha) = X$.*

The other kind of defeat that may be inferred directly from the attack relation of the ASAF is the *indirect defeat*, which captures the intuition that attacks are strictly related to their source, as in the AFRA [1].

Definition 6 (Indirect Defeat)**.** *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF and $\alpha, \beta \in \mathbb{R}$. We say that α* indirectly defeats *β, noted α i-def β, iff α d-def $\mathsf{src}(\beta)$.*

These two kinds of unconditional defeat are grouped together in the following definition and illustrated below.

Definition 7 (Unconditional Defeat)**.** *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF, $\alpha \in \mathbb{R}$ and $X \in (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$. We say that α* unconditionally defeats *X, noted α u-def X, iff α d-def X or α i-def X.*

Example 2. *Given the ASAF Δ_1 from Ex. 1, the following unconditional defeats occur. The direct defeats: α d-def $\mathscr{B}$, ε d-def $\mathscr{F}$, η d-def $\mathscr{G}$, λ d-def $\mathscr{H}$, τ d-def $\mathscr{J}$, κ d-def $\mathscr{N}$, δ d-def β, φ d-def δ, π d-def ω; and the indirect defeats: ε i-def η, τ i-def κ, κ i-def τ.*

3.2. Conditional Defeats

As mentioned before, the coexistence of attacks and supports may lead to having additional conflicts between the elements of the ASAF. These conflicts will be identified as *conditional defeats* since, unlike the defeats defined in Section 3.1, their existence depends on the consideration of the support relation of the ASAF. Following the necessary interpretation of support, such conflicts are handled in [11] by characterizing the notion of *extended attack*, which reinforces the acceptability constraints presented in Section 2: given an attack $\mathscr{A} \longrightarrow \mathscr{B}$ and a sequence of necessary supports $\mathscr{B} \Longrightarrow \ldots \Longrightarrow \mathscr{C}$, there is an extended attack from $\mathscr{A}$ to $\mathscr{C}$.

The intuitions presented above are captured in the ASAF by defining the notion of *extended defeat*. In particular, we will distinguish the *support sequence* involved in this kind of defeat, and the corresponding supports will be referred to as the *support set*.

Definition 8 (Support Sequence and Support Set)**.** *Let* $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ *be an ASAF and* $X \in (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$. *We say that* $\Sigma = [\mathscr{A}_1, \ldots, \mathscr{A}_n]$ *is a* support sequence for X $(n \geq 2)$ *iff* $\mathscr{A}_n = X$ *and for every* $\mathscr{A}_i$ $(1 \leq i \leq n-1)$ *it holds that* $(\mathscr{A}_i, \mathscr{A}_{i+1}) \in \mathbb{S}$. *We define the* support set of Σ *as* $\mathbf{S} = \bigcup_{i=1}^{n-1} S_i$, *with* $S_i = (\mathscr{A}_i, \mathscr{A}_{i+1})$.

Definition 9 (Extended Defeat)**.** *Let* $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ *be an ASAF,* $\alpha \in \mathbb{R}$, $X \in (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ *and* $\mathbf{S} \subseteq \mathbb{S}$. *We say that* α extendedly defeats X given $\mathbf{S}$, *noted* α *e-def* X *given* $\mathbf{S}$, *if exists a support sequence* $\Sigma = [\mathscr{A}_1, \ldots, X]$ *for* X *s.t.* $\mathsf{trg}(\alpha) = \mathscr{A}_1$ *and* $\mathbf{S}$ *is the support set of* Σ.

Extended defeats in the ASAF are illustrated by the following example.

Example 3. *Let* Δ_1 *be the ASAF from Ex. 1. Then, we have the following extended defeats:* α *e-def* $\mathscr{C}$ *given* $\{\beta\}$, α *e-def* $\mathscr{D}$ *given* $\{\beta, \gamma\}$, α *e-def* $\mathscr{L}$ *given* $\{\beta, \gamma, \omega\}$, λ *e-def* δ *given* $\{\theta\}$, *and* τ *e-def* θ *given* $\{\mu\}$.

It can be noted that Def. 9 explicitly identifies the support sequence originating the extended defeat. Therefore, as shown by the following Proposition, adding a support link to a support sequence results in a new extended defeat.

Proposition 1. *Let* $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ *be an ASAF,* $\alpha \in \mathbb{R}$, $\beta \in \mathbb{S}$ *and* $\mathbf{S} \subseteq \mathbb{S}$. *If* α *e-def* $\mathsf{src}(\beta)$ *given* $\mathbf{S}$, *then* α *e-def* $\mathsf{trg}(\beta)$ *given* $\mathbf{S} \cup \{\beta\}$.

Proof. If α e-def $\mathsf{src}(\beta)$ given $\mathbf{S}$, then, by Def. 9, there exists a support sequence $\Sigma = [\mathscr{A}_1, \ldots, \mathsf{src}(\beta)]$ for $\mathsf{src}(\beta)$ s.t. $\mathbf{S}$ is the support set of Σ. Since by hyp. $\beta = (\mathsf{src}(\beta), \mathsf{trg}(\beta)) \in \mathbb{S}$, by Def. 8, $\Sigma' = [\mathscr{A}_1, \ldots, \mathsf{src}(\beta), \mathsf{trg}(\beta)]$ is a support sequence for $\mathsf{trg}(\beta)$ and $\mathbf{S} \cup \{\beta\}$ is the support set of Σ'. Thus, by Def. 9, α e-def $\mathsf{trg}(\beta)$ given $\mathbf{S} \cup \{\beta\}$. □

Given that the ASAF combines intuitions and results from the AFRA [1] and the AFN [11], its is reasonable to combine the intuitions behind the notions of indirect defeat and extended defeat to identify additional conflicts between the elements of the ASAF. In other words, similarly to the indirect defeat, we define the notion of *extended-indirect defeat* where an extended defeat on an argument is propagated to the attacks it originates. This kind of defeat is also conditional since it relies on the existence of an extended defeat, hence on the existence of supports.

Definition 10 (Extended-Indirect Defeat). *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF, $\alpha, \beta \in \mathbb{R}$ and $\mathbf{S} \subseteq \mathbb{S}$. We say that α* extended-indirectly defeats *β* given **S**, *noted α ei-def β given **S**, iff α e-def $\mathsf{src}(\beta)$ given **S**.*

This is illustrated by the following example.

Example 4. *Given the ASAF Δ_1 from Ex. 1, the only extended-indirect defeat is α ei-def ε given $\{\beta, \gamma\}$. This is because, as shown in Ex. 3, α e-def $\mathscr{D}$ given $\{\beta, \gamma\}$ and, as it can be observed in Ex. 1, $\mathscr{D} = \mathsf{src}(\varepsilon)$.*

Then, similarly to the case of unconditional defeats, the extended and extended-indirect defeats are grouped together in the following definition.

Definition 11 (Conditional Defeat). *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF, $\alpha \in \mathbb{R}$, $X \in (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ and $\mathbf{S} \subseteq \mathbb{S}$. We say that α* conditionally defeats *X* given **S**, *noted α c-def X given **S**, iff α e-def X given **S** or α ei-def X given **S**.*

4. Acceptability Semantics of the ASAF

Having identified the situations in which defeats between the elements of the ASAF occur, in this section we will characterize the acceptability semantics of the ASAF following an extension-based approach. In particular, as stated in [7], the extensions of the ASAF may not only include arguments, but also attacks and supports. This is to reflect the fact that attacks and supports may be affected by other interactions and thus, the presence of an attack or a support in an extension of the ASAF will imply that it is "active".

Following the methodology of [8], in Section 4.1 we will first define some basic semantic notions for the ASAF. In particular, we will show that the notion of acceptability complies with the constraints imposed by the attack and support relations of the ASAF. Moreover, we will show that results from [8] regarding the notions of acceptability and admissibility also hold for the ASAF. Then, in Section 4.2, we will define the acceptability semantics of the ASAF by characterizing its complete, preferred, stable and grounded extensions. Furthermore, we will show that the ASAF satisfies the relationships between the complete, preferred, stable and grounded extensions given in [8].

4.1. Semantic Notions

Analogously to [8], the notion of conflict-freeness establishes the minimum requirements a set of elements of the ASAF should satisfy in order to be collectively accepted.

Definition 12 (Conflict-Freeness). *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF and $\mathbf{S} \subseteq (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$. We say that **S*** is conflict-free *iff:*

- $\nexists \alpha, X \in \mathbf{S}$ *s.t. α u-def X; and*
- $\nexists \beta, Y \in \mathbf{S}$, $\nexists \mathbf{S}' \subseteq \mathbf{S}$ *s.t. β c-def Y given $\mathbf{S}'$.*

Example 5. *Let Δ_1 be the ASAF from Ex. 1. Some conflict-free sets of Δ_1 are: $\emptyset$, $\{\mathscr{M}, \omega\}$, $\{\mathscr{N}, \mathscr{J}\}$, $\{\lambda, \delta\}$, $\{\mu, \mathscr{E}, \delta\}$, $\{\alpha, \beta, \varepsilon\}$, $\{\mathscr{A}, \mathscr{B}, \mathscr{C}, \mathscr{D}, \mathscr{E}, \mathscr{F}, \mathscr{G}, \mathscr{H}, \mathscr{I}, \mathscr{J}, \mathscr{K}, \mathscr{L}, \mathscr{M}, \mathscr{N}, \beta, \gamma, \omega, \theta, \mu\}$ and $\{\mathscr{A}, \alpha, \gamma, \mathscr{M}, \pi, \mathscr{L}, \mathscr{I}, \lambda, \mathscr{K}, \varphi, \beta, \mathscr{F}, \eta, \mathscr{E}, \mu\}$. In contrast, the sets $\{\alpha, \mathscr{B}\}$, $\{\lambda, \theta, \delta\}$, $\{\pi, \omega\}$ and $\{\tau, \kappa\}$, among others, are not conflict-free.*

As expressed in Def. 12, if a set $\mathbf{S}$ includes all the elements required for the existence of a defeat in the ASAF, then $\mathbf{S}$ will not be conflict-free. This implies that, in particular, any set of elements of an ASAF which does not include an attack will be conflict-free. This is the case of the set $\{\mathscr{A},\mathscr{B},\mathscr{C},\mathscr{D},\mathscr{E},\mathscr{F},\mathscr{G},\mathscr{H},\mathscr{I},\mathscr{J},\mathscr{K},\mathscr{L},\mathscr{M},\mathscr{N},\beta,\gamma,\omega,\theta,\mu\}$ illustrated in Ex. 5, which includes every argument and support of the ASAF Δ_1 but none of its attacks. Moreover, when considering conditional defeats, all the elements required for the existence of a defeat must be included in a non-conflict-free set. Hence, if one of the supports in the corresponding support sequence is missing, the resulting set is conflict-free. This situation is illustrated by the conflict-free sets $\{\lambda,\delta\}$ and $\{\alpha,\beta,\varepsilon\}$ in Ex. 5.

Then, we define the notion of acceptability in the context of an ASAF, which characterizes the defense by a set of arguments, attacks and supports against the occurrence of defeats on its elements. Hence, since the ASAF allows for unconditional and conditional defeats, we need to consider all the defeats that may occur, as well as the different ways for providing defense against them.

Definition 13 (Acceptability). *Let* $\Delta = \langle \mathbb{A},\mathbb{R},\mathbb{S}\rangle$ *be an ASAF,* $X \in (\mathbb{A}\cup\mathbb{R}\cup\mathbb{S})$ *and* $\mathbf{S} \subseteq (\mathbb{A}\cup\mathbb{R}\cup\mathbb{S})$. *We say that* X is acceptable w.r.t. $\mathbf{S}$ *iff it holds that:*

1. $\forall\alpha\in\mathbb{R}$ *s.t.* α *u-def* X*, either:*
   (a) $\exists\beta\in\mathbf{S}$ *s.t.* β *u-def* α*; or*
   (b) $\exists\beta\in\mathbf{S}$, $\exists\mathbf{S}'\subseteq\mathbf{S}$ *s.t.* β *c-def* α *given* $\mathbf{S}'$.
2. $\forall\alpha\in\mathbb{R}$, $\forall\mathbf{T}\subseteq\mathbb{S}$ *s.t.* α *c-def* X *given* $\mathbf{T}$*, either:*
   (a) $\exists\beta\in\mathbf{S}$ *s.t.* β *u-def* α*;*
   (b) $\exists\beta\in\mathbf{S}$, $\exists\gamma\in\mathbf{T}$ *s.t.* β *u-def* γ*;*
   (c) $\exists\beta\in\mathbf{S}$, $\exists\mathbf{S}'\subseteq\mathbf{S}$ *s.t.* β *c-def* α *given* $\mathbf{S}'$*; or*
   (d) $\exists\beta\in\mathbf{S}$, $\exists\mathbf{S}'\subseteq\mathbf{S}$, $\exists\gamma\in\mathbf{T}$ *s.t.* β *c-def* γ *given* $\mathbf{S}'$.

As the preceding definition shows, defense against an unconditional defeat may only be achieved by defeating the corresponding attack. On the other hand, a conditional defeat may be repelled by defeating the corresponding attack or one of the supports required by the conditional defeat. In either case, defense can be provided by both unconditional and conditional defeats. Moreover, it should be noted that, although Def. 13 accounts for a set $\mathbf{S}$ of arguments, attacks and supports of an ASAF, the only elements contributing to the defense are the attacks and supports. This is because attacks and supports are ones leading to the existence of defeats (see Defs. 7 and 11). In other words, similarly to the AFRA, defense through an unconditional defeat can only be provided by an attack. In contrast, defense by a conditional defeat is given by an attack and a set of supports. These intuitions are illustrated in the following example.

Example 6. *For instance, given the ASAF* Δ_1 *from Ex. 1,* $\mathscr{A}$ *and* φ *are acceptable w.r.t.* $\emptyset$*,* β *is acceptable w.r.t.* $\{\varphi\}$*,* $\mathscr{N}$ *is acceptable w.r.t.* $\{\tau\}$*,* $\mathscr{D}$ *is acceptable w.r.t.* $\{\delta\}$*,* θ *is acceptable w.r.t.* $\{\kappa\}$*, and* $\mathscr{F}$ *and* η *are acceptable w.r.t.* $\{\alpha,\beta,\gamma\}$*. In contrast, for example,* $\mathscr{B}$ *is not acceptable w.r.t.* $\emptyset$ *and* δ *is not acceptable w.r.t.* $\{\kappa\}$*.*

The following proposition shows that, like in the AFRA, the acceptability of an attack implies the acceptability of its source.

Proposition 2. *Let* $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ *be an ASAF,* $\alpha \in \mathbb{R}$ *and* $\mathbf{S} \subseteq (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$. *If* α *is acceptable w.r.t.* $\mathbf{S}$*, then* $\mathsf{src}(\alpha)$ *is acceptable w.r.t.* $\mathbf{S}$.

Proof. Suppose by contradiction that α is acceptable w.r.t. $\mathbf{S}$ and $A = \mathsf{src}(\alpha)$ is not acceptable w.r.t. $\mathbf{S}$. Then, either (a) $\exists \beta \in \mathbb{R}$ s.t. β u-def A, and $\nexists \gamma \in \mathbf{S}$, $\nexists \mathbf{S}' \subseteq \mathbf{S}$ s.t. γ u-def β or γ c-def β given $\mathbf{S}'$; or (b) $\exists \beta \in \mathbb{R}$, $\exists \mathbf{T} \subseteq \mathbb{S}$ s.t. β c-def A given $\mathbf{T}$, and $\nexists \gamma \in \mathbf{S}$, $\nexists \mathbf{S}' \subseteq \mathbf{S}$, $\nexists \delta \in \mathbf{T}$ s.t. γ u-def β, γ c-def β given $\mathbf{S}'$, γ u-def δ or γ c-def δ given $\mathbf{S}'$.

(a) By Def. 4, it holds that $A = \mathsf{src}(\alpha) \in \mathbb{A}$. Then, if β u-def A, by Defs. 7 and 5, it must be the case that β d-def A. Therefore, by Def. 6, β i-def α.

(b) By Def. 4, it holds that $A = \mathsf{src}(\alpha) \in \mathbb{A}$. Then, if β c-def A given $\mathbf{T}$, by Defs. 11 and 9, β e-def A given $\mathbf{T}$. Therefore, by Def. 10, β ei-def α given $\mathbf{T}$.

Then, by Def. 13, α would not be acceptable w.r.t. $\mathbf{S}$, contradicting the hypothesis. □

The following proposition shows that the notion of acceptability complies with the constraints imposed by the necessary interpretation of support adopted by the ASAF.

Proposition 3. *Let* $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ *be an ASAF,* $\mathbf{S} \subseteq (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ *a conflict-free set and* $\alpha \in \mathbb{S}$ *acceptable w.r.t.* $\mathbf{S}$. *If* $\mathsf{trg}(\alpha)$ *is acceptable w.r.t.* $\mathbf{S}$*, then* $\mathsf{src}(\alpha)$ *is acceptable w.r.t.* $\mathbf{S}$*; equivalently, if* $\mathsf{src}(\alpha)$ *is not acceptable w.r.t.* $\mathbf{S}$*, then* $\mathsf{trg}(\alpha)$ *is not acceptable w.r.t.* $\mathbf{S}$.

Proof. If $A = \mathsf{src}(\alpha)$ is not acceptable w.r.t. $\mathbf{S}$, then it holds that either (a) $\exists \beta \in \mathbb{R}$ s.t. β u-def A, and $\nexists \gamma \in \mathbf{S}$, $\nexists \mathbf{S}' \subseteq \mathbf{S}$ s.t. γ u-def β or γ c-def β given $\mathbf{S}'$; or (b) $\exists \beta \in \mathbb{R}$, $\exists \mathbf{T} \subseteq \mathbb{S}$ s.t. β c-def A given $\mathbf{T}$, and $\nexists \gamma \in \mathbf{S}$, $\nexists \mathbf{S}' \subseteq \mathbf{S}$, $\nexists \delta \in \mathbf{T}$ s.t. γ u-def β, γ c-def β given $\mathbf{S}'$, γ u-def δ or γ c-def δ given $\mathbf{S}'$.

(a) By Def. 4, it holds that $A = \mathsf{src}(\alpha) \in \mathbb{A}$. Then, if β u-def A, by Defs. 7 and 5, it must be the case that β d-def A. Therefore, by Def. 9, β e-def $\mathsf{trg}(\alpha)$ given $\{\alpha\}$.

(b) By Def. 4, it holds that $A = \mathsf{src}(\alpha) \in \mathbb{A}$. Then, if β c-def A given $\mathbf{T}$, by Defs. 11 and 9, it must be the case that β e-def A given $\mathbf{T}$. Therefore, by Prop. 1, β e-def $\mathsf{trg}(\alpha)$ given $\mathbf{T} \cup \{\alpha\}$.

Since by hyp. α is acceptable w.r.t. $\mathbf{S}$ and $\mathbf{S}$ is conflict-free, $\nexists \lambda \in \mathbf{S}$, $\nexists \mathbf{S}'' \subseteq \mathbf{S}$ s.t. λ u-def α or λ c-def α given $\mathbf{S}''$. As a result, by Def. 13, $\mathsf{trg}(\alpha)$ is not acceptable w.r.t. $\mathbf{S}$. □

The following proposition shows that the notion of acceptability is monotonic with respect to set inclusion.

Proposition 4. *Let* $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ *be an ASAF,* $X \in (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ *and* $\mathbf{S} \subseteq (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$. *If* X *is acceptable w.r.t.* $\mathbf{S}$*, then* $\forall \mathbf{S}' \subseteq (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ *s.t.* $\mathbf{S} \subseteq \mathbf{S}'$*:* X *is acceptable w.r.t.* $\mathbf{S}'$.

Proof. Suppose by contradiction that X is acceptable w.r.t. $\mathbf{S}$ and $\exists \mathbf{S}' \subseteq (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ s.t. $\mathbf{S} \subseteq \mathbf{S}'$ and X is not acceptable w.r.t. $\mathbf{S}'$. Then, it holds that either (a) $\exists \alpha \in \mathbb{R}$ s.t. α u-def X and $\nexists \beta \in \mathbf{S}'$, $\nexists \mathbf{S}'' \subseteq \mathbf{S}'$ s.t. β u-def α or β c-def α given $\mathbf{S}''$; or (b) $\exists \alpha \in \mathbb{R}$, $\exists \mathbf{T} \subseteq \mathbb{S}$ s.t. α c-def X given $\mathbf{T}$ and $\nexists \beta \in \mathbf{S}'$, $\nexists \mathbf{S}'' \subseteq \mathbf{S}'$, $\nexists \gamma \in \mathbf{T}$ s.t. β u-def α, β c-def α given $\mathbf{S}''$, β u-def γ or β c-def γ given $\mathbf{S}''$. Thus, since $\mathbf{S} \subseteq \mathbf{S}'$, by Def. 13, X would not be acceptable w.r.t. $\mathbf{S}$, contradicting the hypothesis. □

Next, like in [8], admissible sets of the ASAF are defined by combining the notions of conflict-freeness and acceptability.

Definition 14 (Admissibility)**.** *Let* $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ *be an ASAF and* $\mathbf{S} \subseteq (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$*. We say that* $\mathbf{S}$ is admissible *iff it is conflict-free and* $\forall X \in \mathbf{S}$*: X is acceptable w.r.t.* $\mathbf{S}$.

Example 7. *Let* Δ_1 *be the ASAF from Ex. 1. Some admissible sets of* Δ_1 *are* $\emptyset$*,* $\{\alpha, \beta, \gamma, \varphi, \mathscr{F}, \mathscr{M}\}$ *and* $\{\mathscr{A}, \alpha, \gamma, \mathscr{M}, \pi, \mathscr{L}, \mathscr{I}, \lambda, \mathscr{K}, \varphi, \beta, \mathscr{F}, \eta, \mathscr{E}, \mu, \tau, \mathscr{N}\}$*. In contrast, for instance, the sets* $\{\beta, \theta, \lambda, \mathscr{J}, \kappa\}$ *and* $\{\varepsilon, \mathscr{G}\}$ *are not admissible; the former because* β *is not defended against the direct defeat by* δ*, whereas the latter because* ε *is not defended against the extended-indirect defeat by* α*.*

The following proposition shows that the notions of acceptability and admissibility allow for Dung's fundamental lemma to hold in the context of an ASAF.

Lemma 1. *Let* $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ *be an ASAF,* $\mathbf{S} \subseteq (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ *an admissible set of* Δ*, and* $X, Y \in (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ *s.t. X and Y are acceptable w.r.t.* $\mathbf{S}$*. Then, it holds that* (1) $\mathbf{S}' = \mathbf{S} \cup \{X\}$ *is admissible, and* (2) *Y is acceptable w.r.t.* $\mathbf{S}'$*.*

Proof.

1. To prove that $\mathbf{S}'$ is admissible we have to prove that X is acceptable w.r.t. $\mathbf{S}'$ and $\mathbf{S}'$ is conflict-free. Since $\mathbf{S} \subseteq \mathbf{S}'$ and, by hypothesis, X is acceptable w.r.t. $\mathbf{S}$, by Prop. 4, X is acceptable w.r.t. $\mathbf{S}'$. Now, suppose by contradiction that $\mathbf{S}'$ is not conflict-free. Then, since by hypothesis $\mathbf{S}$ is admissible, it must be the case that $\exists W, Z \in \mathbf{S}$, $\exists \mathbf{T} \subseteq \mathbf{S}$ s.t. either (a) X u-def W; (b) W u-def X; (c) X c-def W given $\mathbf{T}$; (d) W c-def X given $\mathbf{T}$; or (e) W c-def Z given $\mathbf{T} \cup \{X\}$.
   - (a) If X u-def W, since by hypothesis $\mathbf{S}$ is admissible, it must be the case that $\exists \alpha \in \mathbf{S}$, $\exists \mathbf{S_1} \subseteq \mathbf{S}$ s.t. α u-def X or α c-def X given $\mathbf{S_1}$. Furthermore, since by hypothesis X is acceptable w.r.t. $\mathbf{S}$, it must be the case that $\exists \beta \in \mathbf{S}$, $\exists \mathbf{S_2} \subseteq \mathbf{S}$, $\exists \gamma \in \mathbf{S_1}$ s.t. β u-def α, β c-def α given $\mathbf{S_2}$, β u-def γ, or β c-def γ given $\mathbf{S_2}$. As a result, the set $\mathbf{S}$ would not be conflict-free, contradicting the hypothesis that $\mathbf{S}$ is admissible.
   - (b) If W u-def X, since by hypothesis X is acceptable w.r.t. $\mathbf{S}$, then $\exists \alpha \in \mathbf{S}$, $\exists \mathbf{S_1} \subseteq \mathbf{S}$ s.t. α u-def W or α c-def W given $\mathbf{S_1}$. As a result, in each case, the set $\mathbf{S}$ would not be conflict-free, contradicting the hypothesis that $\mathbf{S}$ is admissible.
   - (c) If X c-def W given $\mathbf{T}$, since by hypothesis $\mathbf{S}$ is admissible, it must be the case that $\exists \alpha \in \mathbf{S}$, $\exists \mathbf{S_1} \subseteq \mathbf{S}$, $\exists \gamma \in \mathbf{T}$ s.t. either (i) α u-def X, (ii) α c-def X given $\mathbf{S_1}$, (iii) α u-def γ or (iv) α c-def γ given $\mathbf{S_1}$. Cases $(c.i)$ and $(c.ii)$ are analogous to case (b) and thus, $\mathbf{S}$ would not be conflict-free, contradicting the hypothesis that $\mathbf{S}$ is admissible. In cases $(c.iii)$ and $(c.iv)$, since $\alpha \in \mathbf{S}$, $\gamma \in \mathbf{T} \subseteq \mathbf{S}$ and $\mathbf{S_1} \subseteq \mathbf{S}$, the set $\mathbf{S}$ would not be conflict-free, contradicting the hypothesis that $\mathbf{S}$ is admissible.
   - (d) This case is analogous to case (b) and thus, $\mathbf{S}$ would not be conflict-free, contradicting the hypothesis that $\mathbf{S}$ is admissible.
   - (e) If W c-def Z given $\mathbf{T} \cup \{X\}$, since by hypothesis $\mathbf{S}$ is admissible, then $\exists \alpha \in \mathbf{S}$, $\exists \mathbf{S_1} \subseteq \mathbf{S}$, $\exists \gamma \in \mathbf{T}$ s.t. either (i) α u-def W, (ii) α c-def W given $\mathbf{S_1}$, (iii) α u-def γ, (iv) α c-def γ given $\mathbf{S_1}$, (v) α u-def X or (vi) α c-def X given $\mathbf{S_1}$. Thus, in cases $(e.i)$-$(e.iv)$, the set $\mathbf{S}$ would not be conflict-free, contradicting the hypothesis that $\mathbf{S}$ is admissible. In cases $(e.v)$ and $(e.vi)$, similarly to case (a), since by hypothesis X is acceptable w.r.t. $\mathbf{S}$, it would be the case that $\exists \beta \in \mathbf{S}$, $\exists \mathbf{S_2} \subseteq \mathbf{S}$, $\exists \lambda \in \mathbf{S_1}$ s.t. β u-def α, β c-def α given $\mathbf{S_2}$, β u-def λ or β c-def λ given $\mathbf{S_2}$; in all cases, the set $\mathbf{S}$ would not be conflict-free, contradicting the hypothesis that $\mathbf{S}$ is admissible.
2. Since $\mathbf{S} \subseteq \mathbf{S}'$ and, by hyp., Y is acceptable w.r.t. $\mathbf{S}$, by Prop. 4, Y is acceptable w.r.t. $\mathbf{S}'$.

□

4.2. Extensional Semantics of the ASAF

Starting from the semantic notions defined in Section 4.1, we characterize the complete, preferred, stable and grounded extensions of the ASAF as follows.

Definition 15 (ASAF Extensions). *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF and $\mathbf{S} \subseteq (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$.*

- $\mathbf{S}$ *is a* complete extension *of Δ iff it is admissible and $\forall X \in (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$: if X is acceptable w.r.t. $\mathbf{S}$, then $X \in \mathbf{S}$.*
- $\mathbf{S}$ *is a preferred extension of Δ iff it is a maximal (w.r.t. $\subseteq$) admissible set of Δ.*
- $\mathbf{S}$ *is a stable extension of Δ iff it is conflict-free and $\forall X \in (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S}) \backslash \mathbf{S}$: $\exists \alpha \in \mathbf{S}$, $\exists \mathbf{S}' \subseteq \mathbf{S}$ s.t. α u-def X or α c-def X given $\mathbf{S}'$.*
- $\mathbf{S}$ *is the grounded extension of Δ iff it is the smallest (w.r.t. $\subseteq$) complete extension of Δ.*

Example 8. *Let us consider the ASAF Δ_1 from Ex. 1 and the grounded and preferred semantics. The grounded extension of Δ_1 is $\mathbf{G} = \{\mathscr{A}, \alpha, \gamma, \mathscr{M}, \pi, \mathscr{L}, \mathscr{I}, \lambda, \mathscr{K}, \varphi, \beta, \mathscr{F}, \eta, \mathscr{E}, \mu\}$. Note that although $\mathsf{src}(\mu)$ is involved in an attack cycle that is not resolved when considering the grounded semantics, the support μ is active and thus, $\mu \in \mathbf{G}$. Then, when considering the preferred semantics, there are two alternatives for resolving the attack cycle involving $\mathsf{src}(\mu)$, leading to the existence of two preferred extensions of Δ_1: $\mathbf{P_1} = \mathbf{G} \cup \{\tau, \mathscr{N}\}$ and $\mathbf{P_2} = \mathbf{G} \cup \{\kappa, \mathscr{J}, \theta\}$. In particular, even though $\{\tau, \mu\} \subseteq \mathbf{P_1}$ defends δ against the extended defeat by λ given $\{\theta\}$, $\mathbf{P_1}$ does not defend δ against the direct defeat by φ; therefore, $\delta \notin \mathbf{P_1}$.*

Next, we will show that the ASAF semantics from Def. 15 fulfill the relationships between the corresponding semantics proposed in [8]. The following lemma illustrates the relationship between the preferred and complete extensions of an ASAF.

Lemma 2. *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF. Every preferred extension of Δ is also a complete extension of Δ, but not vice-versa.*

Proof. Suppose that $\exists \mathbf{S} \subseteq (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ s.t. $\mathbf{S}$ is a preferred extension of Δ but not a complete extension of Δ. Then, by Def. 15, it would be the case that $\exists X \in (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ s.t. X is acceptable w.r.t. $\mathbf{S}$ and $X \notin \mathbf{S}$. By Lemma 1, $\mathbf{S} \cup \{X\}$ is admissible. Therefore, $\mathbf{S}$ would not be a maximal admissible set, contradicting the assumption that $\mathbf{S}$ is a preferred extension of Δ. To show that the reverse does not hold let us consider the ASAF $\Delta = \langle \mathbb{A}, \mathbb{R}, \emptyset \rangle$, with $\mathbb{A} = \{\mathscr{A}, \mathscr{B}\}$ and $\mathbb{R} = \{(\mathscr{A}, \mathscr{B}), (\mathscr{B}, \mathscr{A})\}$. By Def. 15, $\emptyset$ is a complete extension of Δ, whereas the only preferred extensions of Δ are $\{\mathscr{A}\}$ and $\{\mathscr{B}\}$. □

Similarly, the following lemma relates the stable and preferred extensions of an ASAF.

Lemma 3. *Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF. Every stable extension of Δ is also a preferred extension of Δ, but not vice-versa.*

Proof. It is clear that every stable extension of Δ is a maximal (w.r.t. $\subseteq$) admissible set of Δ, hence a preferred extension of Δ. To show that the reverse does not hold, let us consider the ASAF $\Delta = \langle \mathbb{A}, \mathbb{R}, \emptyset \rangle$, with $\mathbb{A} = \{\mathscr{A}\}$ and $\mathbb{R} = \{(\mathscr{A}, \mathscr{A})\}$. By Def. 15, $\emptyset$ is a preferred extension of Δ but not a stable extension of Δ. □

Finally, by Def. 15, the grounded extension of an ASAF is also its complete extension.

5. Related Work and Conclusions

In this work we have proposed an approach for characterizing the acceptability semantics of the ASAF introduced in [7]. On the one hand, similarly to [7], we adopted an extension-based approach. On the other hand, differently from [7], we did not make use of a translation into a Dung's AF for obtaining the extensions of the ASAF; instead, we characterized the acceptability semantics directly on the ASAF. This constitutes the main contribution of the paper.

In order to do this, we first identified the different defeats that may occur between the elements of the ASAF. We distinguished between those defeats that can be inferred directly from the attack relation and those that require the consideration of the support relation (respectively, the unconditional and conditional defeats). Therefore, when defining the notion of acceptability, it was necessary to account for all the ways in which defense against a defeat can be provided: either by defeating the corresponding attack or, in the case of conditional defeats, by defeating one of the involved supports. Finally, using the basic notions defined in Section 4.1, a characterization of the acceptability semantics of the ASAF was given in Section 4.2.

Another difference between our approach for obtaining the extensions of the ASAF and the one proposed in [7] regards the presence of supports in the corresponding extensions. For instance, let us consider the ASAF Δ_1 from Ex. 1, whose grounded and preferred extensions were illustrated in Ex. 8. As explained before, even though the source of μ is involved in an attack cycle that is not resolved by the grounded semantics, the support μ is still active. This intuition is captured by our characterization of the ASAF semantics since μ belongs to the grounded extension $\mathbf{G}$ of Δ_1. In contrast, if we consider the same scenario following the approach given in [7], the ASAF would be translated into an AF such that no support-argument related with μ is in the grounded extension, thus failing to capture the intuition that μ is active.

Our work relates to [1] since the characterization of the ASAF semantics follows the methodology adopted by the AFRA. In particular, when considering an ASAF with an empty support relation (*i. e.*, an AFRA) and the complete, preferred, stable or grounded semantics, the results obtained following the approach by [1] and ours coincide. This is because given such an ASAF only direct and indirect defeats will occur, and the definition of those defeats in Section 3 follows the intuitions of [1]. Moreover, like in the AFRA, when considering defense against defeats in such an ASAF we will only have to account for direct and indirect defeats. Therefore, the formalization of the ASAF can be seen as an extension of the AFRA.

An ASAF where only attacks and supports between arguments may occur can be considered as an AFN [11]. Differently from [11], where arguments attack (here, defeat) other arguments, in our approach only attacks are able to defeat other elements of the ASAF. Nevertheless, the definition of acceptability in both approaches follows the same intuitions. Whereas in the AFN defense against a defeat from an argument $\mathscr{A}$ is provided by defeating $\mathscr{A}$, in the ASAF this is achieved by defeating the attacks $\mathscr{A}$ originates (through indirect defeats). Hence, following the approach of [11], the extensions of such an ASAF will coincide with the ones obtained through our approach after filtering out the attacks and supports.

In [16] the authors present a formalism that, similarly to ours, extends Dung's framework by adding a support relation and a second-order attack relation that can target attacks and supports. However, in contrast with our approach, their second-order attack relation only allows for attacks to first-order supports and attacks. That is, the interaction is fixed, not being able to combine and nest the attack and support relations at any level. In addition, their second-order attack relation and the attack relation of the ASAF differ in that the former can be originated from first-order attacks, whereas the latter originates only from arguments. Another difference between their approach and ours regards the treatment of support. In contrast with our support relation, in [16] only supports between arguments are allowed. Also, they adopt a deductive interpretation of support which, as shown in [4], corresponds to a dual interpretation of our necessary support.

In this work we defined the complete, preferred, stable and grounded semantics of the ASAF, which correspond to the four classical semantics given in [8]. In particular, Lemmas 2 and 3 show that our characterization of these semantics satisfies the relationships proposed in [8]. Notwithstanding this, the results shown in this work could be extended to other semantics such as semi-stable or ideal; we aim to address this as future work. We also plan to formalize the relationship between the ASAF and the AFRA, as well as the relationship between the ASAF and the AFN. Finally, we intend to study the relationship between the outcome obtained by following the approach of [7] and the outcome obtained by following the approach for determining the extensions of the ASAF proposed in this paper. Moreover, we aim at exploring the computational cost of the acceptability calculus in both approaches, and contrast the results.

References

[1] P. Baroni, F. Cerutti, M. Giacomin, and G. Guida. AFRA: Argumentation Framework with Recursive Attacks. *Int. J. Approx. Reasoning*, 52(1):19–37, 2011.

[2] M. W. A. Caminada and L. Amgoud. On the evaluation of argumentation formalisms. *Artif. Intell.*, 171(5–6):286–310, 2007.

[3] C. Cayrol and M-C. Lagasquie-Schiex. On the acceptability of arguments in bipolar argumentation frameworks. In *Proc. of ECSQARU*, pages 378–389. LNAI 3571, Springer, 2005.

[4] C. Cayrol and M-C. Lagasquie-Schiex. Bipolarity in argumentation graphs: Towards a better understanding. *Int. J. Approx. Reasoning*, 54(7):876–899, 2013.

[5] C. Cayrol and M-C. Lagasquie-Schiex. An axiomatic approach to support in argumentation. In *Proc. of TAFA*, pages 74–91. LNAI 9524, Springer, 2015.

[6] A. Cohen, S. Gottifredi, A. J. García, and G. R. Simari. A survey of different approaches to support in argumentation systems. *Knowledge Eng. Review*, 29:513–550, 2014.

[7] A. Cohen, S. Gottifredi, A. J. García, and G. R. Simari. An approach to abstract argumentation with recursive attack and support. *J. Applied Logic*, 13(4):509–533, 2015.

[8] P. M. Dung. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artif. Intell.*, 77(2):321–358, 1995.

[9] S. Modgil. Reasoning about preferences in argumentation frameworks. *Artif. Intell.*, 173:901–934, 2009.

[10] F. Nouioua. AFs with necessities: further semantics and labelling characterization. In *Proc. of SUM*, pages 120–133. LNAI 8078, Springer, 2013.

[11] F. Nouioua and V. Risch. Argumentation frameworks with necessities. In *Proc. of SUM*, pages 163–176. LNAI 6717, Springer, 2011.

[12] N. Oren and T. J. Norman. Semantics for evidence-based argumentation. In *Proc. of COMMA*, pages 276–284. FAIA 172, IOS Press, 2008.

[13] S. Polberg and N. Oren. Revisiting support in abstract argumentation systems. In *Proc. of COMMA*, pages 369–376. FAIA 266, IOS Press, 2014.

[14] H. Prakken. On support relations in abstract argumentation as abstraction of inferential relations. In *Proc. of ECAI*, pages 735–740. FAIA 263, IOS Press, 2014.

[15] I. Rahwan and G. R. Simari. *Argumentation in Artificial Intelligence*. Springer, 2009.

[16] S. Villata, G. Boella, D. M. Gabbay, and L. W. N. van der Torre. Modelling defeasible and prioritized support in bipolar argumentation. *Ann. Math. Artif. Intell.*, 66(1-4):163–197, 2012.