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Non-Utilitarian Coalition Structure Generation

Oskar Skibski[1] and **Henryk Michalewski**[2] and **Andrzej Nagórko**[3] and **Tomasz P. Michalak**[4,5] and **Andrew Dowell**[6] and **Talal Rahwan**[7] and **Michael Wooldridge**[8]

Abstract. The coalition structure generation problem is one of the key challenges in multi-agent coalition formation. It involves partitioning a set of agents into coalitions so that system performance is optimized. To date, the multi-agent systems literature has focused exclusively on the *utilitarian* version of this problem which seeks to maximize the sum of the values of the coalitions involved. However, there are many examples of situations in which other performance metrics are of interest. In particular, in games with non-transferable utility, we may be more interested in an egalitarian optimal coalition structure, or in minimizing the difference between the utilities of the most affluent and poorest agents. In this paper, we present a number of exact algorithms to solve such non-utilitarian formulations of the coalition structure generation problem.

1 INTRODUCTION

The coalition structure generation problem involves partitioning the set of agents into coalitions so that the performance of the system is optimized. It has been advocated for a variety of potential applications, including improving the surveillance of common areas by autonomous sensors [10]; reducing the uncertainty of the expected energy output of virtual power plants [3]; and increasing the throughput of cognitive radio networks [13].

To date, the multi-agent systems literature has entirely focused on the *utilitarian* version of the coalition structure generation problem in which the objective function to be maximized is the sum of the values of the coalitions involved.[9] Here, the underlying model is typically a game in which every coalition is assigned a single numerical value that represents its utility. Such games are called *transferable utility* (TU) games as an implicit assumption is that a coalition's utility is transferable among agents in the coalition.

While the ability to transfer payoff occurs naturally in many settings, it is not universal. For instance, in goal-oriented systems, agents may derive utility from accomplishing specific goals (e.g. saving lives in disaster response [11]) and one agent may not profit from another agent's goal. Similarly, in resource allocation problems, indivisible and non-transferable resources can be assigned to agents, not to coalitions [7, 6]. Also, non-transferable payoffs occur in various environmental and economic problems [15]. All such domains can be modelled with *non-transferable utility* (NTU) games, in which the utility of a coalition is expressed as a *vector* of real numbers with each entry representing the non-transferable utility of the particular agent within this coalition.

The utilitarian formulation of the coalition structure generation problem can be applied not only to the TU games, but also to NTU ones. In particular, it is enough to sum up the utility vectors for all the coalitions (see the next section for details). However, such an approach is based on an assumption that all agents are completely benevolent and, in the worst case, they agree to sacrifice all their individual utilities. This is because, under utilitarianism, the maximisation of the system welfare is achieved without any regard to the situation of individual agents. For instance, in the NTU with the following non-zero values of coalitions: $V(\{a_1\}) = 9$; and $v(\{a_1, a_2, a_3\}) = [3-\epsilon, 3-\epsilon, 3-\epsilon]$; two coalition structures are considered optimal: $\{\{a_1\}, \{a_2\}, \{a_3\}\}$ and $\{\{a_1\}, \{a_2, a_3\}\}$. In both, the entire value is generated by $\{a_1\}$, while all other coalitions have value zero. On the other hand, if the coalition structure $\{\{a_1, a_2, a_3\}\}$ was chosen then the value of the entire system would be only marginally lower, i.e., $9 - 3\epsilon$, but it would be equally distributed.

Against this background, in our research, we studied two non-utilitarian coalition structure generation problems: *Egalitarian* and *Balanced*. In the former one, the objective function to be *maximized* is the value of the smallest agent utility in the coalition structure (i.e., the poorest agent). In the latter one, the objective function to be *minimized* is the difference between the smallest and the largest agent utilities in the coalition structure (i.e., the difference between the richest and the poorest agents). While the notion of the egalitarian welfare is very well known in social sciences and has been already extensively discussed in the multi-agent systems literature, albeit in different contexts [8, 7, 9, 5], the concept of the balanced welfare is inspired by the economic concept of the Richest/Poorest average income ratio [2], i.e., the relative income difference between the (typically 10%) cohorts of the poorest and the richest members of human societies.[10] We first analysed exact dynamic programming for both problems. Next, we investigated how to solve them under two concise representations of coalitional games, namely MC-nets [12] and Decision Diagrams [17, 1], using linear programming techniques.

[1] University of Warsaw, Poland
[2] University of Warsaw, Poland
[3] University of Warsaw, Poland
[4] University of Warsaw, Poland
[5] University of Oxford, UK
[6] Independent Researcher, UK
[7] Masdar Institute of Science and Technology, UAE
[8] University of Oxford, UK

[9] A detailed overview of the literature on the coalition structure generation problem can be found in the work by [16].

[10] We note that the problem of balanced welfare optimization was also analysed in the OR literature [14] but under the assumption that the input may be *incomplete*, i.e. that not all coalitions are feasible. This is fundamentally different from the standard models of coalitional games, where all coalitions are feasible. We thank one of the anonymous reviewers for highlighting the work by Martell et al.

2 Preliminaries

Let $N = \{a_1, a_2, \ldots, a_n\}$ be a set of agents. A characteristic function game with *transferable utility (TU)* is a pair, (N, v), where $v : 2^N \to \mathbb{R}$ is the utility function that assigns a real value to every coalition $C \subseteq N$. As the utility is transferable, it is shared by all members of any coalition.

Conversely, a characteristic function game with *non-transferable utility (NTU)* is a pair, (N, V), where $V : 2^N \to \mathbb{R}^N$ is the utility function that for every coalition $C \subseteq N$ assigns a n-bit vector $(x_1, x_2, \ldots, x_n) \in \mathbb{R}^N$. Here, x_i denotes the individual utility of agent $a_i \in C$. We will refer to x_i in $V(C)$ as $V_i(C)$ and assume that agents outside a coalition have a zero utility, *i.e.*, $V_i(C) = 0$ for every $a_i \notin C, C \subseteq N$.

A *coalition structure* over N is a partition of the agents to coalitions, $CS = \{C_1, C_2, \ldots, C_k\}$ such that $\bigcup_{i \in \{1,\ldots,k\}} C_i = N$, and $C_i \cap C_j = \emptyset$ for any $i, j \in \{1, \ldots, k\}$ where $i \neq j$. The set of all coalition structures over N is denoted Π^N.

The (standard) utilitarian coalition structure generation (CSG) problem is defined as follows:

Definition 1 UTILITARIAN CSG: *Given a game (N, V), find a coalition structure CS^* that maximizes the sum of values of all agents. Formally:*

$$CS^* \in \arg\max_{CS \in \Pi^N} \left(\sum_{C_j \in CS} \sum_{i \in C_j} V_i(C_j) \right).$$

While the characteristic functions can encode any game, their exponential size means that they can be used to model only relatively small systems. Given this, a number of concise representations of coalitional games have been proposed in the literature.[11] In Section 5, we consider two such concise formalism: MC-nets [12] and Decision Diagrams [17] on which we focused in our research.

3 PROBLEM DEFINITIONS

In this paper, we consider two non-utilitarian CSG problems for the NTU games: EGALITARIAN CSG and BALANCED CSG. The former one assesses the performance of a coalition structure with respect to the value of smallest agent utility within it.

Definition 2 EGALITARIAN CSG: *Given a game with non-transferable utility, (N, V), find a coalition structure CS^* with the maximal value of the smallest agent utility. Formally:*

$$CS^* \in \arg\max_{CS \in \Pi^N} \left(\min_{i \in C_j \in CS} V_i(C_j) \right).$$

Conversely, the BALANCED CSG problem assesses the performance of a coalition structure with respect to the difference between the value of an agent with biggest utility and value of an agent with the smallest utility.

Definition 3 BALANCED CSG: *Given a game with non-transferable utility, (N, V), find a coalition structure CS^* that minimizes the difference between values of the smallest and the largest agents utilities. Formally:*

$$CS^* \in \arg\min_{CS \in \Pi^N} \left(\max_{i \in C_j \in CS} V_i(C_j) - \min_{i \in C_j \in CS} V_i(C_j) \right).$$

[11] For more on concise representations of coalitional games see the book by [4].

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