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Minisum and Minimax Committee Election Rules for General Preference Types

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Abstract. In committee elections it is often assumed that voters only (dis)approve of each candidate or that they rank all candidates, as it is common for single-winner elections. We suggest an intermediate approach, where the voters rank the candidates into a fixed number of groups. This allows more diverse votes than approval votes, but leaves more freedom than in a linear order. A committee is then elected by applying the minisum or minimax approach to minimize the voters' dissatisfaction. We study the axiomatic properties of these committee election rules as well as the complexity of winner determination and show fixed-parameter tractability for our minimax rules.

1 Introduction

A central point in computational social choice is the analysis of voting systems, see for example the book chapter by Zwicker [11]. Whereas the initial focus was mainly on single-winner elections, the study of committee elections recently received considerable attention. In a committee election a winner is a subset of candidates of a predefined size.

Most voting rules require the voters to either rank all candidates in a strict linear order, which might be impossible given a large set of candidates, or to divide them into two groups, i. e., approval ballots, which might be too rough to fully express the voters' preferences. As an intermediate approach, we propose ℓ-ballots. Voters group the candidates into a fixed number of groups, where all candidates in one group are tied. We use this type of ballot – a slight variant of the model proposed by Obraztsova et al. [10]– to define committee election rules that minimize the voters' dissatisfaction and study computational and axiomatic properties of these rules. To that end, we apply the well-known minisum method where the sum of the distances to the individual votes is minimized, and the minimax method where the maximal distance to an individual vote is minimized. Originally, the minisum and minimax methods have been applied to approval votes by Brams et al. [4]. The most relevant papers for our study are those by Baumeister et al. [2, 3] who extended this approach to determine winning committees for different forms of votes, namely trichotomous votes as well as complete and incomplete linear orders. Elkind et al. [6] studied axiomatic properties such as consistency, monotonicity, and solid coalitions for different multiwinner voting rules, including STV, Bloc, k-Borda and different variants of the Chamberlin-Courant and Monroe's rule. We adapt some of these properties to our setting and study them for the class of ℓ-group rules. The parameterized complexity of minimax voting rules has been studied by Misra et al. [9] for approval votes as well as by Liu and Guo [8] for trichotomous votes and linear and partial orders. In both papers it is shown that, for their respective voting rules, winner determination is W[2]-hard when parameterized by the size of the committee and that computing a winning committee is fixed-parameter tractable with respect to a distance parameter.

2 Definitions

Let $C = \{c_1, \dots, c_m\}$ be a set of candidates and $V = (v_1, \dots, v_n)$ a profile, i. e., a list of voters represented by their vote. In an *ℓ-ballot* over C, a vote is given as a list of ℓ pairwise disjoint sets of candidates, which may also be empty: $v = (G_1, \dots, G_\ell)$ where $G_i \cap G_j = \emptyset$ for $1 \leq i, j \leq \ell$ and $i \neq j$, and $\bigcup_{1 \leq i \leq \ell} G_i = C$. Considering a set of candidates $C = \{c_1, c_2, c_3, c_4\}$, a possible 3-ballot is $(\{c_3, c_4\}, \{\}, \{c_1, c_2\})$ which means that candidates c_3 and c_4 are preferred to all other candidates, and candidates c_1 and c_2 are the most disliked ones.

A very similar ballot model has been introduced by Obraztsova et al. [10]. The predefined ℓ groups correspond to their preference levels. In contrast to our model, they assume that the first and last group are never empty and that at least one voter specifies no empty group. However, these are only technical requirements that are not crucial for our results.

A *committee* is a subset of C. Let $F_k(C)$ denote the set of all committees of size k. A *committee election* is a triple $E = (C, V, k)$, where C is the set of candidates, V is a list of voters, represented by ℓ-ballots for some fixed constant ℓ over C, and $k \in \mathbb{N}$ denotes the committee size. A *committee election rule* $\mathcal{R}$ is a function that, given a committee election, returns a set of tied winning committees.

Now we introduce the ℓ-group voting rules discussed in this paper. For this sake we define $\delta_\ell(v, W) = \sum_{c \in C} |v(c) - W(c)|$ as the *dissatisfaction* (or *distance*) between an ℓ-ballot v and a committee $W \in F_k(C)$ where $W(c) = 1$ for a candidate $c \in W$, and $W(c) = \ell$ for a candidate $c \notin W$, and where $v(c)$ denotes the group number of a candidate c. For the case of $\ell = 2$ this distance corresponds to the Hamming distance between the vote and the committee. The following two rules elect the winning committee(s) for profiles consisting of ℓ-ballots.

Definition 1 (minisum/minimax ℓ-group rule)
- *Minisum ℓ-group rules are functions f^ℓ_{sum} so that $f^\ell_{sum}((C,V,k)) = \text{argmin}_{W \in F_k(C)} \sum_{v \in V} \delta_\ell(v, W)$, i. e., f^ℓ_{sum} minimizes the sum of the voters' dissatisfaction to the winning committees.*
- *Minimax ℓ-group rules are functions f^ℓ_{max} so that $f^\ell_{max}((C,V,k)) = \text{argmin}_{W \in F_k(C)} \max_{v \in V} \delta_\ell(v, W)$, i. e., f^ℓ_{max} minimizes the dissatisfaction of the least satisfied voter with the winning committees.*

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Note that the minisum/minimax voting rules defined by Baumeister and Dennisen [2] correspond to our minisum/minimax ℓ-group rules for $\ell = 2, 3$, and m, and without allowing empty groups.

3 Results

Due to space restrictions we present only the results of our work. For the axiomatic study, we first adapt the existing definitions for some properties to handle the more general input type of ℓ-ballots. Then we can show that the minisum ℓ-group rules satisfy nearly all properties at hand, whereas the minimax ℓ-group rules violate some of them. An overview of our results is given in Table 1.

Properties	ℓ-group rules minisum	ℓ-group rules minimax
Non-imposition, Homogeneity	✓	✓
Consistency	✓	×
Independence of clones	✓	×
Committee monotonicity	✓	×
(Candidate) monotonicity	✓	✓
Positive responsiveness	✓	×
Pareto criterion	✓	✓
(Committee) Condorcet consistency	×	×
Solid coalitions, Consensus committee	×	×
Unanimity	strong	strong

Table 1: Properties for minisum and minimax ℓ-group rules

Next, we study the complexity of computing a winning committee for minisum and minimax ℓ-group rules. For the minisum rule the problem can be solved in polynomial time, as it can be shown that the candidates c with the lowest score $\sum_{v \in V} v(c)$ form a winning committee.

For the study of minimax rules we need the following auxiliary decision problem.

Minimax ℓ-Score	
Given:	A committee election $E = (C, V, k)$, and a nonnegative integer d.
Question:	Is there a committee $W \in F_k(C)$ such that $\max_{v \in V} \delta_\ell(v, W) \leq d$?

LeGrand et al. [7] show that a problem corresponding to our Minimax 2-Score is NP-hard, a result that can be generalized to every greater value of ℓ. On these grounds we resort to the study of parameterized complexity. Thus, our goal is to formulate an efficient algorithm when certain parameters of the problem are small, i. e., can be treated as a constant.[3] For approval voting Misra et al. [9] show that the problem is W[2]-hard, when parameterized by the size of the committee. This hardness result also applies to Minimax ℓ-Score. Hence, an attempt to tune an algorithm with respect to the size of the committee is most likely going to result in failure.

As a positive result we give an algorithm that efficiently solves the Minimax ℓ-Score problem when the parameter d is treated as a constant, which proofs the following theorem.

Theorem 1 *There is an algorithm solving* Minimax ℓ-Score *whose running time is in* $O\left((mn + m \log m)\left(\frac{\sqrt{33}}{2}d\right)^d\right)$. *In particular,* Minimax ℓ-Score *is fixed-parameter tractable when parameterized by d.*

[3] For formal definitions and background regarding parameterized complexity we refer to the book of Downey and Fellows [5].

4 Conclusion

We have introduced different ways of expressing the voters' preferences in committee elections, namely ℓ-ballots, an intermediate between approval votes and linear orders. In addition to axiomatic properties, we have studied the computational complexity of winner determination. While in the minisum case computing a winning committee under ℓ-group rules can be done efficiently, Minimax ℓ-Score is NP-hard. However, there exists a fixed-parameter tractable algorithm that determines a winning committee.

Note that the input type of ℓ-ballots is only one form of a more general vote. In our setting the differences in scores between two groups are always equivalent and there may be situations where for example the first two groups are of greater importance than the other ones. So as a very general framework one could consider that each voter reports two dissatisfaction values (a, b) to each candidate, one for the case that the candidate is in the committee, the other one for the case where the candidate is not in the committee. We call the resulting voting rules minisum/minimax (a, b)-rules. Obviously our ℓ-group rules are obtained as a special case of such (a, b)-rules, when we restrict the input to $a + b = l - 1$ for each voter. More interestingly, we can show that under some mild restrictions the results obtained in this paper even hold for the very general class of (a, b)-rules.

As a task for future work we propose to identify other interesting special cases of (a, b)-rules and provide a characterization for them. Furthermore, we want to consider different rules for these types of input and identify which of the properties from Table 1 are satisfied, and especially find rules that fulfill Condorcet consistency and committee Condorcet consistency. Closely related to the setting of minisum and minimax elections are the systems of proportional representation, which themselves are related to the interesting concept of justified representation [1]. Thus, a task for future research is to redefine and study these concepts for more general types of votes.

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