

On Truthful Auction Mechanisms for Electricity Allocation

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Abstract. As technology evolves and electricity demand rises, more and more research focus on the efficient electricity allocation mechanisms so as to make consumer demand adaptive to the supply of electricity at all times. In this paper, we formulate the problem of electricity allocation as a novel combinatorial auction model, and then put forward a directly applicable mechanisms. It is proven that the proposed mechanism is equipped with some useful economic properties and computational traceability. Our works offer potential avenues for the study about efficient electricity allocation methods in smart grid.

1 INTRODUCTION

Recently, there is a sudden demand for electricity with the economic development of science and technology, then the task to make consumer demand adaptive to electricity supply at all times becomes especially challenging[1], which can reduce the risk of disastrous electricity network collapses, and bring financial and environmental benefits—as some generators can be run on idle, or long time overload may damage the generators, which even exerts a bad influence on the electricity network[2]. In order to overcome the challenge, we are motivated to design the mechanisms for electricity allocation, which is mainly faced with three obstacles: (1) *Truthfulness*; (2) *System Efficiency*; (3) *Low execution time*. Many excellent mechanisms for making consumer demand adaptive to electricity supply have been designed in the existing literatures (e.g. [3]–[5]), in particular, two strains of thought seem to dominate the effort to deal with this problem: one is abridging customers' consuming activities for electricity, the other is shifting customers' consuming activities for electricity to off-peak hours in order to reduce peak-to-average ratio (PAR).

In this paper, we present a novel game model of multi time slots combinatorial auction for the electricity allocation problem, in which electricity consumers can bid for the electricity in multiple time slots, and the electricity in each time slot can be simultaneously sold to multiple electricity consumers, and then propose a combinatorial auction mechanism with dynamic price called TAMEA-DP. In addition, we make it possible that the proposed mechanism satisfies individual rationality, budget balance and truthfulness.

2 THE MODEL AND OBJECTIVE

We assume that the large utility company (i.e., seller) is trustworthy, and has a set of time slots $M = \{1, 2, \dots, m\}$, in which the quanti-

ties Q_j of the electricity (i.e. electricity capacity) sold to buyers. The electricity in each time slot can be simultaneously sold to multiple buyers, only if its electricity capacity is not reached. Furthermore, the seller declares the discrete price curve $p^j(\cdot)$, which indicates the minimum unit price of electricity that the utility company wishes to sell. We denote the discrete price curve $p^j(\cdot)$ as a two-level decreasing function: $p^j(< h_j) = p_j^H$ and $p^j(\geq h_j) = p_j^L$, with $p_j^L < p_j^H$. Here, h_j is the price threshold in time slot j , p_j^H denotes the normal unit price, and p_j^L is similar to a discount price or group price. In addition, we also further assume that the discount price in any time slot is less than the normal price in any other time slot. The seller's offer is defined by $O = \{< Q_j, p^j(\cdot) > \}_{j \in M}$.

We also assume that there is a set $N = \{1, 2, \dots, n\}$ of potential electricity consumers (i.e., buyers). Each buyer $i \in N$ requests her electricity demand for every time slot and has a valuation v_i on the requested electricity. Let \mathbf{R} be an $N * M$ matrix where each row of the matrix, \mathbf{r}_i represents the requested electricity demand of buyer i . Each entry r_{ij} is the electricity demand of buyer i for time slot j . The total aggregated demand in time slot j is $S_j = \sum_{i \in N} r_{ij}$, and the total demand of each agent during the whole planning period is $\tau_i = \sum_{j \in M} r_{ij}$. The electricity valuation v_i is a private information to the buyer i , and then to join the auction, each buyer i should also submit her bid b_i on the requested electricity. Obviously, if $b_i = v_i$, then buyer i is truthful, otherwise she lies about her valuation. In this auction, the buyers simultaneously submit their sealed offers, denoted by $B = (B_1, B_2, \dots, B_n)$, where $B_i = (r_i, b_i)$. To keep the production line running, we assume that each buyer submits a single bid and is single-minded in the auction. In addition, we denote the charge of buyer $i \in N$ by p_i , and define the utility u_i of buyer i to be the difference between her valuation v_i and the charge p_i , i.e., $u_i = v_i - p_i$. As stated above, the electricity allocation problem (EAP) can be defined as four constraints:

- (1) $\sum_{i \in W} z_{ij} \leq Q_j \quad \forall j \in M$; (2) $g(\mathbf{z}_i) < p_i \leq b_i \quad \forall i \in W$;
- (3) $\mathbf{z}_i = \mathbf{0}, p_i = 0 \quad \forall i \notin W$; (4) $\sum_{j \in M} z_{ij} = \tau_i \quad \forall i \in W$;

In this paper, we aim to design a Truthful Auction Mechanism for Electricity Allocation With Dynamic Price (TAMEA-DP), denoted by $\psi = (B, O)$, where given B and O , the auctioneer determines the winning buyer set W , the electricity allocation \mathbf{z}_i for each winning buyer $i \in W$ and the charge p_i for each buyer $i \in N$, such that: (1) For each buyer i , u_i is maximized when bidding v_i . (2) For each buyer, $p_i \leq b_i$; (3) $\sum_{i \in W} p_i \geq \sum_{j \in M} p^j(S_j) * S_j$, which implies that the profit of auctioneer $U_{auc} \geq 0$

3 DESIGN OF TAMEA-DP

TAMEA-DP consists of three schemes: winner determination scheme which is used for deciding who will be the winning buyer,

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allocation scheme which aims to how to allocate the electricity, and charging scheme which determines the payment for buyers.

1) *Winner Determination Scheme* receives the seller's offer and buyers' bid as its input, and aims to determine a set of winning buyers. This algorithm firstly initializes that all buyers are losing buyers, and then ranks the buyers in non-increasing order of their bid density d_i , which can be calculated as: $d_i = \frac{v_i}{|\sum_{j \in M} r_{ij} k_j|^q}$, where $q >$

0. The following steps are the key process: this algorithm checks whether each buyer i ' requested electricity go beyond the available electricity of each time slot. If not, buyer i will be added to the set W . Next, for each buyer $i \in W$, we assess whether her bid satisfies constraint (2), i.e. buyer i ' bid exceeds the weighted sum of dynamic prices for all the requested electricity. If not, we will remove buyer i from the set W , and add her to the set L in which buyers' bids satisfy constraint (1) but not satisfy constraint (2). This process will be repeated until there is no buyer removed from the set W .

2) *Allocation Scheme* collects the set of winning buyers, and sellers' offer. To begin, this algorithm separates time slots in M into two sets: $M^+ = \{j | j \in M \text{ and } S_j > h_j\}$, $M^- = \{j | j \in M \text{ and } S_j \leq h_j\}$, and then the total electricity which goes beyond / within the price threshold can be calculated as $T^+ = \sum_{j \in M^+} (S_j - h_j)$ and $T^- = \sum_{j \in M^-} (h_j - S_j)$ respectively. Next, we should consider two cases: $T^+ \leq T^-$ and $T^+ > T^-$, and the algorithm proceeds as follows: If $T^+ \leq T^-$, the algorithm then ranks time slots in M^- by $l_j = S_j - h_j$ in non-decreasing order. Next, starting from the time slot with lowest l_j value in M^- , we increase its total aggregated electricity S_j until it is equal to its price threshold. This process is repeated until whole electricity which goes beyond the price threshold is transferred, i.e., $T^+ = 0$. Similarly, if $T^+ > T^-$, the algorithm firstly sorts time slots in M^+ by p_j^L in non-increasing order, and then greedily reduces its total aggregated electricity S_j in the sorted order until it is equal to its price threshold. We continue this way until there is no time slot in which the total electricity goes within its price threshold, i.e., $T^- = 0$. Finally, the adjusted electricity in each time slot will be equally shared by the winning buyers.

3) *Charging Scheme* collects the seller's offer, buyers' bid and the set of winners, and aims to determine the prices that each buyers should pay. The idea of our charging scheme is that the losing buyers pay nothing, and the winning buyer should pay her critical value, which is the lowest possible price that she should claim for her requested electricity in order to still be the winning buyer. Therefore, the process of calculating the price that buyer i pays can be conducted as follows: If buyer i is a winning buyer, the algorithm firstly excludes her bid b_i to construct a new market setting B' . Next, it runs Winner Determination Scheme with B' , and then only selects the new winning buyers compared to the initial winners to form the set LW_i of losing competitors. If LW_i is not empty, the algorithm calculates the competitive price p_i^{comp} as the product of $|\sum_{j \in M} r_{ij} k_j|^q$ and the maximum bid density in LW_i , otherwise p_i^{comp} is 0. Finally, the highest value between the competitive price p_i^{comp} and the allocation-specific reserve price $g(z_i)$ is determined as the prices that winning buyer i should pay, and losing buyer should pay nothing.

4 SIMULATIONS

The simulation code is written in C# with .NetFramework 4.0 and run on a local machine. We consider linear bid density with $q = 1$, and randomly generate the buyers $N = \{1, 2, \dots, 10\}$ who can randomly request between 11 and 20 electricity. In addition, buyer's valuation is $v_i = \beta \sum_{j \in M} r_{ij} * p^j(1)$, which is generated as a random

value based on the unit price of each time slot and the scale factor β distributed over $[0.8, 1.2]$. In this experiment, we also assume that the ratio of buyer's bid to her valuation is δ , then it represents that the buyer is truthful when $\delta = 1$, otherwise it means that the buyer submits a mendacious bid (It indicates that the buyer states lower and inflated bids when setting $\delta < 1$ and $\delta > 1$ respectively). As shown in Figure 1, we note that for each buyer $i \in N$, she can gain the highest utility when $\delta = 1$, i.e., no buyer can improve her utility by bidding untruthfully.

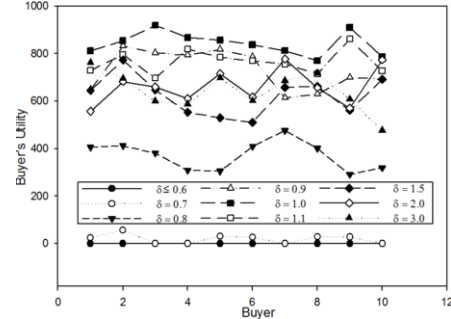


Figure 1. Truthfulness for TAMEA-DP

5 CONCLUSION

In this paper, we propose a truthful combinatorial auction mechanisms with dynamic price for allocating the electricity to achieve the goal of making consumer demand adaptive to electricity supply at all times. It is verified that the proposed mechanisms can simultaneously achieve three important economic properties including individual rationality, truthfulness and budget balance. In our future work, we will explore the impacts of different bid density on the proposed mechanisms with the different market setting, and further improve the proposed mechanism to prevent electricity consumers from collaborating with each other.

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