## **DA-BSP:** Towards Data Association Aware Belief Space Planning for Robust Active Perception

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Belief space planning (BSP) and decision-making under uncertainty are fundamental problems in robotics and artificial intelligence, with applications including autonomous navigation, object grasping and manipulation, active SLAM, and robotic surgery. In the presence of uncertainty, such as in robot motion and sensing, the true state of variables of interest (e.g. robot poses), is unknown and can only be represented by a probability distribution over possible states, given available data. This distribution, the belief space, is inferred using probabilistic approaches based on incoming sensor observations and prior knowledge. The corresponding BSP problem is an instantiation of a partially observable Markov decision problem (POMDP) [4].

Existing BSP approaches (e.g. [2,5,9,11]) typically assume data association to be given and perfect, i.e. the robot is assumed to correctly perceive the environment to be observed by its sensors, given a candidate action. However, this assumption, denoted for brevity as DAS, can be harder to justify while operating in ambiguous and perceptually aliased environments (see Figure 1), and in the presence of different sources of uncertainty (uncertainty due to stochastic control and imperfect sensing).

Indeed, in the presence of ambiguity, DAS may lead to incorrect posterior beliefs and as a result, to sub-optimal actions. More advanced approaches are thus required to enable reliable operation in ambiguous conditions, approaches often referred to as (active) robust perception. Yet, existing robust perception approaches (e.g. [1, 3, 6, 10]) focus on the passive case, where robot actions are externally determined and given.

In this work we develop a general data association aware belief space planning (DA-BSP) framework capable of better handling complexities arising in a real world, possibly perceptually aliased, scenarios. We rigorously incorporate reasoning about data association within belief space planning (and inference), while also considering other sources of uncertainty (motion, sensing and environment). In particular, we show that due to perceptual aliasing, the posterior belief becomes a mixture of probability distribution functions, and design cost functions that measure the expected level of ambiguity and posterior uncertainty. Using these and standard costs (e.g. control penalty, distance to goal) within the objective function, yields a general framework that reliably



Figure 1: (a) Generative graphical model. Standard BSP approaches assume data association (DA) is given and perfect (DAS). We in-corporate data association aspects within BSP and thus can reason about ambiguity (e.g. perceptual aliasing) at a decision-making level. (b) Schematic representation of pose, scene and observation spaces. Scenes  $A_1$  and  $A_3$  when viewed from perspective x and x' respectively, produce the same nominal observation  $\hat{z}$ , giving rise to perceptual aliasing.

represents action impact, and in particular, capable of active disambiguation. Our approach is thus applicable to robust active perception and autonomous navigation in perceptually aliased environments. In this short paper, we provide a concise overview of the DA-BSP approach, referring the interested reader to [7, 8] for full details.

## **Concept and Approach Overview**

Given some candidate action  $u_k$  and the belief at planning time k, we can reason about a future observation  $z_{k+1}$  (e.g. an image) to be obtained once this action is executed; its actual value is unknown. All the possible values such an observation can assume should be thus taken into account while evaluating the objective function, which can be written as:

$$J(u_k) \doteq \int_{z_{k+1}} \underbrace{\mathbb{P}(z_{k+1} \mid \mathcal{H}_{k+1}^-)}_{(z_{k+1} \mid \mathcal{H}_{k+1}^-)} c\left(\underbrace{\mathbb{P}(X_{k+1} \mid \mathcal{H}_{k+1}^-, z_{k+1})}_{(z_{k+1} \mid \mathcal{H}_{k+1}^-)}\right), \quad (1)$$

where  $X_{k+1}$  denotes the past and current robot poses  $X_k \doteq$  $\{x_0, \ldots, x_k\}$ , and  $\mathcal{H}_{k+1}^- \doteq \{u_{0:k}, Z_{0:k}\}$ .

The two terms (a) and (b) in Eq. (1) have intuitive meaning: for each considered value of  $z_{k+1}$ , (a) represents how likely is it to get such an observation when both the history  $\mathcal{H}$  and control  $u_k$  are known, while (b) corresponds to the posterior belief given this specific  $z_{k+1}$ .

Existing BSP approaches typically consider data association is solved (DAS), i.e. given and perfect. In other words, DAS

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means we can correctly associate each possible measurement  $z_{k+1}$  with the corresponding scene  $A_i$  it captures and write the corresponding measurement likelihood term  $\mathbb{P}(z_{k+1}|x_{k+1}, A_i)$ . Yet, it is unknown from what future robot pose  $x_{k+1}$  the actual observation  $z_{k+1}$  will be acquired, since the *actual* robot pose  $x_k$  at time k is unknown, the control is stochastic and sensing is imperfect. In inference, we have a similar situation with the key difference that the observation z has been acquired.

Rather than assuming DAS, in this work we incorporate within BSP (and similarly within inference) reasoning about possible scenes or objects that the future observation  $z_{k+1}$ could be generated from, see Figure 1. While this may seem computationally expensive, realistic scenarios typically exhibit parsimonious data association: If the environment has only distinct scenes or objects, then for each specific value of  $z_{k+1}$ , there will be only one scene  $A_i$  that can generate such an observation. In the case of perceptually aliased environments, there could be several other scenes (or objects) that are either completely identical or have a similar visual appearance when observed from appropriate viewpoints. They could equally well explain the considered observation  $z_{k+1}$ . Thus, there are several possible associations  $\{A_i\}$  and due to localization uncertainty determining which association is the correct one is not trivial. As we show in [7], in these cases the posterior (term (b) in Eq. (1)) becomes a Gaussian mixture with appropriate weights that we rigorously compute. Additionally, the weight updates are capable of discriminating against unlikely data-associations, during the planning steps.

We now briefly summarize how terms (a) and (b) in Eq. (1) are calculated while reasoning about data association, referring the reader to [7] for full details.

Computing the term (a):  $\mathbb{P}(z_{k+1}|\mathcal{H}_{k+1}^-)$ : Applying total probability over non-overlapping scene space  $\{A_{\mathbb{N}}\}$  and marginalizing over all possible robot poses, yields

$$\mathbb{P}(z_{k+1}|\mathcal{H}_{k+1}^{-}) \equiv \sum_{i}^{|A_{\mathbb{N}}|} \int_{x} \mathbb{P}(z_{k+1}, x, A_i | \mathcal{H}_{k+1}^{-}) \doteq \sum_{i}^{|A_{\mathbb{N}}|} w_{k+1}^{i}.$$
(2)

As seen from the above equation, to calculate the likelihood of obtaining some observation  $z_{k+1}$ , we consider separately, for each scene  $A_i \in \{A_{\mathbb{N}}\}$ , the likelihood that this observation was generated by scene  $A_i$ . This probability is captured for each scene  $A_i$  by a corresponding weight  $w_{k+1}^i$ ; these weights are then summed to get the actual likelihood of observation  $z_{k+1}$ . As shown in [7], these weights naturally account for perceptual aliasing aspects for each considered  $z_{k+1}$ .

In practice, instead of considering the entire scene space  $\{A_{\mathbb{N}}\}\$  that could be computationally costly, the availability of the belief from the previous time step,  $b[X_{k+1}^-]$ , enables us to consider only those scenes that could be actually observed from the viewpoints with non-negligible probability according to  $b[X_{k+1}^-]$ .

Computing the term  $(b) : \mathbb{P}(X_{k+1}|\mathcal{H}_{k+1}^-, z_{k+1})$ : The term  $(b), \mathbb{P}(X_{k+1}|\mathcal{H}_{k+1}^-, z_{k+1})$ , represents the posterior probability conditioned on observation  $z_{k+1}$ . This term can be similarly calculated, with a key difference: since the observation  $z_{k+1}$  is given, it must have been generated by *one* specific (but unknown) scene  $A_i$  according to an appropriate measurement model. Hence, also here, we consider all possible such scenes and weight them accordingly, with weights  $\tilde{w}_{k+1}^i$  represent-

ing the probability of each scene  $A_i$  to have generated the observation  $z_{k+1}$ .

As shown in [7], the term (b) in Eq. (1) is a GMM with  $M_{k+1}$  components,  $\mathbb{P}(X_{k+1}|\mathcal{H}_{k+1}^{-}, z_{k+1}) = \sum_{r=1}^{M_{k+1}} \xi_{k+1}^{r} b[X_{k+1}^{r+}]$ , where  $b[X_{k+1}^{r+}]$  represents the *r*th component of the belief, and the weights  $\xi_{k+1}^{r}$  are defined recursively (see full details in [7]). Interestingly, the number of components can not only go down  $M_{k+1} \leq M_k$  (as a result of a partially or fully disambiguating action), but could also go up, i.e.  $M_{k+1} > M_k$ .

To summarize the discussion thus far, we have shown that for the myopic case, the objective function (1) can be rewritten as

$$J(u_k) = \int_{z_{k+1}} \left( \sum_{i}^{|A_{\mathbb{N}}|} w_{k+1}^i \right) \cdot c \left( \sum_{r}^{M_{k+1}} \xi_{k+1}^r b[X_{k+1}^{r+1}] \right).$$
(3)

In [7], we present the other ingredients of our approach, including sampling-based simulation of future observations  $\{z_{k+1}\}$  given  $b[X_{k+1}^-]$ , and the design of suitable cost functions to quantify ambiguity level. We also show that DA-BSP considers data-association parsimoniously and a simple thresholding is enough for a scalable application of data-association aware belief space planning, and demonstrate key aspects basic and realistic simulations. Potential directions for future research include extension to non-myopic planning as well as proving the general theoretical properties of DA-BSP.

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