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Computing Extensions' Probabilities in Probabilistic Abstract Argumentation: Beyond Independence

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Abstract. We characterize the complexity of the problem of computing the probabilities of the extensions in probabilistic abstract argumentation. We consider all the most popular semantics of extensions (*admissible*, *stable*, *preferred*, *complete*, *grounded*, *ideal-set*, *ideal* and *semi-stable*) and different forms of correlations that can be defined between arguments and defeats. We show that the complexity of the problem ranges from FP to $FP^{\#P}$-complete, with $FP^{||NP}$-complete cases, depending on the semantics of the extensions and the imposed correlations.

1 INTRODUCTION

In the last decade, several argumentation frameworks have been proposed, with the aim of suitably modeling disputes between two or more parties. Typically, argumentation frameworks model both the possibility of parties to present arguments supporting their theses, and the possibility that some arguments rebut other arguments. Although argumentation is strongly related to philosophy and law, it has gained remarkable interest in AI as a reasoning model for representing dialogues, making decisions, and handling inconsistency and uncertainty [3, 4, 15].

A powerful yet simple argumentation framework is that proposed in the seminal paper [5], called *abstract argumentation framework* (AAF). An AAF is a pair $\langle A, D\rangle$ consisting of a set A of *arguments*, and of a binary relation D over A, whose pairs are called *defeats* or, equivalently, *attacks*. Basically, an argument is an abstract entity that may attack and/or be attacked by other arguments, and an attack expresses the fact that an argument rebuts/weakens another argument.

Example 1 *The defense attorney of Mary and Marc wants to reason about the possible outcome of the trial of the robbery case involving his clients. The arguments of the case are the following, where Anne is a potential witness:*

a: *"Mary says she was at the park when the robbery took place, and therefore denies being involved in the robbery";*
b: *"Marc says he was at home when the robbery took place, and therefore denies being involved in the robbery";*
c: *"Anne says that she is certain that he saw Mary outside the bank just before the robbery took place, and she also thinks that possibly she saw Marc there too".*

The arguments a and b support the innocence of the defendants, and argument c means that a potential witness instills doubts about the innocence of both Mary and Marc. This scenario can be modeled by the AAF $\mathcal{A}$, whose set of arguments is $\{a, b, c\}$, and whose defeat relation consists of the defeats $\delta_{ac} = (a, c), \delta_{ca} = (c, a), \delta_{bc} = (b, c)$ and $\delta_{cb} = (c, b)$, meaning that arguments a and b are both attacked by c and they both counter-attack c.

Several semantics for AAFs, such as *admissible*, *stable*, *preferred*, *complete*, *grounded*, and *ideal-set*, have been proposed [5, 6, 2] to identify "reasonable" sets of arguments, called *extensions*. Basically, each of these semantics corresponds to some properties that "certify" whether a set of arguments can be profitably used to support a point of view in a discussion. For instance, a set S of arguments is an extension according to the *admissible* semantics if it has two properties: it is "*conflict-free*" (that is, there is no defeat between arguments in S), and every argument (outside S) attacking an argument in S is counterattacked by an argument in S. Intuitively enough, the fact that a set is an extension according to the admissible semantics means that, using the arguments in S, you do not contradict yourself, and you can rebut to anyone using an argument outside S to contradict yours. The other semantics correspond to other ways of determining whether a set of arguments would be a "good point" in a dispute.

As a matter of fact, in the real world, arguments and defeats are often uncertain, thus, several proposals have been made to model uncertainty in AAFs, by considering weights, preferences, or probabilities associated with arguments and/or defeats. In this regard, [7, 12, 9, 8, 11, 10, 14, 16] have recently extended the original Dung framework in order to achieve probabilistic abstract argumentation frameworks (prAAFs), where uncertainty of arguments and defeats is modeled by exploiting the probability theory. In particular, [14] proposed a form of prAAF (here denoted as IND, shorthand for "*independence*") where each argument and defeat can be associated with a probability value (and arguments and defeats are viewed as *independent* events), whereas [7] proposed a form of prAAF (here denoted as EX, shorthand for "*extensive*") where uncertainty can be taken into account by extensively specifying a probability distribution function (pdf) over the possible scenarios, as shown in the following example.

Example 2 (continuing Example 1) *In the case of modeling the uncertainty by assigning probabilities to possible scenarios, as done in prAAFs of form* EX*, suppose that the lawyer thinks that only the following 4 scenarios are possible:*
S_1*: "Ann will not testify";*
S_2*: "Ann will testify, and the jury will deem that her argument c undermines those of Mary and Marc (arguments a, b), and vice versa";*
S_3*: "Ann will testify, and the jury will deem that her argument c undermines Mary's and Marc's arguments a, b, while, owing to the bad reputations of Mary and Marc, a and b will be not perceived as strong enough to undermine argument c";*

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S_4: "Ann will testify, and the jury will deem that her argument c undermines Mary's argument a but not Marc's argument b, since Ann was uncertain about Marc's presence. In the other direction, a and b will be not perceived as strong enough to undermine c".

Each S_i is encoded by the AAF α_i in the following list:
$\alpha_1 = \langle\{a, b\}, \emptyset\rangle$, $\alpha_2 = \langle\{a, b, c\}, \{\delta_{ac}, \delta_{ca}, \delta_{bc}, \delta_{cb}\}\rangle$,
$\alpha_3 = \langle\{a, b, c\}, \{\delta_{ca}, \delta_{cb}\}\rangle$, $\alpha_4 = \langle\{a, b, c\}, \{\delta_{ca}\}\rangle$.

Basically, the form of prAAF EX *allows the lawyer to define, one by one, which scenarios are possible, and then to assign a probability to the AAF corresponding to each scenario, on the basis of her/his perception of how likely the scenario is. For instance, the pdf set by the lawyer could be such that: $P(\alpha_1) = 0.1$ and $P(\alpha_2) = P(\alpha_3) = P(\alpha_4) = (1 - P(\alpha_1))/3 = 0.3$, meaning that the lawyer thinks that there is 10% probability that Mary will not manage to testify (owing to her ill-health), and that, in the case she testifies, the other three scenarios are equi-probable.*

Example 3 (continuing Example 1) *In the case that a prAAF of form* IND *is used, the lawyer can associate each argument and defeat with a probability. For instance, the lawyer may set $P(c) = 0.9$ (meaning that there is 10% probability that Mary will not manage to testify) and $P(a) = P(b) = 1$ (meaning that Mary and Marc will certainly testify). Moreover, she/he could set $P(\delta_{ca}) = 1$ (meaning that she/he is certain that the jury will consider Ann's argument as a solid rebuttal of Mary's argument). Analogously, she/he could set $P(\delta_{cb}) = 0.8$ and $P(\delta_{ac}) = P(\delta_{bc}) = 0.4$.*

Given this, since the arguments are considered independent, the possible scenarios modeled by IND *are not only $\alpha_1, \ldots, \alpha_4$ of the previous example, but all the AAFs $\langle A_i, D_i\rangle$ where A_i is a subset of the arguments and D_i a subset of the defeats between the arguments in A_i. Specifically, there are 9 possible AAFs, where 4 out of 9 are equal to $\alpha_1, \ldots \alpha_4$ of the previous example, and the others are $\alpha_5 = \langle\{a, b, c\}, \{\delta_{ac}, \delta_{ca}\}\rangle$, $\alpha_6 = \langle\{a, b, c\}, \{\delta_{ca}, \delta_{bc}\}\rangle$, $\alpha_7 = \langle\{a, b, c\}, \{\delta_{ca}, \delta_{cb}, \delta_{ac}\}\rangle$, $\alpha_8 = \langle\{a, b, c\}, \{\delta_{ca}, \delta_{cb}, \delta_{bc}\}\rangle$, $\alpha_9 = \langle\{a, b, c\}, \{\delta_{ca}, \delta_{ac}, \delta_{bc}\}\rangle$. Moreover, the probability assigned to each AAF $\langle A, D\rangle$ is the result of a product, whose factors are the probabilities (resp., the complements of the probabilities) of the arguments in A (resp., not in A) and the probabilities (resp., the complements of the probabilities) of the defeats between arguments in A that are in D (resp., are not in D). For instance, $P(\alpha_1) = P(a) \times P(b) \times (1 - P(c)) = 0.1$ and $P(\alpha_3) = P(a) \times P(b) \times P(c) \times P(\delta_{ca}) \times P(\delta_{cb}) \times (1 - P(\delta_{ac})) \times (1 - P(\delta_{bc})) = 0.26$.*

2 COMPLEXITY OF PROB$^{sem}_{\mathcal{F}}(S)$

The complexity of the fundamental problem of computing the extensions' probabilities over a prAAF of form $\mathcal{F}$ (denoted, in the following, as PROB$^{sem}_{\mathcal{F}}$) has been thoroughly characterized in [10], in the specific case that $\mathcal{F}$ is IND. Much less is known about the complexity of the same problem over different forms of prAAFs.

We here consider a general form of prAAF (called GEN) with three main amenities: 1) it generalizes EX, since it also enables an "extensive" definition of the pdf over the possible AAFs; 2) it generalizes IND, since it also allows us to impose independence between arguments and defeats; 3) in order to encode a pdf over the possible AAFs, it exploits the representation model of *world-set descriptors* (wsds) and *world-set sets* (ws-sets), that is known to be a succinct and complete model for representing possible worlds and probabilities over them [1, 13]. This paradigm GEN can be exploited to define different syntactic classes of wsds and ws-sets, each allowing different forms of correlations (mutual exclusion, co-occurrence, etc.).

We consider the following well-known semantics: *admissible* (`ad`), *stable* (`st`), *complete* (`co`), *grounded* (`gr`), *preferred* (`pr`), *ideal-set* (`ids`), *ideal* (`ide`), and *semi-stable* (`sst`), and we show that the complexity of PROB$^{sem}(S)$ ranges from FP to $FP^{\#P}$, depending on the semantics of the extensions and the syntactic class of the wsds.

Theorem 1 *Let sem be a semantics in $\{ad, st, gr, co\}$ and $\mathcal{F}$ a prAAF of form* GEN. *The complexity of* PROB$^{sem}_{\mathcal{F}}(S)$ *ranges from FP to $FP^{\#P}$-complete depending on the syntactic class of the wsds.*

Theorem 2 *Let sem be a semantics in $\{pr, ide, ids, sst\}$ and $\mathcal{F}$ a prAAF of form* GEN. *The complexity of* PROB$^{sem}_{\mathcal{F}}(S)$ *ranges from $FP^{||}$-complete to $FP^{\#P}$-complete depending on the syntactic class of the wsds.*

3 CONCLUSION

The problem of characterizing the complexity of the fundamental problem PROB$^{sem}(S)$ of evaluating the probabilities of extensions in probabilistic abstract argumentation frameworks has been addressed, showing that the complexity of PROB$^{sem}(S)$ ranges from FP to $FP^{\#P}$, depending on the semantics of the extensions and the syntactic class of the wsds.

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