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Simple Epistemic Planning: Generalised Gossiping

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Abstract. The gossip problem, in which information (secrets) must be shared among a certain number of agents using the minimum number of calls, is of interest in the conception of communication networks and protocols. We extend the gossip problem to arbitrary epistemic depths. For example, we may require not only that all agents know all secrets but also that all agents know that all agents know all secrets. We give optimal protocols for the generalised gossip problem, in the case of two-way communications, one-way communications and parallel communication. In the presence of negative goals testing the existence of a successful protocol is NP-complete.

1 Introduction

We consider communication problems concerning n agents. We consider that initially, for $i = 1, \ldots, n$, agent i has some information s_i, also known as this agent's secret since, initially, the other agents do not know this information. In many applications, this corresponds to information that agent i wishes to share with all other agents. On the other hand, it may be confidential information which is only to be shared with a subset of the other agents. The simplest version of the problem in which all agents want to communicate their secrets to all other agents (using the minimum number of communications) is traditionally known as the *gossip problem*. Several variants have been studied, and a survey has been published [5].

The gossip problem is a particular case of a multiagent epistemic planning problem. Our main contribution is to study the gossip problem at different epistemic depths. In the classic gossip problem, the goal is for all agents to know all secrets (which corresponds to epistemic depth 1). The equivalent goal at epistemic depth 2 is that all agents know that all agents know all the secrets; at depth 3, all agents must know that all agents know that all agents know all the secrets.

All proofs can be found in the full-length version of this article [3].

2 Epistemic planning and the gossip problem

Dynamic Epistemic Logic DEL [9] provides a formal and very expressive framework for the representation and update of knowledge, and several recent approaches to multi-agent planning are based on it. While DEL provides a very expressive framework, even simple fragments of it have unfortunately been proven to be undecidable [1]. We here consider a simple fragment of the language of DEL where the knowledge operator can only be applied to literals [2].

We use the notation $K_i s_j$ to represent the fact that agent i knows the secret of j, the notation $K_i K_j s_k$ to represent the fact that agent i knows that agent j knows the secret of k, etc. We use the term *positive fluent* for any epistemic proposition of the form $K_{i_1} \ldots K_{i_r} s_j$. If we consider the secrets s_i as constants and that agents never forget, then positive fluents, once true, can never become false. A negative fluent $\neg(K_{i_1} \ldots K_{i_r} s_j)$ can, of course, become false.

A *planning problem* consists of an initial state (a set of fluents I), a set of actions and a set of goals (another set of fluents $Goal$). Each action has a (possibly empty) set of preconditions (fluents that must be true before the action can be executed) and a set of effects (positive or negative fluents that will be true after the execution of the action). A *solution plan* (or protocol) is a sequence of actions which when applied in this order to the initial state I produces a state in which all goals in $Goal$ are true. An example of a goal is $\forall i, j, k \in \{1, \ldots, n\}$, $K_i K_j s_k$, i.e. that all agents know that all agents know all the secrets.

The *gossip problem* on n agents and a graph $G = \langle \{1, \ldots, n\}, E_G \rangle$ is the planning problem in which the actions are $\text{CALL}_{i,j}$ for $\{i, j\} \in E_G$ (i.e. agents i and j can call each other iff there is an edge between i and j in G) and the initial state contains $K_i s_i$ for $i = 1, \ldots, n$ (and implicitly all fluents of the form $K_{i_1} \ldots K_{i_r} s_j$ with $i_r = j$) The action $\text{CALL}_{i,j}$ has no preconditions and its effect is that agents i and j share all their knowledge. We go further and assume that the two agents know that they have shared all their knowledge, so that, if we had $K_i f$ or $K_j f$ before the execution of $\text{CALL}_{i,j}$, for any fluent f, then we have $K_{i_1} \ldots K_{i_r} f$ just afterwards, for any r and for any sequence $i_1, \ldots, i_r \in \{i, j\}$.

Let $\text{Gossip-pos}_G(d)$ be the gossip problem on a graph G in which the goal is a conjunction of positive fluents of the form $(K_{i_1} \ldots K_{i_r} s_j)$ $(1 \leq r \leq d)$. Thus, the parameter d specifies the maximum epistemic depth of goals. $\text{Gossip}_G(d)$ denotes the specific problem in which *all* such goals must be attained. We drop the subscript G to denote the corresponding problem in which the graph G is not fixed but part of the input.

3 Minimising the number of calls for positive goals

In this section we consider the gossip problem at epistemic depth d. For $d = 1$, the minimal number of calls to solve $\text{Gossip}_G(1)$ is either $2n - 4$ if the graph G contains a quadrilateral (a cycle of length 4) as a subgraph, or $2n - 3$ in the general case [4].

Proposition 1 *If the graph G is connected, then for $n \geq 2$ and $d \geq 1$, any instance of $\text{Gossip-pos}_G(d)$ has a solution of length no greater than $d(2n - 3)$ calls.*

For $d \geq 2$, we require considerably less than $d(2n - 3)$ calls for certain graphs since we can often achieve $(d + 1)(n - 2)$. The complete bipartite graph with parts $\{1, 2\}$, $\{3, \ldots, n\}$ is denoted in graph theory by $K_{2,n-2}$. There is a protocol which achieves $(d + 1)(n - 2)$ calls provided G contains $K_{2,n-2}$ as a subgraph. This subsumes a previous result which was given only for the case of a complete graph G [6]. Detecting whether an arbitrary graph G has $K_{2,n-2}$ as a subgraph can be achieved in polynomial time.

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Proposition 2 *For $n \geq 4$, if the n-vertex graph G has $K_{2,n-2}$ as a subgraph, then any instance of Gossip-pos$_G(d)$ has a solution of length no greater than $(d+1)(n-2)$.*

Recall that Gossip$_G(d)$ denotes the version of Gossip-pos$_G(d)$ in which the goal consists of all depth-d positive epistemic fluents. We can, in fact, show that the solution plan given in the proof of Proposition 2 [3] is optimal for Gossip$_G(d)$.

Theorem 3 *The number of calls required to solve Gossip$_G(d)$ (for any graph G) is at least $(d+1)(n-2)$.*

4 One-way communications

We now study a different version of the gossip problem, denote by Directional-gossip, in which communications are one-way. Whereas a telephone call is essentially a two-way communication, e-mails and letters are essentially one-way. The result of CALL$_{i,j}$ is now that agent i shares all his knowledge with agent j but agent i receives no information from agent j. Indeed, to be consistent with communication by e-mail, we assume that after CALL$_{i,j}$, agent i does not even gain the knowledge that agent j knows the information that agent i has just sent in this call (e.g. the e-mail was not read).

Directional-gossip-pos$_G(d)$ can be solved in polynomial time: for example, on an undirected graph G, any solution plan for Gossip-pos$_G(d)$ can be converted into a solution plan for Directional-gossip-pos$_G(d)$ by replacing each two-way call by two one-way calls. What is surprising is that the exact minimum number of calls to solve Directional-gossip-pos$_G(d)$ is often much smaller than this and indeed often very close to the minimum number of calls required to solve Gossip-pos$_G(d)$. We consider, in particular, the hardest version of Directional-gossip-pos$_G(d)$, in which the aim is to establish all epistemic goals of depth d. Let Directional-gossip$_G(d)$ denote the directional gossip problem whose goal is to establish the conjunction of $K_{i_1} \ldots K_{i_d} s_{i_{d+1}}$ for all $i_1, \ldots, i_{d+1} \in \{1, \ldots, n\}$.

In the directional version, the graph of possible communications is now a directed graph G. Let $\overline{G}$ be the graph with the same n vertices as the directed graph G but with an edge between i and j if and only if G contains the two directed edges (i,j) and (j,i). It is known that if the directed graph G is strongly connected, the minimal number of calls for Directional-gossip-pos$_G(1)$ is $2n-2$ [4]. We now generalise this to arbitrary d under an assumption about the graph $\overline{G}$.

Proposition 4 *For all $d \geq 1$, if $\overline{G}$ contains a Hamiltonian path, then any instance of Directional-gossip-pos$_G(d)$ has a solution of length no greater than $(d+1)(n-1)$.*

However, it should be pointed out that determining the existence of a Hamiltonian path in a graph is NP-complete.

The following theorem shows that the protocol given in the proof of Proposition 4 [3] is optimal even for a complete digraph G.

Theorem 5 *The number of calls required to solve Directional-gossip$_G(d)$ (for any digraph G) is at least $(d+1)(n-1)$.*

It is worth pointing out that, by Theorem 3, the optimal number of 2-way calls is only $d+1$ less than the optimal number of one-way calls and is hence independent of n, the number of agents.

5 Parallel communications

An the variant Parallel-gossip-pos$_G(d)$, we consider time steps instead of calls: in each time step each agent can only make one call but several calls can be made in parallel. Parallel-gossip$_G(d)$ is the problem of establishing all depth-d positive epistemic fluents. For Parallel-gossip$_G(1)$ on a complete graph G, if the number of agents n is even, the time taken (in number of steps) is $\lceil \log_2 n \rceil$, and if n is odd, it is $\lceil \log_2 n \rceil + 1$ [7]. We now generalise this.

Proposition 6 *For $n \geq 2$, if the n-vertex graph G has the complete bipartite graph $K_{\lceil n/2 \rceil, \lfloor n/2 \rfloor}$ as a subgraph, then any instance of Parallel-gossip-pos$_G(d)$ has a solution with $d(\lceil \log_2 n \rceil - 1) + 1$ time steps if n is even, or $d\lceil \log_2 n \rceil + 1$ time steps if n is odd.*

Determining whether a n-vertex graph G has the complete bipartite graph $K_{\lceil n/2 \rceil, \lfloor n/2 \rfloor}$ as a subgraph can be achieved in polynomial time [3]. On the other hand, it is known that deciding whether Directional-gossip(1) (the problem in which the digraph G is part of the input) can be solved in a given number of steps is NP-complete [8].

In fact, the following theorem shows that the protocol given in the proof of Proposition 6 [3] is optimal in the number of steps.

Theorem 7 *The number of steps required to solve Parallel-gossip$_G(d)$ (for any graph G) is at least $d(\lceil \log_2 n \rceil - 1) + 1$ if n is even, or $d\lceil \log_2 n \rceil + 1$ if n is odd.*

It can happen that increasing the number of secrets (and hence the number of agents) leads to less steps. Consider the concrete example of 7 or 8 agents. The number of steps decreases from $3d+1$ to $2d+1$ when the number of agents increases from 7 to 8. By adding an extra agent, we actually achieve more calls in less steps.

6 Discussion and conclusion

When we allow negative goals, the gossip problem becomes NP-complete [3]. Nonetheless, we avoid the PSPACE complexity of classical planning. The general conclusion that can be drawn is that many interesting epistemic planning problems are either solvable in polynomial time or are NP-complete, thus avoiding the PSPACE-complete complexity of planning. We consider the gossip problem to be a foundation on which to base the study of richer epistemic planning problems.

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