ECAI 2016
G.A. Kaminka et al. (Eds.)

doi:10.3233/978-1-61499-672-9-1485

False-Name-Proof Mechanisms for Path Auctions in Social Networks

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Abstract. We study path auction mechanisms for buying path between two given nodes in a social network, where edges are owned by strategic agents. The well known VCG mechanism is the unique solution that guarantees both truthfulness and efficiency. However, in social network environments, the mechanism is vulnerable to false-name manipulations where agents can profit from placing multiple bids under fictitious names. Moreover, the VCG mechanism often leads to high overpayment. In this paper, we present core-selecting path mechanisms that are robust against false-name bids and address the overpayment problem. Specifically, we provide a new formulation for the core, which greatly reduces the number of core constraints. Based on the new formulation, we present a Vickery-nearest pricing rule, which finds the core payment profile that minimizes the L_∞ distance to the VCG payment profile. We prove that the Vickery-nearest core payments can be computed in polynomial time by solving linear programs. Our experiment results on real network datasets and reported cost dataset show that our Vickery-nearest core-selecting path mechanism can reduce VCG's overpayment by about 20%.

1 Introduction

We consider the problem of buying shortest paths between two given nodes in a social network. For example, in professional networks like LinkedIn, job seekers might want to buy short paths to potential employers; in social media networks, advertisers might want to buy short paths to target customers. In these examples, edges in the networks are owned by strategic agents, and each agent i has a private cost c_i of being included in the path. The goal is to design a path auction mechanism to determine which path to buy, and how much each agent is paid.

The problem is hard because agents may lie about their costs if lying could increase their utilities. Previous work on path auctions have focused on the well known Vickery-Clark-Groves (VCG) Mechanism [24]. The mechanism pays each agent on the shortest path an amount equal to the highest bid with which the agent could have won. It can be shown that the VCG mechanism is the unique efficient and dominant-strategy truthful path mechanism.

However, the VCG mechanism suffers from two economic problems that make it rarely used in practice. The first is that it can lead to significant overpayment. For example, in Figure 1, VCG selects the bottom path and pays 24, while the cost of the shortest path is only 3. Previous work shows that all truthful path mechanisms can be forced to make arbitrarily high overpayment in the worst case [14].

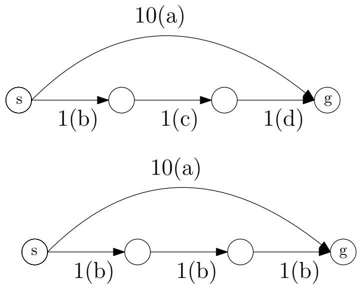

Figure 1: Problems with the VCG path mechanism. Top: VCG pays 8 to agent b, c and d, and the total payment is 24; Bottom: VCG pays 10 to agent b for his three edges, but if agent b places three bids under three different names, he will get $3 * 8 = 24$.

The second economic problem is that the VCG mechanism is vulnerable to false name manipulations [31]. In real world networks, an agent may own multiple edges or even a whole sub-network. Meanwhile, it is also very easy for agents to create fake accounts in social network environments. Then those agents that own many edges can profit from false name manipulations where they place multiple bids under these fictitious names. For example, in Figure 1, VCG pays 10 to agent b for the three edges in the bottom path, however, if agent b submits three separate bids under three different names, he will get a total payment of 24. Therefore, the VCG mechanism is not false-name proof.

In this paper, we are interested in designing efficient path mechanisms to address the economic problems of the VCG mechanism. Based on the framework of core-selecting auctions, we present core-selecting path mechanisms which relax dominant-strategy truthfulness and use the core as the solution concept. We prove that core-selecting path mechanisms are more frugal than the VCG mechanism and are robust against false-name bids. We then give a new core formulation that greatly reduces the number of core constraints, which allows us to compute Vickrey-nearest core payments use linear programming techniques. We further show that the Vickrey-nearest core payments can be computed in polynomial time. We then evaluate core-selecting path mechanisms on real network data and show that the Vickrey-nearest core-selecting path mechanism can greatly reduce VCG's overpayment under realistic bid (reported cost) distributions.

Our main contributions in this work are: (1) We present a new core formulation that reduces the number of core constraints to 2^k, where k is the network diameter. (2) We give a linear program to compute the Vickery-nearest core payments which minimize the L_∞ distance to the VCG payments.

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2 Related Work

The problem of designing mechanisms for path auctions was first studied in [24], where edges in a network are owned by strategic agents, and the cost of the edges is private information of the agent owning the edges. The VCG mechanism is applied to find the shortest paths. The mechanism is shown to be dominant strategy truthful, which means all agents reporting their true costs is a dominant strategy equilibrium. It is also shown that the VCG payments can be computed using n runs of Dijkstras algorithm in $O(nm + n^2 \log n)$ time. It is later shown that if the graph is undirected then the VCG payments can be computed in only $O(m + n \log n)$ time [17].

Although the VCG mechanism for path auction is efficient and truthful, it is found that the VCG mechanism has some undesirable properties. Previous work has found that VCG path mechanism can be forced to make arbitrarily high overpayment in the worst case, in fact the result can be generalized to include all truthful path mechanisms [14, 20]. This led to the study of frugal path mechanisms [2].

Previous work have also studied the VCG overpayment in the Internet inter-domain routing graph [15] and large random graphs [19]. The results show that the VCG overpayment can be intriguingly low when these graphs have unit edge costs. It is also noted the VCG payments can be reduced by removing edges in the graph, but it is NP-hard to determine the optimal set of edges to remove in order to get the lowest VCG payments [13].

False-name manipulations have been studied in a number of anonymous environments, including combinatorial auctions [30, 29, 26, 1], voting [28, 4, 3], matching [25] and social networks [7, 5]. Previous work have also studied first-price path auction mechanism which can be shown to be false-name proof [12, 18].

In addition to the literature on mentioned above, our work is also related to the literature on core-selecting auctions [22, 9, 11, 10, 8]. In particular, we use techniques from [8] to prove that core-selecting path mechanisms are robust against false-name bids.

3 Preliminary

We consider a setting where there is a social network represented by a graph $G = (V, E)$, with $|E| = n$. Each edge in the graph represents a strategic agent e that has a cost $c(e) \in R_{\geq 0}$ of being included in the path, and this cost is the private information of agent e. Given a start node s and a goal node g, the goal is to buy the shortest (least-cost) path from s to g. A solution to this problem is a subset of edges and a payment profile that describes the payments to the agents in the subset.

Since the costs $\boldsymbol{c}$ is private information, we set this up as a mechanism design problem. Each agent e who is a candidate for the path will make a bid $b_e > 0$ and the path mechanism will use an allocation rule $x_e(\boldsymbol{b}) \in \{0, 1\}$ and a payment rule $t_e(\boldsymbol{b}) \geq 0$ to determine whether or not agent e is selected, and the payment to agent e, respectively.

We assume agents are rational and strategic, they will choose bids to maximize their own utilities and may lie about their cost if lying can increase their utilities. Let allocation $\boldsymbol{x} = (x_1, \cdots, x_n)$ and payment profile $\boldsymbol{t} = (t_1, \cdots, t_n)$ denote the outcome of a path mechanism. The utility of agent e is defined through the following quasi-linear function,

$$\pi_e = \begin{cases} t_e - c_e & \text{if agent } e \text{ is selected,} \\ 0 & \text{otherwise.} \end{cases}$$

Note that agents are individual rational, which means that they are willing to participate in a path mechanism only if they are guaranteed a non-negative utility, so we have $\forall e \in E, t_e \geq b_e$.

Denote the auctioneer by 0, its utility $\pi_0 = -\sum_{e \in E} t_e$. The social welfare of $N = E \cup \{0\}$ is then the cost of the chosen path,

$$w(N) = \pi_0 + \sum_{e \in E} \pi_e = -\sum_{e \in E} x_e c_e \tag{1}$$

A path auction mechanism is efficient if it selects a minimum cost path from s to g. The minimum cost is denoted as $d(s, g, G)$. Then the social welfare of an efficient mechanism is: $w^*(N) = -d(s, g, G)$.

3.1 The VCG Mechanism for Path Auctions

One appealing mechanism for the path auction problem is the VCG mechanism [27, 6, 16]. The VCG mechanism is efficient, it chooses a shortest path so that the social welfare is maximized. Bidders that are not included in the chosen path are paid 0. For bidder e in the chosen path, its utility is given by,

$$\pi_{\text{VCG},e} = w^*(N) - w^*(N - \{e\}) \tag{2}$$

where $w^*(N - e)$ is the social welfare if bidder e's bid is ignored.

By the definition of π_e, the VCG payment to agent e can be computed as follows,

$$\begin{aligned} t_{\text{VCG},e} &= \pi_{\text{VCG},e} + b_e \\ &= w^*(N) - w^*(N - \{e\}) + b_e \\ &= -d(s, g, G) + d(s, g, G - \{e\}) + b_e \end{aligned} \tag{3}$$

where $G - \{e\}$ stands for the graph G with edge e removed and $d(s, g, G - \{e\})$ is the cost of the shortest path from s to g in $G - \{e\}$. Alternatively, $d(s, g, G - \{e\})$ can also be considered as the cost of shortest path from s to g in the graph G if we set $c_e = \infty$.

The VCG mechanism is dominant strategy truthful. Note that in Equation 3, as bidder e is in the chosen path, its bid b_e also appears in $d(s, g, G)$, so it can be cancelled with the last term. The VCG payment t_e is thus not dependent on bidder e's reported cost b_e. Therefore, it is a weakly dominant strategy for bidders to report their true costs: $\forall e \in E, b_e = c_e$.

3.2 Problems with the VCG Mechanism

The VCG mechanism is appealing because it is the only auction mechanism that is both truthful and efficient. However, it suffers from two problems that make it rarely used in practice.

The first is its overpayment problem. Consider a graph with two disjoint paths from s to g, the shortest path p_1 with cost 0 and the second shortest path p_2 with cost 1. The VCG mechanism pays 1 to each agent in p_1. If there is $n - 1$ edges in p_1, then VCG will pay $n - 1$ for p_1, which is a $\Theta(n)$ factor more than the cost of the second cheapest path.

In fact, using results from single-parameter mechanism design [23], it can be shown that such a worst-case overpayment is an intrinsic property of any truthful path mechanism [14].

The second problem of the VCG mechanism is that it is vulnerable to false-name bidding or shill bidding, where bidders create fake names and submit multiple bids under these names [31, 12]. This kind of strategic bidding is easy to implement in path auctions on social networks because it is hard to verify all the bidders' identities.

4 Core-Selecting Path Mechanisms

In this section, we propose core-selecting path mechanisms to address the overpayment problem in the VCG mechanism and provide robustness against false-name bids. Since VCG is the unique mechanism to guarantee both allocation efficiency and dominant strategy truthfulness, we have to relax dominant-strategy truthfulness and use alternative solution concepts. The idea is to model path auction as a cooperative game (N, W) and use the core as our solution concept.

The set of agents in the cooperative game is $N = E \cup \{0\}$, which includes all bidders in E and the buyer 0. For a coalition $L \subseteq N$, its coalition value function $W(L)$ is defined as,

$$W(L) = \begin{cases} -d(s, g, L) & \text{if } 0 \in L, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

where $d(s, g, L)$ is the cost of the optimal $s{-}g$ path in the graph formed by L. Note that if a coalition does not include the buyer, then its coalition value equals 0.

We can now define the concept of the core. An outcome is in the core when the total utility of N equals $W(N)$, and the total utility to every coalition L of agents is at least $W(L)$.

Definition 1 (Core outcome)**.** *A core outcome in a path auction mechanism is an allocation and payment profile such that the utility profile $\boldsymbol{\pi} = (\pi_1, \cdots, \pi_n)$ satisfies:*

$$\textit{(C0)}: W(N) = \sum_{i \in N} \pi_i \tag{5}$$

$$\textit{(C1)}: W(L) \leq \sum_{i \in L} \pi_i \quad \forall L \subseteq N \tag{6}$$

The first core constraint $(C0)$ requires that the total utility of N equals $-d(s, g, N)$, which means that the optimal path is always selected. For any coalition L, the second core constraint set $(C1)$ requires that the total utility to L is no less than the value it could obtain, which equals $-d(s, g, L)$.

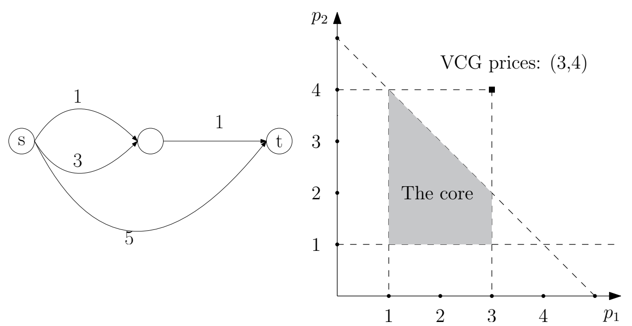

Figure 2: The core of a path auction. Left: There are four bidders with cost $1, 1, 3, 5$. Right: Core payments for bidders with cost 1. The VCG payment profile $(3, 4, 0, 0)$ is not in the core.

Given a bid profile $\boldsymbol{b}$, let $Core(\boldsymbol{b})$ be the set of payment profiles in the core when the bidders' cost profile $\boldsymbol{c}$ equals $\boldsymbol{b}$. It can be verified that $\boldsymbol{b}$ is always in the core: $\boldsymbol{b} \in Core(\boldsymbol{b})$, so the set $Core(\boldsymbol{b})$ is always non-empty. This means that the first-price path auction is core-selecting. We can now define the concept of core-selecting path mechanisms.

Definition 2 (Core-selecting path mechanism)**.** *A path auction mechanism is core-selecting if* (1) *it selects the optimal path; and* (2) *the payment profile* $\boldsymbol{t}$ *is computed so that* $\boldsymbol{t} \in Core(\boldsymbol{b})$.

Example 1. *A path auction is shown in Figure 2. There are four agents, the cost profile* $\boldsymbol{c} = (1, 1, 3, 5)$. *The optimal path with cost* 2 *is selected in both the VCG mechanism and core-selecting mechanisms. The VCG payment profile* $\boldsymbol{t}_{VCG} = (3, 4, 0, 0)$. *The core payments* $Core(\boldsymbol{b})$ *is the convex hull of the set* $\{(1, 1, 0, 0), (1, 4, 0, 0), (3, 1, 0, 0), (3, 2, 0, 0)\}$. *We can see that the VCG payment profile* $\boldsymbol{t}_{VCG} \notin Core(\boldsymbol{b})$, *therefore the VCG mechanism is not a core-selecting path mechanism.*

5 Theoretical Results

In this section, we give several theoretical results for core-selecting path mechanisms. The first is that they are always more frugal than the VCG mechanism, specifically, core payments are never higher than VCG payments.

Theorem 1. *The VCG payment to bidder i equals its highest core payment over all payment profiles in the core. That is,*

$$\forall i \in E, \quad max\{t_i | \boldsymbol{t} \in Core(\boldsymbol{b})\} = t_{VCG,i} \tag{7}$$

Proof. First, note that there is a core outcome in which bidder i gets its VCG payment $t_{\text{VCG},i} = W(N) - W(N - i) + b_i$, while other bidders are paid their cost and get a utility of 0. The buyer's utility $\pi_0 = W(N - i)$. We can verify that this utility profile is in the core. Therefore, we have

$$max\{t_i | \boldsymbol{t} \in Core\} \geq t_{\text{VCG},i} \tag{8}$$

Meanwhile, suppose that in some core outcome bidder i is paid strictly more than its VCG payment, then its utility $\pi_i > t_{\text{VCG},i} - b_i$. For coalition $N - i$, we have

$$\sum_{k \in N - i} \pi_k = W(N) - \pi_i < W(N - i) + b_i \leq W(N - i) \tag{9}$$

The core constraint corresponding to coalition $N - i$ is violated, so we have

$$max\{t_i | \boldsymbol{t} \in Core\} \leq t_{\text{VCG},i} \tag{10}$$

The proof is completed by combining (8) and (10). □

The second result is that core-selecting path mechanisms are robust against false-name bidding. In fact, we show that they are the only type of efficient path mechanisms that are robust against false-name bidding.

Theorem 2. *An efficient path auction mechanism has the property that no bidder can earn more than its VCG payment by bidding with false names if and only if it is a core-selecting mechanism.*

Proof. Let $L \subseteq N$ be a coalition of bidders, it's possible that these bidders are false-name bidders. The condition requires that these bidders can not get more payment than if they were to submit their merged bid in a VCG mechanism, which means $\sum_{i \in L} t_i \leq t_{\text{VCG},L}$. In the VCG mechanism, the payment for coalition L is,

$$t_{\text{VCG},L} = W(N) - W(N - L) + \sum_{i \in L} b_i \tag{11}$$

The condition is therefore

$$\sum_{i \in L} t_i \leq t_{\text{VCG},L} = W(N) - W(N - L) + \sum_{i \in L} b_i \tag{12}$$

Since the path auction mechanism is efficient, we have $W(N) = \pi_0 + \sum_{i\in N} \pi_i$. The condition can be written as,

$$\sum_{i\in(N-L)\cup\{0\}} \pi_i \geq W(N-L) \tag{13}$$

Since L is an arbitrary coalition of bidders, we have that for any coalition $T = N - L$,

$$\sum_{i\in T\cup\{0\}} \pi_i \geq W(T) \tag{14}$$

Therefore all core constraints in $C1$ are satisfied. Combining this with efficiency, we have $\boldsymbol{t} \in Core(\boldsymbol{b})$. □

5.1 Core constraint set formulation

The core of a path auction is defined in terms of coalitions. For a network with n edges, the number of core constraints in $C1$ is 2^n. However, many constraints are redundant, for example, if a coalition L doesn't contain $s{-}g$ path, then $W(L) = -\infty$, the corresponding constraint is redundant and can be removed. We now formulate the core constraint set $C1$ in terms of paths.

$$\text{(C2)}: \sum_{e\in p-p^*} c(e) \geq \sum_{e\in p^*-p} t(e) \quad \forall p \in P \tag{15}$$

where P is the set of all paths from s to g, p^* is the optimal path, and $p - p^* = \{e \in p | e \notin p^*\}$, $p^* - p = \{e \in p^* | e \notin p\}$.

The following proposition shows that constraint set $C1$ and $C2$ describe the same set of core payments.

Proposition 1. *The two sets of constraints $C1$ and $C2$ describe the same core.*

Proof. Let $L \subseteq N$ be a coalition of bidders. By $C1$, we have

$$W(L\cup\{0\}) \leq \pi_0 + \sum_{i\in L} \pi_i \tag{16}$$

Assume that the optimal $s{-}g$ path in the sub-graph formed by L is p. Since all bidders not in p^* are not selected and have utility 0, the constraint can be rewritten as

$$-\sum_{e\in p} c(e) \leq -\sum_{e\in p^*} t(e) + \sum_{e\in p\cap p^*} [t(e) - c(e)] \tag{17}$$

After rearranging the terms, we get

$$\sum_{e\in p^*} t(e) - \sum_{e\in p\cap p^*} t(e) \leq \sum_{e\in p} c(e) - \sum_{e\in p\cap p^*} c(e)$$

which is equivalent to the constraint in $C2$. □

The size of core constraint set is reduced to the number of $s{-}g$ paths. However, there exists graphs with exponentially many $s{-}g$ paths, so there might still be an exponential number of core constraints in $C2$ in the worst case. In fact, it is #P-complete to count the number of $s{-}g$ paths in a general graph.

Note that $p^* - p$ is a subset of p^*, we can enumerate all $p^* - p$ and create a new formulation for the core constraint set in terms of p^*'s subsets.

$$\text{(C3)}: d(s,g,G-x) - d(s,g,G) + \sum_{e\in x} c(e) \geq \sum_{e\in x} t(e) \ \forall x \subseteq p^* \tag{18}$$

where $d(s,g,G-x)$ is the cost of the optimal $s{-}g$ path in the graph G with edges in x removed. Theorem 3 shows that $C3$ still describes the same set of core payments as $C1$ and $C2$.

Theorem 3. *The two sets of constraints $C2$ and $C3$ describe the same core.*

Proof. We will show that for each constraint in one set, there is a constraint in the other set that implies it. Therefore, the two sets describe the same core.

$(C3) \implies (C2)$: Every constraint in $(C2)$ corresponds to a path p. For every such path p, there exists a subset x of p^* such that $x = p^* - p$. By $(C3)$, we have

$$\sum_{e\in p^*-p} t(e) \leq d(s,g,G-(p^*-p)) - d(s,g,G) + \sum_{e\in p^*-p} c(e) \tag{19}$$

As $p^* - p = p^* - (p^* \cap p)$, we get

$$-d(s,g,G) + \sum_{e\in p^*-p} c(e) = -\sum_{e\in p^*\cap p} c(e) \tag{20}$$

Since $p \subset (G - (p^* - p))$, p is a valid path in $G - (p^* - p)$, we get

$$d(s,g,G-(p^*-p)) \leq \sum_{e\in p} c(e) \tag{21}$$

Combining (19), (20), and (21), we have

$$\sum_{e\in p^*-p} t(e) \leq \sum_{e\in p} c(e) - \sum_{e\in p^*\cap p} c(e) = \sum_{e\in p-p^*} c(e) \tag{22}$$

which is p's corresponding constraint in $(C2)$. Therefore every constraint in $(C2)$ also exists in $(C3)$.

$(C2) \implies (C3)$: Every constraint in $(C3)$ corresponds to a subset x of the optimal path p^*. For every such subset x, there exists a path p such that p is the optimal path in the graph $G - x$, so we have $\sum_{e\in p} c(e) = d(s,g,G-x)$.

As $x \subseteq p^*$ and $x \cap p = \emptyset$, we get

$$\sum_{e\in p^*-p} t(e) = \sum_{e\in x} t(e) + \sum_{e\in(p^*-p)-x} t(e) \tag{23}$$

By $C2$, we have

$$\sum_{e\in x} t(e) + \sum_{e\in(p^*-p)-x} t(e) \leq \sum_{e\in p} c(e) - \sum_{e\in p\cap p^*} c(e) \tag{24}$$

For each $e \in p^*$, we have $c(e) \leq t(e)$,

$$\sum_{e\in x} t(e) \leq \sum_{e\in p} c(e) - \sum_{e\in p\cap p^*} c(e) - \sum_{e\in(p^*-p)-x} c(e) \tag{25}$$

$$= \sum_{e\in p} c(e) - \sum_{e\in p^*} c(e) + \sum_{e\in p^*-p} c(e) - \sum_{e\in(p^*-p)-x} c(e) \tag{26}$$

$$= d(s,g,G-x) - d(s,g,G) + \sum_{e\in x} c(e) \tag{27}$$

Therefore, the constraint corresponding to p in $C2$ implies the constraint corresponding to x in $C3$. □

The theorem indicates that for a shortest path with k edges, the number of core constraints in $(C3)$ is 2^k. Real social networks often have small diameter (longest shortest path length k), which allows us to compute core payments efficiently using linear programming techniques.

6 Pricing Algorithms

As core-selecting path mechanisms are not dominant-strategy truthful, it is important to provide incentives for bidders to bid truthfully. In this section, we present Vickrey-nearest pricing rule for core-selecting path mechanisms, which is shown to have good incentive properties. The idea is to find core payments that maximize the total payment and are as close to the VCG payments as possible.

6.1 Maximum Payment

We employ a two-step optimization algorithm to determine core payments. First, we maximize the total payment over the core. Recall the core constraint set $C3$, for each subset x of the optimal path p^*, we have

$$\sum_{e\in x} t(e) \le d(s,g,G-x) - d(s,g,G) + \sum_{e\in x} c(e) \tag{28}$$

Let $\beta_x = d(s,g,G-x) - d(s,g,G) + \sum_{e\in x} c(e)$, and denote the vector of all β_x values as $\boldsymbol{\beta}$, we have

$$A\boldsymbol{t} \le \boldsymbol{\beta} \tag{29}$$

where A is a $2^k \times k$ matrix, k is the number of edges in p^*. A_{ij} equals 1 if bidder j is in the i-th subset and equals 0 otherwise. The maximum total payment α can then be found using the following linear program (LP-0):

$$\begin{aligned} \text{(LP-0)} : \alpha = \max\ & \boldsymbol{t}\cdot\mathbf{1} \\ \text{subject to: } & A\boldsymbol{t} \le \boldsymbol{\beta} \\ & \boldsymbol{t} \ge \boldsymbol{c} \end{aligned} \tag{30}$$

The linear program LP-0 has $2^k + k$ constraints, which is exponential in the number of variables in LP-0, however, we can still prove that LP-0 can be solved in polynomial time.

Proposition 2. *The maximum core payment α can be computed in time polynomial in k by solving the linear program LP-0.*

Proof. We first show that there is a polynomial time separation oracle, which given a core payment profile $\boldsymbol{t}$, answers that $\boldsymbol{t}$ satisfies $A\boldsymbol{t} \le \boldsymbol{\beta}$ and $\boldsymbol{t} \ge \boldsymbol{c}$, or returns an inequality that is not satisfied by $\boldsymbol{t}$.

The second set of constraints $\boldsymbol{t} \ge \boldsymbol{c}$ is easy to verify. To check that whether $\boldsymbol{t}$ satisfied the first set of constraints $A\boldsymbol{t} \le \boldsymbol{\beta}$, we set the cost of edges in the original optimal path p^* to be $\boldsymbol{t}$ and then compute a new optimal path p'. If the optimal path stays the same, $p^* = p'$, then the first set of constraints is satisfied; otherwise assume that $pp = p^* - p'$, then the following constraint corresponding to pp is not satisfied: $\sum_{e\in pp} t(e) \le \beta_{pp}$.

Since the separation oracle runs in polynomial time, the ellipsoid method can give solutions to LP-0 in time polynomial in k. □

6.2 VCG-Nearest Payments

In the second step, we find core payments to minimize the L_∞ distance to the VCG payments. The L_∞ between two payment vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ is defined as

$$L_\infty(\boldsymbol{x},\boldsymbol{y}) = \max\{|x_1 - y_1|, |x_2 - y_2|, \cdots, |x_k - y_k|\}$$

Besides the core constraints in (LP-0), we add one more constraint that the sum of payments should equal the maximum total payment: $\boldsymbol{t}\cdot\mathbf{1} = \alpha$. Then we can find the Vickrey-nearest payments by solving the following problem:

$$\begin{aligned} \text{(NLP-1)}: r = \min\ & \|\boldsymbol{t} - \boldsymbol{t}_{\text{VCG}}\|_\infty \\ \text{subject to: } & A\boldsymbol{t} \le \boldsymbol{\beta} \\ & \boldsymbol{t} \ge \boldsymbol{c} \\ & \boldsymbol{t}\cdot\mathbf{1} = \alpha \end{aligned} \tag{31}$$

It is hard to optimize (31) directly. We reformulate it as a linear program by adding a new variable y. Meanwhile, for each edge i in the optimal path, we add a new constraint: $|t_{\text{VCG},i} - t_i| \le y$. y can be understood as the the maximum difference between t_i^{VCG} and t_i. As $t_i \le t_{\text{VCG},i}$, the new constraints can be rewritten as: $\boldsymbol{t} + y\mathbf{1} \ge \boldsymbol{t}_{\text{VCG}}$. Then we can find the Vickrey-nearest payments $\boldsymbol{t}$ by solving the following linear program:

$$\begin{aligned} \text{(LP-2)}: r = \min\ & y \\ \text{subject to: } & A\boldsymbol{t} \le \boldsymbol{\beta} \\ & \boldsymbol{t} \ge \boldsymbol{c} \\ & \boldsymbol{t}\cdot\mathbf{1} = \alpha \\ & \boldsymbol{t} + y\mathbf{1} \ge \boldsymbol{t}_{\text{VCG}} \end{aligned} \tag{32}$$

For an optimal path with k edges, the linear program (LP-2) has k decision variables and $2^k + 2k + 1$ constraints.

Theorem 4. *The Vickrey-nearest core payments can be computed in polynomial time by solving linear program LP-2.*

Proof. Given a core payment profile $\boldsymbol{t}$ and y, we already give a polynomial time separation oracle for $A\boldsymbol{t} \le \boldsymbol{\beta}$ and $\boldsymbol{t} \ge \boldsymbol{c}$ in Proposition 2. The third set of constraints $\boldsymbol{t}\cdot\mathbf{1} = \alpha$ is trivial to check. The last set of constraints can also be sequentially verified in $O(k)$ time. Therefore, we can also solve LP-2 in polynomial time using the ellipsoid method. □

For the path auction in Figure 2, the maximum total payment α is 5, and the Vickerey-nearest core payments is $(2, 3, 0, 0)$.

To analyze the incentive property of the Vickerey-nearest pricing rule, we assume bidders know each others' costs and analyze the Nash equilibrium. Denote $\hat{b}_{-i}$ as the bids of bidders other than i. Given any $\hat{b}_{-i}$, let $\hat{t}_{\text{VCG},i}$ be the VCG payment to bidder i when i is truthful. Then we have the following result,

Proposition 3. *$\hat{t}_{VCG,i}$ is a best response by bidder i to the bids $\hat{b}_{-i}$ of others.*

Proposition 3 holds because if i's bid is more than $\hat{t}_{\text{VCG},i}$, then the core constraint corresponding to coalition $N - \{i\}$ is violated. Define the regret of a bidder as the difference between his utility submitting a best-response bid to the bids of others and his utility when bidding truthfully. We can prove the following result,

Theorem 5. *The Vickerey-nearest core-selecting path mechanism minimizes the maximum regret for bidders across all core-selecting path mechanisms.*

Proof. Assume that all bidders are truthful. Fixing bids b_{-i} of others, the best response of bidder i is to bid his VCG payment $t_{\text{VCG},i}$. Then bidder i's regret is $t_{\text{VCG},i} - t_i$, where t_i is the VCG-nearest core payment for bidder i. As the Vickerey-nearest payments $\boldsymbol{t}$ minimizes the L_∞ distance to the VCG payments $\boldsymbol{t}_{\text{VCG}}$, the maximum regret for all bidders is minimized. □

7 Experiment Results

In this section, we evaluate the performance of false-name proof path mechanisms.

7.1 Datasets and Problem Generation

First we describe the network and cost(bidding) datasets used in the following experiments. The network datasets we used are from the SNAP datasets [21], which consist of four social networks:

- Facebook network. The dataset consists of friends lists from Facebook. The data was collected from survey participants using Facebook app [3].
- Google+ network. This dataset consists of circles (friends lists) from Google. The data was collected from users who had manually shared their circles.
- Twitter network. This dataset consists of friend list from Twitter. The data was crawled from public sources.
- Wikipedia voting network. The network contains voting data for Wikipedia administrators elections . Nodes in the network represent wikipedia users and a directed edge from node i to node j represents that user i voted on user j.

The detailed network statistics are given in table 1.

Networks	Nodes	Edges	$d_{\max}$	90-percentile $d_{\max}$
Facbook	4,039	88,234	8	4.7
Google+	107,614	13,673,453	6	3.0
Twitter	81,306	1,768,149	7	4.5
Wiki-Vote	7,115	103,689	7	3.8

Table 1: Network statistics, $d_{\max}$ is the network diameter, or the maximum shortest path length, the last column is the 90-th percentile of shortest path length distribution.

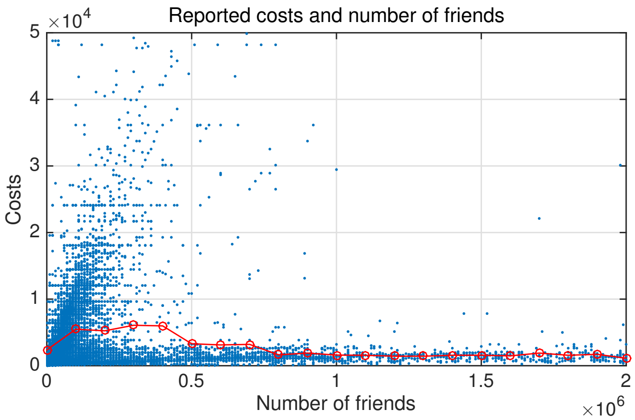

Figure 3: Reported costs plotted against number of friends. The red line shows average reported costs.

Compared with network data, the true costs of bidders are difficult to obtain. In our experiments, we use reported cost data from a microblog advertising platform weiboyi [4], where microbloggers are asked to report their costs to make recommendations to friends in their social network. Figure 3 shows reported costs and the number of friends for 15082 microbloggers. As the correlation between reported costs and the number of friends is rather small (-0.005), we use a simulation approach to generate realistic bids. Each time we take a cost at random from the microblogger bidding data and assign it to an edge from our networks. We also use another unit cost distribution where all costs are set to 1.

The compared path mechanisms include the VCG mechanism (VCG) and the Vickrey-nearest core-selecting mechanism (VNC). For each network and each path mechanism, we generate 3000 problem instances where the start node s and goal node g are selected uniformly at random from all nodes. All problem instances are executed using CPLEX 12.6 on a 3.1 GHz Intel Core i5 processor.

Networks	Avg. shortest path cost	Avg. VCG payments	Avg. VNC payments	Avg. maximum regret
Facebook	3.63	5.66	5.03	0.26
Wikipedia	2.99	3.78	3.43	0.18
Google+	3.19	3.79	3.52	0.08
Twitter	4.81	6.07	5.46	0.18

Table 2: Average payment and average maximum regret under unit cost distribution.

Networks	Avg. shortest path cost	Avg. VCG payments	Avg. VNC payments	Avg. maximum regret
Facebook	871.07	2113.19	1697.70	81.50
Wikipedia	1038.11	2982.92	2599.35	106.56
Google+	673.71	1607.24	1479.95	35.16
Twitter	1380.82	3584.91	2982.22	162.17

Table 3: Average payment and average maximum regret under reported cost distribution.

7.2 Payment Performance

We first study the payment performance of path mechanisms. In Table 2 and 3, we show average payments for the unit cost distribution and the reported cost distribution respectively. We can see that VNC payments are always less than VCG payments. Under the unit cost distribution, average overpayment is not very high for both mechanisms. However, under the reported cost distribution, VCG overpayments become significant in all four networks. In particular, the VCG mechanism overpays by 2304 in the twitter network when the shortest paths cost only 1380.

The performance measure we used is the overpayment factors. The overpayment factor of a path mechanism M is defined as the ratio between its total payment and the true cost of the shortest path p^*.

$$OF = \frac{\sum_{e \in p^*} t_M(e)}{cost(p^*)} \tag{33}$$

We give detailed average overpayment factor results in Figure 4, where average overpayment factors are plotted against the number of edges in p^*. We can see that under the reported distribution, the VCG mechanism overpays by a factor of 2.5 in the Facebook, Google+ and Twitter networks, and overpays by a factor of 3 in the Wikipedia voting network. Meanwhile, the VNC mechanism only overpays by a factor of 2 in these networks. Meanwhile, under unit cost distribution, there is little difference between the overpayment factors for VCG and VNC mechanisms, this is because the overpayments are already very low. Note that when the shortest path contains only one edge ($k = 1$), the VNC payment is equal to the VCG payment, so

[3] https://www.facebook.com/apps/application.php?id=201704403232744
[4] http://www.weiboyi.com

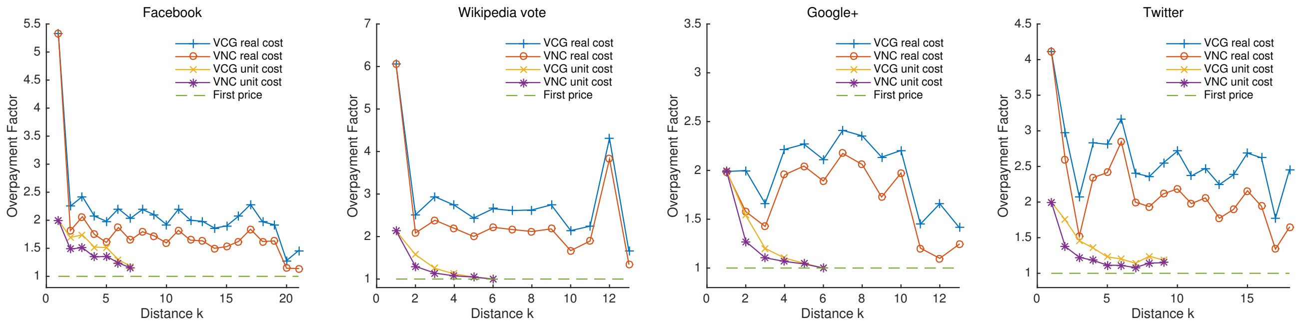

Figure 4: Comparison of average overpayment factors under different cost distributions.

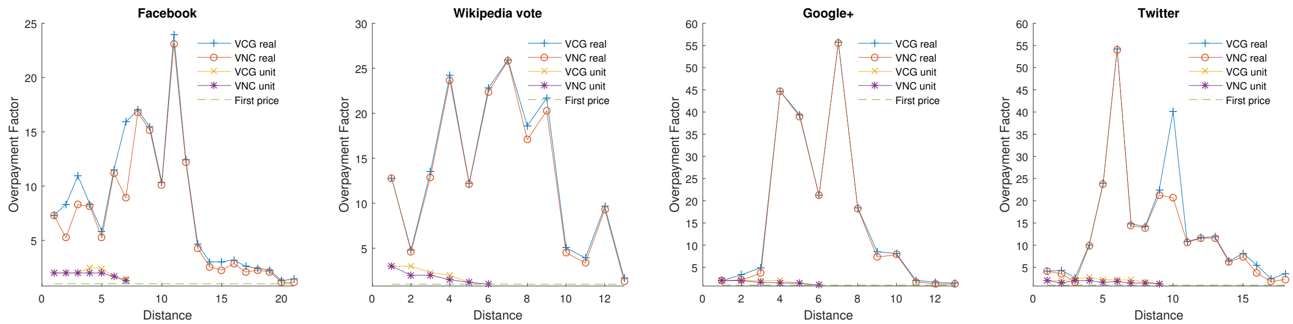

Figure 5: Comparison of worst case overpayment factors under different cost distributions.

the overpayment factors are the same, this is confirmed by Figure 4. We also plot worst case overpayment factor results in Figure 5. We can see that under reported cost distributions, VNC's overpayment factors and VCG's overpayment factors are very close in the worst case.

We next evaluate bidders' maximum regrets under the two cost distributions. Define the maximum regret ratio as the ratio between the maximum regret and the shortest path cost. Under the unit cost distribution, the maximum regret ratio is less than 3% in the Google+ network and around 6% in the others. Under the reported cost distribution, the maximum regret ratio is around 5% in the Google+ network and around 10% in the others.

Networks	first-price	VCG	maxpay	VNC
Facbook	0.004	0.050	0.102	0.104
Google+	0.156	5.382	6.921	6.922
Twitter	0.026	0.790	1.309	1.310
Wiki-Vote	0.001	0.042	0.055	0.056

Table 4: Average time performance (in seconds) under unit cost for first-price mechanism (first-price), the VCG mechanism (VCG), maximum payments (maxpay) and Vickrey-nearest payments (VNC).

7.3 Time Performance

In the next, we evaluate the time performance of path mechanisms. Table 4 shows the average time performance of four different path mechanisms under unit cost. We can see that the time performance of the maximum core payment and the Vickrey-nearest payment is very close. Moreover, in the Facebook network and the Twitter network, core-selecting path mechanisms have comparable time performance with the first-price mechanism and the VCG mechanism. In the Google+ network and the Wikipedia voting network, core-selecting path mechanisms use about twice the time compared with the VCG mechanism, however, note that all mechanisms use less than 1 seconds.

Networks	first-price	VCG	maxpay	VNC
Facbook	0.005	0.073	12.892	27.530
Google+	0.220	6.220	29.420	29.431
Twitter	0.040	1.001	151.043	175.300
Wiki-Vote	0.001	0.054	0.330	0.382

Table 5: Average time performance (in seconds) under reported cost for first-price mechanism (first-price), the VCG mechanism (VCG), maximum payments (maxpay) and Vickrey-nearest payments (VNC).

Table 5 shows the average time performance of four different path mechanisms under the reported cost distribution. We can see from the table that the first-price path mechanism and the VCG path mechanism only need less than 1 seconds in all networks except the Google+ network, where it take 6 seconds on average to compute the payments. Meanwhile, the maximum core payments and VCG-nearest core payments need much more time to compute, in the Wikipdia voting network the time performance is comparable with that of the first-price mechanism and the VCG mechanism. In the Twitter network, the core-selecting mechanisms have a very bad time performance, where it takes more than 2 minutes on average to compute the core payments. We believe the reason is that some agent reported a very high cost, which make the linear program LP-0 very hard to solve by CPLEX. This indicates one possible future research direction that we may need to find more efficient algorithms for computing the maximum core payments when very high cost (bids) can be reported.

8 Conclusions

In this paper, we propose false-name-proof path mechanisms to address the overpayment problem of the VCG path mechanism and provide robustness against false-name manipulations. Based on a novel core constraint formulation, we give a polynomial time core-selecting path mechanism which finds the core payment profile that minimizes the L_∞ distance to the VCG payment profile. We show that the Vickrey-nearest core-selecting path mechanism has good incentive properties. Experiment results on real network and reported cost data show that the overpayment of the VCG mechanism can be very high, while the Vickrey-nearest core-selecting path mechanism can reduce VCG's overpayment by about 20%. Therefore, Vickrey-nearest core-selecting path mechanism can be considered as an appealing candidate mechanism for path auctions in social network environments where false-name manipulations are too common to ignore.

In this paper, we assume that all possible subsets of agents can form coalitions, however, in real world applications it is reasonable to assume that only connected agents can form coalitions. It is an interesting direction to devise efficient payment algorithms for this kind of environment.

ACKNOWLEDGEMENTS

The authors would like to thank the anonymous reviewers for their helpful suggestions. This paper is supported by the National Natural Science Foundation of China (Grant No. 61375069,61403156, 61502227) and this research is partially supported by the Collaborative Innovation Center of Novel Software Technology and Industrialization, Nanjing University.

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