# Managing Energy Markets in Future Smart Grids Using Bilateral Contracts

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**Abstract.** Future smart grids will empower home owners to buy energy from real-time markets, coalesce into energy cooperatives, and sell energy they generate from their local renewable energy sources. Such interactions by large numbers of small prosumers (that both consume and produce) will engender potentially unpredictable fluctuations in energy prices which could be detrimental to all actors in the system. Hence, in this paper, we propose negotiation mechanisms to orchestrate such interactions as well as pricing mechanisms to help stabilise energy prices on multiple time scales. We then prove 1) that our solution guarantees that, while prices fluctuations can be constrained, 2) that it is individually rational for agents to join energy cooperatives and 3) that the negotiation mechanisms we employ result in pareto-optimal solutions.

# **1 INTRODUCTION**

Future smart grids aim to allow the seamless integration of distributed renewable energy (wind or solar) to provide clean and renewable energy. Moreover, as smart meters are deployed as part of smart grid initiatives, home owners will be able to participate in energy markets to, not only buy and store energy, but also shift their consumption according to real-time prices as well as sell the surplus energy they generate from their local energy sources, acting as *prosumers* [8]. Crucially, with smarter communication technologies and home energy management systems, prosumers will be able to form collectives to have a greater say in energy markets.

The smart grid will therefore engender an influx of such new and smaller actors into the energy markets trading alongside larger existing players energy producers that manage large energy sources (nuclear or gas). Experience from existing energy wholesale markets and commodity stock markets, indicate that, in contrast to markets with a few large suppliers, prices in these open markets will tend to fluctuate unpredictably whenever imbalances exist between demand and supply (see figure 1). This may lead to speculation in the market, which exacerbates the situation. In such circumstances, the complexity of coping with fluctuating prices will make it even more difficult for consumers to save money and manage their energy. Moreover, the difficulty of predicting demand and supply could lead to dangerous imbalances that could cause blackouts. Previous work has investigated allocating demand according to supply using specific pricing signals [11, 7], which incentives consumers to shift their needs to times where supply is high. Moreover, they have proposed the use of batteries and the exchange of energy [3, 14] to buy and stock when



**Figure 1.** The evolution of the spot price. The red (resp. blue) line represents the price on the German (resp. French) market in 2013 ( $\in$  /MWh)

energy is cheap and consume or store when energy is expensive. Finally, some authors propose to gather consumers into cooperatives [2] or coalitions [12] to benefit from cheaper prices in the forward market and to buy less as possible on the spot market. Unfortunately, these different works assume that the grid is always able to provide energy, if the other sources (renewable generators or batteries) cannot.

In this paper, we study the problem of forming such cooperatives without such assumptions and provide negotiation-based solutions to the settlement of contracts between prosumers. We consider cooperatives because they significantly improve the buying power of small prosumers. To help manage such cooperatives, we assume that they are set up and led by individual aggregators that buy from selected providers to supply energy at the cheapest rate to the cooperative. Similar to [12], aggregators do so using predictions of energy consumption provided by individual prosumers. Moreover, to ensure that prosumers are incentivised to predict their behaviours accurately, the aggregator penalises any deviations from such predictions. These penalties reflect charges they would incur from the provider should they over or under consume. However, in so doing, a key challenge they face is that individual consumers may want to consume more to avoid being penalised by the aggregator (and in turn the producer). Moreover, despite the formation of large cooperatives, there is no guarantee that prices will stabilise in the long run.

Against this background, we propose an approach that uses pricing signals while attempting to control the high price volatility when there is (or when agents forecast and speculate) an imbalance between supply and demand in the energy market. To address such issues, we propose a model based on bilateral contracts that constrain

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the retail price as well as the demand. In a similar way to existing energy trading protocols (e.g., in the UK or Netherlands), our framework breaks down the creation of energy supply contracts according to three different timescales: yearly, daily, and hourly. By so doing, contracts account for different levels of perceived uncertainty in demand on these timescales. Given this scenario, this paper advances the state of the art in the following ways:

- We propose a new mechanism to manage energy markets through the use of constraints on energy prices and demand for energy. We prove that the negotiation of the annual contract (the first of the three levels), between providers and aggregators, where these constraints are set, leads to pareto-optimal solutions.
- 2. We propose a novel pricing scheme that we apply at the two lower levels (i.e., daily and hourly). Our pricing scheme incentivises prosumers to (i) make predictions to know the amount of energy they need during the different parts of the day ahead allowing them to optimise their costs (by shifting the use of their appliances, using their batteries, etc...) according to the price signal and (ii) avoid deviating from their predicted hourly consumption. We prove the monotonicity of the price with respect to consumption (i.e. agents are not incentivised to consume in order to avoid under-consumption penalties) and provide a closed-form formula to compute expected payments for any prosumer in a cooperative.
- 3. We develop an algorithm based on mathematical programming, that allows agents to minimise their cost given the price signal they receive and the limits of consumption that they are committed to holding. We prove that, in our mechanism, prosumer agents who use this algorithm, are encouraged to join a cooperative.

The rest of the paper is structured as follows. Section 2 presents the state of the art. Section 3 gives the model of each type of agents and Section 4 gives their behaviour. Section 5 shows the negotiation mechanism. Section 6 analyses the different properties of the mechanism, Section 7 describes the experiments and we conclude in section 8.

# 2 RELATED WORK

To tackle the problem of peak demand, several works proposed incentive tariff schemes which aim to balance supply and demand. In Demand-Response models [4], suppliers change their tariffs when the demand is high, encouraging consumers to reduce their consumption. Several mechanisms based on Demand-Response have been proposed: Time-of-use (TOU) in which price of energy is high at peak times and low at off-peak times (typically after 11pm). Crit*ical peak pricing* [6] imposes a much higher price at peak times compared to tariffs with TOU. Real Time pricing (RTP) [10] varies the price across the day in line with supply and demand at individual time points. Recently, a new tariff scheme has been proposed by [13], called Prediction-of-use (POU). In this model, consumers give a forecast of their baseline consumption against which they are given a price for predicted consumption. If they deviate from this forecast based consumption, they will be penalised. This tariff scheme incentives consumers to forecast their consumption accurately. In our model, we combine TOU with POU schemes to both account for varying levels of demand and supply during the day and to obtain accurate predictions of consumption from prosumers.

Other works study the formation of prosumer (or consumer) agent coalitions or cooperatives with the aim to reduce energy costs and increase storage efficiency. [2] presents a scheme for electricity consumption shifting. Agents participating in the scheme are motivated to form cooperatives, in order to reduce their electricity bills via lower group prices granted for sizable consumption shifting from high to low demand time intervals. Even though this work uses agents cooperatives to shift peak load, it doesn't focus on price volatility. [11] proposes a new multiagent coordination algorithm to shape the energy consumption of the cooperative. To coordinate individual consumers, they introduce a virtual signal sent by a central coordinator, to induce consumers to shift demand. They show that their algorithm is scalable with respect to the number of agent, but they use a two-level threshold rate, which is a high constraint to deal with the price volatility. In our work, the price has an upper and lower bound which are negotiated according to agents' preferences. Among these, we note the work of [12] that proposes a coalition formation mechanism in which agents set up a coalition which simulates a Virtual Energy Consumer (VEC). The VEC buys an important amount of energy on the forward market where prices are fixed and quite cheap. If the VEC needs more, it buys more on the spot market, where prices are higher. Being in a cooperative, agents buy more energy on the forward market (compared to the spot market) and compensate for their deviation within the cooperative. However, this work focuses on stable coalition formation rather than controlling price volatility. In [9], authors study the POU scheme and analyse the case of efficient buyer groups. They study the structure groups should take to buy in an efficient way in POU context. In contrast, in our model, one level of our architecture uses the TOU approach to negotiate a baseline price. This price is taken as reference to use POU scheme.

# **3 MODEL OF AGENTS**

In this work, we consider three sets of agents.  $\mathcal{A}_U$  denotes the set of prosumer agents,  $\mathcal{A}_A$  denotes the set of aggregator agents and  $\mathcal{A}_F$  denotes the set of provider agents. We provide the model of each type of agents. In the following sections, we describe the model of prosumer agents which may own a storage capacity, a generator and smart meters allowing them to manage their demand to fit the production signal. As well as prosumers, producers can estimate their production (with smart meters technologies) and produce a signal that foster the best behaviour of their customers, i.e. stalling the level of demand on the production. In our model, the ability of the prosumer/consumer to fit the production signal leads to the control of the price volatility. Next, we assume that a day is divided into a set  $\mathcal{T}$  of hourly slots such that each agent needs to decide its behaviour for each slot.

## 3.1 Model of prosumer agents

Let  $ps_u$  be an agent, representing a prosumer u, belonging to a cooperative. At each slot,  $ps_u$  requests for an amount of energy  $q_{ps}$ , from the cooperative, to support its needs.  $ps_u$  is committed itself to respect limits of requested energy with  $Q_{ps}^{min}$  (resp.  $Q_{ps}^{max}$ ), the minimum (resp. maximum) energy requested by  $ps_u$  from the cooperative  $\forall t \in \mathcal{T}$ .  $b_{ps}^t$  is the energy needed by  $ps_u$  at slot t.  $b_{ps}$  is different from  $q_{ps}$  as it only takes into account the appliance consumptions but not the possible production or use of storage energy owned by the prosumer.  $s_{ps}$  is the storage capacity of  $ps_u$ .  $soc_{ps}^-$  represents the state of charge and  $soc_{ps}^+$  represents the remaining storage capacity, i.e.  $soc_{ps}^- + soc_{ps}^+ = s_{ps}$ .  $p_{ps}$  represents the energy produced by the prosumer's renewable generator,  $p_{ps} \leq p_{ps}^{max}$ , with  $p_{ps}^{max}$  the maximum production capacity of the generator.  $\sigma_{ps}^b$  is the forecast mean error of  $ps_u$  consumption. The deviation

between the forecast of  $ps_u$ 's needs and its real needs follows a normal law  $\mathcal{N}(b_{ps}, \sigma_{ps}^b)$  (see [5] for explanation).  $\sigma_{ps}^p$  is the forecast mean error of  $ps_u$  production. The deviation between the forecast of the prosumer's production and its real production follows the normal law  $\mathcal{N}(p_{ps}, \sigma_{ps}^p)$ .  $ps_u$  can always sell its over-supply to its cooperative, even if its predictions are not accurate. We next prove in Property 3 that it is interesting for a prosumer to belong in a cooperative.

#### 3.2 Model of provider agents

Each provider agent  $pv_f \in \mathcal{A}_F$  has a production capacity  $p_{pv} \in [0; p_{pv}^{MAX}]$  where  $p_{pv}^{MAX}$  is its maximum production capacity. We assume that each  $pv_f$  has an optimal production capacity  $p_{pv}^{OPT}$  which allows it to maximise its profit, with  $0 < p_{pv}^{OPT} < p_{pv}^{MAX}$ . The tariff proposed by the provider is minimal when the demand is equal to  $p_{pv}^{OPT}$  and increases as the demand deviates from it.  $A_{pv}$  is the annual subscription cost of  $pv_f$  and  $pen_{pv}$  represents its penalty costs. Moreover, each  $ps_f$  has a maximal tariff  $tr_{pv}^{MAX}$  above which nobody is willing to buy its energy. The properties 1 and 2 show that  $p_{pv}^{OPT}$  and  $tr_{pv}^{MAX}$  lead to bounded tariffs.

## 3.3 Model of aggregator agents

Let  $ag_a$  be an aggregator agent, with  $\mathcal{Q}_{ag}^{MIN}$  (resp.  $\mathcal{Q}_{ag}^{MAX}$ ), the minimum (resp. maximum) amount of energy requested by the cooperative from its providers  $\forall t \in \mathcal{T}$ . If at slot t the energy  $q_{ag}^t$  requested by  $ag_a$  is higher than  $\mathcal{Q}_{ag}^{MAX}$ , this involves that some prosumers request more than they can. In so doing, the prosumers who overconsume have to contract with providers to account for their overconsumption. Property 4 shows that in this way the cooperative is not penalised.  $\sigma_{ag}^b$  is the forecast mean error of the cooperative consumption.  $Coop_{ag}$  represents the set of agents managed by the aggregator and  $\mathcal{P}_{ag}$  represents the set of providers it contracts with.

# **4 BEHAVIOUR OF AGENTS**

In this section, we describe the behaviour of each type of agents. First, the prosumer behaviour is modelled by a linear program that minimises its day energy cost, given its needs, production, storage and shifting capacities and the price signal, for each slot. Second, we propose a tariff computation formula used by the provider to set the price according to the requested energy and its production. Third, we introduce the algorithm of the aggregator behaviour that computes the best amount to request for the annual contract negotiations with several providers and distributes the penalty of the cooperative among its prosumers.

## 4.1 Prosumer agents behaviour

The optimisation of  $ps_u$  cost is held on three time scales: (i) at the annual contract level,  $ps_u$  announces its limits  $Q_{ps}^{min}$  and  $Q_{ps}^{max}$  which make him pay the cheapest subscription cost and lower the maximum tariff allowing the cooperative to satisfy its needs, (ii) at the daily contract level,  $ps_u$  announces the vector  $\langle q_{ps}^t, \ldots, q_{ps}^{t+n} \rangle$  which minimises its cost taking into account its storage capacities, its production, its possible shifting and the price signal, (iii) at the hourly contract level,  $ps_u$  should not deviate from  $q_{ps}^t$  to avoid penalties. Every day, prosumer minimises equation (2) subject to constraints  $\{c_0, ..., c_{13}\}$ :

$$\min \sum_{t=1}^{24} tr^t . q^t$$
 (2)

With  $tr^t$ , the tariff at slot t and  $q^t$  the energy requested at slot t by the agent.

$$c_0: q^t = b_{ps}^t - p_{ps}^t + qs_{ps}^{+,t} - qs_{ps}^{-,t} + ef_{ps}^{+,t} - ef_{ps}^{-,t} c_1: q^t \ge b_{ps}^t - p_{ps}^t - be_{ps}^t - soc_{ps}^{-,t}$$

This guarantees that the requested energy at slot t is enough to support the prosumer's need according to the available energy in its battery, its production and shifting, with  $be_{ps}^t \in [0, b_{ps}^t]$  the part of  $b_{ps}^t$  which is shiftable at slot t.  $ef_{ps}^{+,t}$  represents shifting that can be made in advance, i.e. increase the demand at slot t and  $ef_{ps}^{-,t} \leq be_{ps}^t$  shifting that can be done later, i.e. decrease the demand at slot t and  $qs_{ps}^{-,t}$  the energy extracted from the battery at slot t.

$$c_2: q^t \leq \mathcal{Q}_{ps}^{max} \\ c_3: q^t \geq \mathcal{Q}_{ps}^{min}$$

The above two constraints guarantee that the requested energy at slot t respects the upper and lower limits.

$$c_4: soc_{ps}^{-,t} \ge qs_{ps}^{-,t} - qs_{ps}^{+,t}$$

This constraint guarantees that the extracted energy from the battery at slot t is lower than the remaining energy in the battery.

$$c_5: soc_{ps} \ge qs_{ps} - qs_{ps}$$
  
This constraint guarantees that the remaining storage capacity is  
higher than the amount of energy stored at slot t.

$$c_6: soc_{ps}^{-,1} = socinit_{ps}$$
  
$$c_7: soc_{ps}^{+,1} = s_{ps} - socinit_{ps}$$

The above two constraints initialize the program with the energy available (and remaining capacity) in the battery at the first slot.

$$c_{8}: soc_{p,t}^{-,t} + soc_{p,t}^{+,t} = s_{ps}$$

$$c_{9}: soc_{p,t}^{+,t+1} = soc_{+,t}^{-,p} - qs_{p,t}^{+,t} + qs_{p,s}^{-,r}$$

$$c_{10}: soc_{-,t}^{-,t+1} = soc_{-,t}^{-,t} + qs_{-,s}^{+,t} - qs_{-,s}^{-,r}$$

 $c_{10}: soc_{ps}^{-,t+1} = soc_{ps}^{-,t} + qs_{ps}^{+,t} - qs_{ps}^{-,t}$ These three constraints guarantee the battery integrity.

$$c_{11}: soc_{ps}^{-,24} = socr_{ps}$$

This guarantees that a specific level of energy,  $socr_{ps}$ , will remain at the end of the day.

$$\begin{aligned} c_{12} &: \sum_{t \in T} ef_{ps}^{+,t} = \sum_{t \in T} ef_{ps}^{-,t} \\ c_{13} &: \sum_{t \in T} ef_{ps}^{+,t} \le \sum_{t \in T} be_{ps}^{t} \end{aligned}$$

The above two constraints guarantee the management of the shifting. This linear program allows prosumers to benefit from their battery and their shifting possibilities to adjust their consumption according to the price signal and take advantage of the lower price during the day.

**Prosumer agent states** : (i) Each prosumer is initialised with parameters  $Q_{ps}^{min}, Q_{ps}^{max}, s_{ps}, p_{ps}^{max}, \sigma_{ps}^{b}, \sigma_{ps}^{p}$  and sends its profile to its aggregator. (ii) At the end of each day, ps computes  $socr_{ps}$ , solves (2) and sends the resulted schedule of hourly demand to the aggregator. (iii) When aggregator returns the price signal, ps solves (2) again and sends back the new schedule.

#### 4.2 Provider agent behaviour

The provider uses a function  $\mathcal{F} : \mathcal{Q} \to \mathcal{T}r$  where  $\mathcal{Q}$  is an amount of energy and  $\mathcal{T}r$  a tariff.

 $\mathcal{F}_{pv}(q_{pv}^{tot}) = tr_{pv}^{OPT} + tr_{ag}^{max}(1 - e^{\left(\frac{-|p_{pv}^{OPT} - q_{pv}^{tot}|\right)}{tr_{ag}^{oPT}}}) (3)$   $tr_{pv}^{OPT} \text{ is the tariff proposed by the provider when } p_{pv}^{OPT} = q_{pv}^{tot}$ and  $tr_{ag}^{max} \in [0, tr_{pv}^{MAX}]$  is the coefficient negotiated in the annual contract.  $q_{pv}^{tot} = \sum_{ag \in \mathcal{A}_A} q_{ag}$ , with  $q_{ag}$  the amount of energy requested by the aggregator  $ag_a$  who has an annual contract with  $pv_f$ . Designed in this way, the tariff function allows having a monotonous and continuous increasing behaviour between  $tr_{pv}^{OPT}$ and  $tr_{pv}^{OPT} + tr_{ag}^{max}$  and incentivised prosumers to adjust their consumption to the production. **Property 1**: Under the hypothesis that the provider computes the tariffs according to (3), the maximal tariff he will apply to a cooperative is less or equal to  $tr_{pv}^{OPT} + tr_{ag}^{max}$ .

**Proof.** To prove that, we study the evolution of the price when the difference between the supply and demand goes toward infinity. We will show that  $\lim_{|p_{vv}^{OPT}-q_{pv}^{tot}|\to+\infty} \mathcal{F}_{pv}(q_{pv}^{tot}) = tr_{pv}^{OPT} + tr_{ag}^{max}$ . First,

$$\begin{split} &\lim_{|p_{pv}^{OPT} - q_{pv}^{tot}| \to +\infty} tr_{ag} = \\ &\lim_{|p_{pv}^{OPT} - q_{pv}^{tot}| \to +\infty} tr_{pv}^{OPT} + tr_{ag}^{max} (1 - e^{\left(\frac{-|p_{pv}^{OPT} - q_{pv}^{tot}|\right)}{tr_{ag}^{max}}\right)}) \\ &e^{-x} \to 0 \text{ when } x \to +\infty \text{ so, by substitution, we get:} \\ &\lim_{|p_{pv}^{OPT} - q_{pv}^{tot}| \to +\infty} tr_{pv}^{OPT} + tr_{ag}^{max} (1 - e^{\left(\frac{-|p_{pv}^{OPT} - q_{pv}^{tot}|\right)}{tr_{ag}^{max}}\right)}) = \\ &\lim_{|p_{pv}^{OPT} - q_{pv}^{tot}| \to +\infty} tr_{pv}^{OPT} + tr_{ag}^{max} (1 - 0) \\ &\text{As a result} \lim_{|p_{pv}^{OPT} - q_{pv}^{tot}| \to +\infty} \mathcal{F}_{pv}(q_{pv}^{tot}) = tr_{pv}^{OPT} + tr_{ag}^{max}. \end{split}$$

The first property guarantees that supply cannot rationally rise without an increase of the demand, because the lower the prices, the less profitable is the mechanism.

**Property 2**: Under the hypothesis that the provider computes the tariffs according to (3), the minimal tariff it will apply to a cooperative is  $tr_{pv}^{OPT}$ .

**Proof.** The tariff is minimal if the total amount of requested energy is equal to the optimal production of the provider, i.e.  $p_{pv}^{OPT} = q_{pv}^{tot}$ . To prove that, we study the level of the price when the supply is equal to the demand.

if 
$$p_{pv}^{OPT} = q_{pv}^{tot}$$
, we have  $e^{\left(\frac{-|p_{pv}^{OPT} - q_{tot}^{tot}|}{tr_{ag}^{max}}\right)} = e^{\left(\frac{0}{tr_{ag}^{max}}\right)} = 1$  and  $tr_{pv}^{OPT} + tr_{ag}^{max} \cdot (1-1) = tr_{pv}^{OPT}$ 

The second property guarantees that prices are lower when all produced energy is requested.

## 4.3 Aggregator agent behaviour

An aggregator agent manages supply and demand within a cooperative. This agent contracts with providers to meet the demand of the cooperative (the sum of the demands of the agents in the cooperative) allowing to benefit from competition between providers to decrease the energy price<sup>4</sup>. We differentiate three kinds of contracts: (i) the annual contract sets the maximum tariff applicable by a provider and the maximal and minimal demands requested by the cooperative at each slot, (ii) the daily contract fixes a set of hourly contracts a day ahead, (iii) the hourly contract matches an amount of energy and a tariff at a given slot. POU tariff scheme is applied on hourly contracts taking as baseline negotiated earlier in the daily contract. Indeed, some prosumers may under-consume while others may over-consume. Individually, each agent will pay penalties if it underconsumes or over-consumes. However, inside the cooperative, under consumption of some prosumer will balance over-consumption of others and vice versa. As shown in section 4.4, it allows cancelling or reducing agent penalties and limiting the need for agents to request the market to sell the energy they under-consume or to buy the energy they over-consume, thus, limiting the volatility in the energy market.

The aggregator will spread the need of the cooperative between several contracts negotiated with potential suppliers, using the following linear program. This linear program computes the upper and lower bound to negotiate with each potential provider with the purpose of minimising the cost over the year, considering the subscription cost and the penalty cost of each provider.

min  $\sum_{pv \in \mathcal{P}_{ag}} A_{pv} \cdot x_{ag}^{pv} + pen_{pv} \cdot \sigma_{ag} \cdot \mathcal{H} \cdot y_{ag}^{pv}$  (5) With  $x_{ag}^{pv} = \mathcal{Q}_{ag,pv}^{max}$  the maximal amount of energy requested by the aggregator agent to provider  $pv_f$ ,  $y_{ag}^{pv} = \mathcal{Q}_{ag,pv}^{max} - \mathcal{Q}_{ag,pv}^{min}$  the width of the energy band of the aggregated demand of the cooperative to the provider pv,  $\mathcal{H}$  is the number of hours in a year and  $\sigma_{ag}$  the mean deviation consumption of the cooperative

deviation consumption of the cooperative. s.t.  $\sum_{pv \in \mathcal{P}_{ag}} x_{ag}^{pv} = \mathcal{Q}_{ag}^{MAX}$ , with  $\mathcal{Q}_{ag}^{MAX}$  the upper bound of the aggregated demand of the cooperative.

The sum of the maximal amount of energy in  $ag_a$ 's contract must allow providing  $Q_{aq}^{MAX}$  to the cooperative.

s.t. 
$$\sum_{pv \in \mathcal{P}_{ag}} x_{ag}^{pv} - y_{ag}^{pv} = \mathcal{Q}_{ag}^{MIN}$$

This sum guarantees that the deviation sum  $\sum_{pv \in \mathcal{P}_{ag}} \mathcal{Q}_{ag,pv}^{max} - \mathcal{Q}_{ag,pv}^{min} = \mathcal{Q}_{ag}^{MAX} - \mathcal{Q}_{ag}^{MIN}$  compensates the total deviation of the cooperative.

s.t.  $\forall pv \in \mathcal{P}_{ag}, x_{ag}^{pv} \geq y_{ag}$ This inequality is equivalent to  $\mathcal{Q}_{ag,pv}^{max} \geq \mathcal{Q}_{ag,pv}^{max} - \mathcal{Q}_{ag,pv}^{min}$ . It allows not having  $\mathcal{Q}_{ag,pv}^{max} < \mathcal{Q}_{ag,pv}^{min}$ . With  $\mathcal{Q}_{ag,pv}^{max}$  (resp.  $\mathcal{Q}_{ag,pv}^{min}$ ) the maximum (resp. minimum) energy requested by  $ag_a$  from  $pv_f$ .

#### 4.3.1 Prosumers production tariffication

We consider the case where a prosumer agent  $ps_u$  produces more than its needs i.e.  $p_{ps}^t > b_{ps}^t$ . It first meets its needs, then it will sell the oversupply to other members of the cooperative, knowing that the sale price is lower than the providers' prices. The agent which belongs to the cooperative makes sure that it sells its energy. In return, the cooperative can benefit from cheaper energy than the providers one. The remuneration of  $ps_u$ , computed by  $ag_a$ , can be formulated as:

$$tr_{ps}^{t} = \mathcal{F}_{ag}(q_{ag}^{t} + p_{ps}^{pred,t}) \cdot (1 - e^{-|p_{ps}^{pred,t} - p_{ps}^{real,t}|}) (4)$$

where  $tr_{ps}^{t}$  is the sale tariff of  $ps_{u}$  at slot t.  $\mathcal{F}_{ag}(q_{ag}^{t} + p_{ps}^{t})$  is the mean tariff applied by the set of providers  $\mathcal{P}_{ag}$  to  $ag_{a}$  for demand  $q_{ag}^{t} + p_{ps}^{pred}$ , with  $q_{ag}^{t}$  the cooperative demand and  $p_{ps}^{pred,t}$  the forecast production of  $ps_{u}$ . This tariff is only effective if the agent produces exactly what it forecasts. If not, the tariff is multiplied by  $(1 - e^{|p_{ps}^{pred,t} - p_{ps}^{real,t}|)$ . The more the deviation between the forecast and the real production is high, the less  $ps_{u}$  will be rewarded, because the deviation involves possible penalties for the cooperative. The formulation of (4) incentives prosumers to forecast accurately their production.

**Property 3**: Under the hypothesis that  $ag_a$  negotiates with several providers, and considering an agent  $ps_u$ , who forecasts that it will produce an oversupply  $p_{ps}^{pred}$  at slot t, agents in the cooperative find it preferable to consume the locally produced energy, i.e. consume the oversupply of  $ps_u$ , each time it's possible.

**Proof.** As 
$$(1 - e^{-|p_{ps}^{pred,t} - p_{ps}^{real,t}|}) \in [0, 1[$$
, we have the inequality  $\mathcal{F}_{ag}(q_{ag}^t + p_{ps}^t) \cdot (1 - e^{-|p_{ps}^{pred,t} - p_{ps}^{real,t}|}) \leq \mathcal{F}_{ag}(q_{ag}^t + p_{ps}^t)$ 

The third property guarantees that agents will exchange energy between them before resorting to the grid, involving less exchanges on the market.

<sup>&</sup>lt;sup>4</sup> We suppose the financial profit brought by these multiple subscriptions should be higher than the involved costs.

#### 4.3.2 Prosumers and providers interactions

When  $ps_u$  announces its consumption limits, it commits to the cooperative to not deviate from these limits at each slot. If  $ps_u$  doesn't respect this limit commitment and the amount request by the aggregator become higher than the upper bound negotiated in the annual contract, it has to directly pass a contract with a provider for each slot where the limits are not respected.

**Property 4**: The agents belonging to a cooperative are not penalised by an agent who consumes more than its higher limit at one slot, i.e.  $q_{ps}^{real} > Q_{ps}^{max}$  involves  $q_{ag} > Q_{ag}^{MAX}$ , since it will contract for its over demand (demand  $> Q_{ps}^{max}$  without available balancing into the cooperative).

**Proof.** Let  $q_{ag/\{ps\}} < Q_{ag/\{ps\}}^{MAX}$  and  $q_{ps}^{real} > Q_{ps}^{max}$  the deviation of the agent such that  $q_{ag/\{ps\}} + q_{ps}^{real} > Q_{ag/\{ps\}}^{max} + Q_{ps}^{max}$ .  $p_{su}$  will pass a contract directly with a provider for the quantity  $q_{ps}^{real} - Q_{ps}^{max}$ . So the quantity bought by the cooperative becomes  $q_{ag/\{ps\}} + q_{ps}^{real} - (q_{ps}^{real} - Q_{ps}^{max}) = q_{ag/\{ps\}} + Q_{ps}^{max}$  which is less than  $Q_{ag/\{ps\}}^{MAX} + Q_{ps}^{max}$  as  $q_{ag/\{ps\}} < Q_{ag/\{ps\}}^{MAX}$ .

Property 4 guarantees an incentive for prosumers to respect their commitment towards the cooperative.

## 4.4 Penalty distribution

Before introducing the computation formula of penalty distribution, we denote:  $\Delta q_{ag} = \sum_{ps \in Coop} q_{ps}^{pred} - q_{ps}^{real}$ : the deviation consumption of the cooperative. If  $\Delta q_{ag} > 0$  (resp.  $\Delta q_{ag} < 0$ ), the cooperative under-consumes (resp. over-consumes). Let  $\Delta q_{ps} = |q_{ps}^{pred,t} - q_{ps}^{real,t}|$ : the deviation consumption of  $ps_u$ ,  $Pen_a = \sum_{pv \in \mathcal{P}_{ag}} Pen_{pv}$ : the penalty of the cooperative,  $pen_{ps}$ : the penalty of  $ps_u$ ,  $\mathcal{C}^+ = \sum_{ps \in Coop} f(q_{ps}^{pred}, q_{ps}^{real}, dif_{ag})$  with:

$$f(x, y, z) = \begin{cases} 1 & \text{if } z > 0 \text{ and } x > y \\ & \text{or } z < 0 \text{ and } x < y \\ 0 & \text{else} \end{cases}$$

The function f returns 1 if  $ps_u$  contributes to penalise the cooperative, i.e. it over-consumes (resp. under-consumes) when the cooperative over-consumes (resp. under-consumes). The sum enumerates the agents who penalise the cooperative  $Q^- = \sum_{ps \in Coop} g(q_{ps}^{pred,t}, q_{ps}^{real,t}, dif_{ag}) \cdot \Delta q_{ps}$  with:

$$g(x, y, z) + f(x, y, z) = 1$$

The function g returns 1 when  $ps_u$  contributes to decrease the deviation of the cooperative, i.e. it over-consumes (resp. under-consumes) when the cooperative under-consumes (resp. over-consumes). The product  $g().\Delta q_{ps}$  computes the amount of energy which was under-consumed (resp. over-consumed) when the cooperative overconsumed (resp. under-consumed). The penalties distribution formula applied by the aggregator agent to the prosumers of the cooperative is then:

$$pen_{ps} = \begin{cases} Pen_{ag} \cdot \frac{\Delta q_{ps} - \frac{Q^{-}}{C^{+}}}{\sum_{\Delta} ag} & \text{if f(.)=1} \\ 0 & \text{else} \end{cases}$$

The fraction  $\frac{Q^-}{C^+} \left( = \frac{\sum_{ps \in Coop} g(q_{ps}^{pred,t}, q_{ps}^{real,t}, dif_{ag}).dif_{ps}}{\sum_{ps \in Coop} f(q_{ps}^{pred,t}, q_{ps}^{real,t}, dif_{ag})} \right)$  corresponds to the compensation of the cooperative which is spread be-

sponds to the compensation of the cooperative which is spread between all the agents in a fair way. Now that the aggregator knows the amount it will request for each supplier, it will negotiate with each of them, to fix the tariff and the lower bound of the contracts. The following section provides the description of the interactions between the agents. The algorithm 1 gives a global view of aggregators' behaviour detailed in section 5.

Algorithm 1: Algorithm of an aggregatror  $ag_a$ 

1 if all the prosumer profile are received then  $\mathcal{Q}_{ag}^{MIN} \leftarrow \sum_{ps \in \mathcal{C}oop} \mathcal{Q}_{ps}^{min}, \mathcal{Q}_{ag}^{MAX} \leftarrow \sum_{ps \in \mathcal{C}oop} \mathcal{Q}_{ps}^{max},$ 2  $\sigma_{ag} \leftarrow \sqrt{\sum_{ps \in \mathcal{C}oop} (\sigma_{ps}^b)^2}$  and solve (5); for  $pv \in \mathcal{P}_{ag}$  do  $tr_{pv}^{max} \leftarrow 0, \mathcal{Q}^{min} \leftarrow \mathcal{Q}_{pv,ag}^{min}, \mathcal{Q}^{max} \leftarrow \mathcal{Q}_{ag,pv}^{max}$  (5);  $ag_a$  submits the proposal to  $pv_f$ ; 3 4 5 **6** if  $ag_a$  receives a proposal (Annual-Contract:  $\mathcal{X}_{ag}, \mathcal{X}_{pv}, id$ ) then  $ag_a$  computes  $\mathcal{U}_{aq}$  and  $\mathcal{U}_{pv}$ ; 7 8 if  $\mathcal{U}_{ag} \geq \mathcal{U}_{pv}$  then 9  $ag_a$  accepts (Annual-Contract: $\mathcal{X}_{ag}, \mathcal{X}_{pv}, id$ ); 10 for  $ps \in Coop$  do  $\begin{vmatrix} \Delta_{ps} = \mathcal{Q}_{ps}^{max} - \mathcal{Q}_{ps}^{min}, \Delta_{aq} = \sum ps \in \mathcal{C}oop\Delta_{ps}; \end{vmatrix}$ 11 else 12  $ag_a$  computes  $\mathcal{Q}_{pv}^{max'}$ ,  $\mathcal{Q}_{pv}^{min'}$ ,  $tr_{pv}^{max'}$  such that 13  $\mathcal{Z}_{ag} > \mathcal{Z}_{pv};$  $\mathcal{X}_{ag} \leftarrow \{\mathcal{Q}_{pv}^{max'}, \mathcal{Q}_{pv}^{min'}\}, \mathcal{X}_{pv} \leftarrow \{tr_{pv}^{max'}\}; ag_a \text{ proposes the new}$ 14 15 Annual-Contract( $\mathcal{X}_{ag}, \mathcal{X}_{pv}, id$ ); 16 if the plannings of all the prosumers are received then

17for  $ps \in Coop, t \in T$  do18 $\[ schedule[t] = schedule[t] + q_t^{pv}; \]$ 19if there is no difference in the plannings then20 $\[ ag_a \text{ accepts the daily contract;} \]$ 21else22 $\[ ag_a \text{ sends the new Planning;} \]$ 23if  $ag_a$  receives all the tariffs from its providers then24 $\[ ag_a \text{ sends the price signal to the prosumers;} \]$ 

25 if end of slot then

$$Pen_{ag} = \sum_{pv \in \mathcal{P}} pen_{pv} * |q_{ag}^{real} - q_{ag}^{preal}|;$$
  
for  $ps \in Coop$  do

28 | if f(ps) == 1 then 29 |  $pen_{ps} \leftarrow Pen_{ag} \cdot \frac{\Delta Q_{ps} - \frac{Q^{-}}{C^{+}}}{\Delta Q_{Coop}}$ 

30

## **5** NEGOTIATION MECHANISM

In this section, we present the negotiation mechanism, used for contracts over the three time scales, and formalise the different types of contracts handle in each scale. At the first level, providers and prosumers agree on tariff and demand constraints, that lower levels have to satisfy. The goal of the first level is to guarantee to the provider a bounded demand, while it guarantees to prosumers a tariff range. The bounded demand allows to easily know what would be the demand in the future, thus, decreasing the speculation possibilities. Formally, an annual contract is a triplet: { $Q^{min}, Q^{max}, tr^{max}$ }. At the second level, providers and aggregators contract on a daily contract, formalised by a vector of *n* hourly contracts:  $\langle H^t, H^{t+1}, \ldots, H^{t+n} \rangle$ . An hourly contract is a triple  $\langle q_{ag}, tr_{pv}, t \rangle$  with  $q_{ag}$  the forecast demand selling at  $tr_{pv}$  at slot *t*.

## 5.1 Negotiation of the annual contract

Over the period set out in the contracts, the aggregator will spread the demand of the cooperative among several providers. The maximum amount requested on each provider results from (5). Hence, aggregator and providers will negotiate. They will do concessions on the coefficient  $tr_{max}$  and the lower boud using MCP<sup>5</sup> (there are no concessions on  $Q_{max}$  since the cooperative has to be sure to get enough energy at each slot). In the MCP, agents submit round by round proposals making a concession at each new proposal. An agreement is reached when,  $u_{ag}(x_{ag}) \leq u_{pv}(x_{ag})$ , or  $u_{pv}(x_{pv}) \leq u_{ag}(x_{pv})$ , with  $x_{ag}$  the proposal of the aggregator agent,  $x_{pv}$  the proposal of the provider agent,  $u_{ag}$  the aggregator utility function and  $u_{pv}$  the provider utility function. The Zeuthen strategy indicates the agent which has to make a concession during the next round by calculating the Zeuthen index,  $Z_i = \frac{u_i(x_i) - u_i(x_j)}{u_i(x_i)}$ . The agent with the lower  $Z_i$  has to make a concession. To negotiate the annual contract, aggregators  $ag_a$  and providers  $pv_f$  use the following utility functions:

•  $u_{ag} = 2 \cdot \mathcal{Q}_{ag,pv}^{max} - \mathcal{Q}_{ag,pv}^{min} - tr_{ag}^{max}$ 

• 
$$u_{pv} = \mathcal{Q}_{aq,pv}^{min} + tr_{aq}^{max}$$

The utility of aggregators is high if the negotiated energy band is high, i.e.  $\mathcal{Q}_{ag,pv}^{max} - \mathcal{Q}_{ag,pv}^{min}$  is high and when  $tr_{ag}^{max}$  is low. Moreover, the utility is high if  $\mathcal{Q}_{ag,pv}^{max}$  is high, allowing a bigger amount of energy for the cooperative. The utility of the provider is high when  $\mathcal{Q}_{ag,pv}^{min}$  is high, i.e. when the negotiated energy band is narrow (as  $\mathcal{Q}_{ag,pv}^{max}$  is constant the abilities to predict the demand is facilitated) and when the tariff is high. To guarantee the pareto-optimality of negotiated solution with the MCP, both utility functions have to be symmetric, i.e. if  $u_i(\mathcal{X}_1) = u_i(\mathcal{X}_2)$  then  $u_j(\mathcal{X}_1) = u_j(\mathcal{X}_2)$ . So, let  $u_i(x_1) = 2.\mathcal{Q}_{x_1}^{max} - \mathcal{Q}_{x_1}^{min} - tr_{ag}^x$  and  $u_i(x_2) = 2.\mathcal{Q}_{x_2}^{max} - \mathcal{Q}_{x_2}^{min} - tr_{ag}^{x_1}$  and  $u_j(x_2) = \mathcal{Q}_{x_2}^{min} + tr_{ag}^{x_2}$ . If  $u_i(x_1) = u_i(x_2)$  then  $2.\mathcal{Q}_{x_1}^{min} - \mathcal{Q}_{x_1}^{min} - tr_{x_3}^{x_1} = 2.\mathcal{Q}_{x_2}^{max} - \mathcal{Q}_{x_2}^{min} - tr_{ag}^{x_2} = 2.\mathcal{Q}_{x_2}^{max} - \mathcal{Q}_{x_2}^{min} + tr_{ag}^$ 

The concession strategy is the following one for the provider:

• 
$$tr_{t+1}^{max} = tr_t^{max} \cdot \frac{\mathcal{Q}^{max} - \mathcal{Q}_t^{min} + 1}{\mathcal{Q}^{max} - \mathcal{Q}_t^{min} + t}$$
, with  $tr_{t+1}^{max} < tr_t^{max} \forall t$ 

•  $\mathcal{Q}_{t+1}^{min} = \mathcal{Q}_t^{min} \cdot \frac{\mathcal{Q}_t - \mathcal{Q}_t + 1}{\mathcal{Q}_{t-1}^{max} - \mathcal{Q}_t^{min} + t}$ ; with  $\mathcal{Q}_{t+1}^{min} < \mathcal{Q}_t^{min} \forall t$ 

We can see that the concessions on  $tr_t^{max}$  and  $Q_t^{min}$  are monotonic and decreasing. The concession strategy for aggregator are the following one:

• 
$$tr_{t+1}^{max} = tr_t^{max} + \sigma_a^b$$
, with  $tr_{t+1}^{max} > tr_t^{max} \forall t$   
•  $\mathcal{Q}_{t+1}^{min} = (\mathcal{Q}_t^{min} + 1) \cdot \frac{\mathcal{Q}_t^{max}}{\sigma_t^b + pen_t, \mathcal{Q}_t^{max}}$ , with  $\mathcal{Q}_{t+1}^{min} > \mathcal{Q}_t^{min} \forall t$ 

We can see that the concessions on  $tr_t^{max}$  and  $Q_t^{min}$  are monotonic and increasing. Since (i) the utility functions are symmetric and follow a monotonic behaviour (according to the concession functions), (ii) the use of Zeuthen index, (iii) the MCP context, then the negotiation will converge on a pareto-optimal solution. Moreover, in a context where a cooperative can negotiate with several providers, there are no incentive to lie on the price for the provider. Indeed, if he proposes a higher price, the cooperative will negotiate with other providers and if he proposes a lower price he will do less benefit.

#### 5.1.1 Annual contract steps

(i) Each prosumer agent sends its consumption profile to the aggregator. (ii) The aggregator computes the cooperative profile and the distribution of the demands between the providers. It submits the annual contract proposals. (iii) The provider accepts or counters proposals. (iv) The negotiation between the agents continues following the MCP protocol.

#### 5.2 Negotiation of the daily contract

The negotiation of the daily contract follows the following steps:

1. Each prosumer agent sends its consumption profile of the day to the aggregator

 $< q_{ps}^{t}, q_{ps}^{t+1}, \dots, q_{ps}^{t+n} >.$ 

- 2. The aggregator computes the profile of the cooperative  $\langle \sum_{ps \in Coop} q_{ps}^t, \sum_{ps \in Coop} q_{ps}^{t+1}, \dots, \sum_{ps \in Coop} q_{ps}^{t+n} \rangle$  and requests providers.
- 3. Providers send their tariffs  $\langle tr_{pv}^t, tr_{pv}^{t+1}, \ldots, tr_{pv}^{t+n} \rangle$ .
- 4. Aggregators transfer pricing signal as proposed in [7, 14, 11]. Prosumers can reschedule their planning to reduce their bill.
- 5. Go to step 2 if there are reschedules.

# 5.3 The hourly contract

There are no negotiations for the hourly contract. Agents pay for their consumption with penalties according to their deviation.

We will show in the next section that, according to their expected payment, prosumers have an incentive to participate in a cooperative.

# 6 THEORETICAL ANALYSIS

This section studies three properties of the model. First, we show that prosumers reduce their cost when they consume less, even if they have to pay penalties. Then, we give the expected bill of an agent who is a member of a cooperative. We take up this property to show that, in our model, prosumers pay fewer penalties in a cooperative. Finally, we prove the volatility minimisation.

#### 6.1 Monotonicity with respect to consumption

A prosumer may not be incited to consume unnecessarily in order to avoid paying penalties. The perceived benefit by an agent who doesn't consume has to be higher than any penalty:  $q_{ps}^t tr_{ag}^t > q_{ps}^{t'} tr_{ag}^t + pen_{pv} (|q_{ps}^t - q_{ps}^{t'}|)$ . Let  $q_{ps}^t$  be the energy requested and  $q_{ps}^{t'}$  the energy consumed:

$$\begin{aligned} q_{ps}^{t}.tr_{ag}^{t} &> q_{ps}^{t}.tr_{ag}^{t} + pen_{pv}.(q_{ps}^{t} - q_{ps}^{t}) \\ q_{ps}^{t}.tr_{ag}^{t} &> q_{ps}^{t'}.tr_{ag}^{t} + pen_{pv}.q_{ps}^{t} - pen_{pv}.q_{ps}^{t'} \\ q_{ag}^{t}.tr_{ag}^{t} - pen_{pv}.q_{ps}^{t} &> q_{ps}^{t'}.tr_{ag}^{t} - pen_{pv}.q_{ps}^{t'} \\ q_{ag}^{t}.(tr_{ag}^{t} - pen_{pv}) &> q_{ps}^{t'}.(tr_{ag}^{t} - pen_{pv}) \\ q_{ag}^{t} &> q_{ag}^{t'}.(tr_{ag}^{t} - pen_{pv}) > 0 \end{aligned}$$

The first inequality shows that our system does not incite agents to consume to avoid under consumption penalties subject to the condition  $tr_{ag}^t - pen_{pv} > 0$ ; i.e. the tariff is higher than the penalties. This shows that the property coming from POU is kept in our case, thus agents are not incited to request energy on the grid to avoid penalties.

<sup>&</sup>lt;sup>5</sup> For these negotiations, agents adopte, in the first instance, the MCP and the Zeuthen strategy. We choose to apply the MCP but our model allows to use other protocols.

#### 6.2 Expected payment

We can formulate the expected penalty paid by an agent by:

$$\underline{pen_{ps} = pen_{ag}(\sigma_{ag}^b - \frac{\sqrt{\sum_{ps \in \mathcal{C}oop}(\sigma_{ps}^b)^2}}{\mathcal{C}^+})}$$

with  $\sqrt{\sum_{ps\in\mathcal{C}oop} (\sigma_{ps}^b)^2}$  the mean deviation of the consumption of the set of agents who limit the deviation, with  $\sum_{ps\in\mathcal{C}oop} f(x,y,z) = \mathcal{C}^+$ , the number of agents in this set. Let  $x_{ag}$  be the variable representing the deviation of the cooperative,  $x_{ps}$  the variable representing the deviation of  $ps_u$ ,  $\mu_{ag}$  the mean consumption of the cooperative and  $\mu_{ps}$  the mean consumption of the prosumer. The prosumer will pay a penalty only in two of these cases:  $P(x_{ag} > \mu_{ag}).P(x_{ps} > \mu_{ps}) + P(x_{ag} < \mu_{ag}).P(x_{ps} < \mu_{ps}) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ . The expected penalty of an agent is:

$$pen_{ps} = \frac{1}{2} \cdot pen_a (\sigma_{ps}^b - \frac{\sqrt{\sum_{ps \in Coop} (\sigma_{ps}^b)^2}}{C^+})$$
  
The expected payment of an agent becomes :

$$\sum_{t=0}^{n} tr_{ag}^{t}.q_{ps}^{t} + \frac{1}{2}.pen_{ag}(\sigma_{ps}^{b} - \frac{\sqrt{\sum_{ps \in Coop} (\sigma_{ps}^{b})^{2}}}{C^{+}})$$

## 6.3 Individual rationality

We will show that an agent has interest to be a member of a cooperative, by demonstrating that its expected payment is lower when the agent is in a cooperative. Hence, we demonstrate the following inequality:

$$\sum_{t=0}^{n} tr_{ag}^{t}.q_{ps}^{t} + \frac{1}{2}.pen_{ag}.(\sigma_{ps}^{b} - \frac{\sqrt{\sum_{ps\in\mathcal{C}oop} (\sigma_{ps}^{b})^{2}}}{\sum_{ps\in\mathcal{C}oop} f(x_{ps}, y_{ps}, z)})$$

$$< \sum_{t=0}^{n} tr_{ag}^{t}.q_{ps}^{t} + pen_{ps}.\sigma_{ps}^{b}$$

$$(1)$$

Suppose that  $pen_{ag} = pen_{ps}$ , in this case the inequality becomes:

$$\begin{split} &\frac{1}{2} \cdot \left( \sigma_{ps}^{b} - \frac{\sqrt{\sum_{ps \in Coop} (\sigma_{ps}^{b})^{2}}}{\sum_{ps \in Coop} f(x_{ps}, y_{ps}, z)} \right) < \sigma_{ps}^{b} \\ &\frac{\sigma_{ps}^{b}}{2} - \frac{\sqrt{\sum_{ps \in Coop} (\sigma_{ps}^{b})^{2}}}{2 \cdot \sum_{ps \in Coop} f(x_{ps}, y_{ps}, z)} < \sigma_{ps}^{b} \\ &- \frac{\sqrt{\sum_{ps \in Coop} (\sigma_{ps}^{b})^{2}}}{2 \cdot \sum_{ps \in Coop} f(x_{ps}, y_{ps}, z)} < \frac{\sigma_{ps}^{b}}{2} \\ &- \frac{\sqrt{\sum_{ps \in Coop} (\sigma_{ps}^{b})^{2}}}{\sum_{ps \in Coop} f(x_{ps}, y_{ps}, z)} < \sigma_{ps}^{b} \end{split}$$

 $\sqrt{\sum_{ps\in\mathcal{C}oop} (\sigma_{ps}^b)^2} \ge 0$  and  $\sigma_{ps}^b \ge 0$  by definition of standard deviation.  $\sum_{ps\in\mathcal{C}oop} f(x_{ps}, y_{ps}, z) \ge 1$  because  $f(x_{ps}, y_{ps}, z) = 1$  for prosumer agent  $ps_u$ . The negative sign in front of the ration of two positive numbers guarantees the inequality. Joining the third property encourage agents to exchange energy in the cooperative before requesting the grid.

## 6.4 Volatility minimisation

According to function (3), the tariff applied by a provider  $pv_f$  is constant and equal to  $tr^{opt}$  if its requested amount of energy  $q_{pv}^{tot} = p_{pv}^{opt}$ . Thus, in case of any pricing signal that encourages consumers to request an amount of energy equal to the optimal production of producers, we have the sum  $\sum_{i=1}^{24} \sqrt{(q_{pv}^{tot} - p_{pv}^{opt})^2}$  which is minimising. This leads to  $\sum_{i=1}^{24} \sqrt{(tr_{pv}^i - tr_{pv})^2} \rightarrow 0$ , with  $\overline{tr_{pv}}$  the mean tariff, due to (3). The volatility is the standard deviation of the price. As  $\sum_{i=1}^{24} \sqrt{(tr_{pv}^i - \overline{tr_{pv}})^2}$  is the standard deviation of the price we can say that the volatility is going toward 0.

#### 7 EMPIRICAL EVALUATION

In this section, we first describe the initialization of our data. Then, we present and discuss the results of our evaluation. Our goal is to show the price evolution according to the tariff scheme we propose. Thus, we look at the tariff evolution according to the demand evolution (shifting and use of storage capacity to store) and the level of the penalties inside a cooperative which may add volatility in the final price paid by the prosumer.

# 7.1 Experimental setup

We begin by considering a scenario where each agent is in a cooperative and has a generator, a storage capacity, some loads, some shifting possibilities, some forecast capacities and expected consumption limits.

**Consumption limits** :  $Q^{max}$  (resp.  $Q^{min}$ ) is drawn randomly in  $[Q^{min}, 2.Q^{min}]$  (resp. [6.5, 11.5]) as in [11].

**Generator** : Each agent is equipped with a generator with a maximal production drawn randomly in [1, 6] 6 KW is the maximum delivery power for suitable domestic<sup>6</sup> wind turbine or solar panel.

**Storage capacity** : Each agent is equipped with a storage capacity which is drawn randomly in [0; 6, 4], the 6.4 KWh correspond to the powerwall storage capacity<sup>7</sup>. At the beginning of the simulation the storage capacity starts with an amount of energy which is drawn randomly in  $[0, s_{ps}]$ .

**Loads** : At each slot, the forecast consumption of a consumer is drawn randomly in  $[0, Q^{max}]$ . Then the real consumption is drawn randomly following the law  $\mathcal{N}(q^{pred}, \sigma^b)$ , so the real consumption can be higher than  $Q^{max}$ .

**Forecast capacities** :  $\sigma^b$  is randomly drawn between 3.59 and 17.9 percent of  $Q^{max}$ , 3.59 represents poor predictors and 17.9 good predictors in [13].  $\sigma^p$  is randomly drawn between 0 and 5 percent of  $P^{max}$ .

**Shifting possibilities** : For each slot, the shifting possibility of the slot is drawn randomly between 0 and 50 percent of the forecast consumption of the slot.

**Number of agents** : We begin by testing the model with one cooperative. The number of agents in the cooperative is set to 10.

**LP Solver** : For the different linear programs we use the glpk solver [1].

# 7.2 Empirical results

Figure 3 shows the deviation harmonization effect. We can observe that the deviation is lower when the agents are in a cooperative than when they are alone. The harmonization effect leads to a reduction of 63% of the deviation, i.e. the under-consumption (or over-consumption) of some agents is compensated at 63% of the over-consumption (or under-consumption) of the others. Thus, this abates the level of penalties, and so doing, the volatility of the rate paid by the prosumers. The mechanism uses the combination of storage capacities and shifting possibilities. The use of this combination allows to shift the demand of the cooperative towards slots where tariffs are low. As figure 2a shows at slot 8, we can observe a peak of demand from the cooperative. At the same slot on figure 2b, we can observe that this peak of demand is in part due to a shift of demand, agents move their shiftable needs and store energy on this slot. On figure 2c, we can note that at slot 8, tariff is not moving so much far from the

<sup>&</sup>lt;sup>6</sup> http://www.energysavingtrust.org.uk/domestic/wind-turbines

<sup>&</sup>lt;sup>7</sup> http://www.teslamotors.com/powerwall



Figure 2. Evolution of the demand of a cooperative and the associated prices



Figure 3. The harmonization effect on consumer deviation

mean price (81.7 for a mean of 84.17). On a more global vision, Figure 2a and 2b show quite high flows in the cooperative management of energy. The demand of the cooperative moves between 50 and 200 kWh/slot during a day. On the shifting side, they evolve from 30 to -90 kWh/slot in the day (-90 denotes that the consumption is scheduled earlier in the day). Against that, figure 2c shows that the range of price is quite low, moving between 81.7 and 86.7 per slot. The volatility of the price in the day is 1.92 (standard deviation) with a mean of 84.17. For comparaison, in 2014, a report<sup>8</sup> outlines a volatility of 11.51 between the beginning of 2009 and the end of 2012.

# 8 CONCLUSION

In this paper, we attack the problem of designing a sustainable mechanism to limit the pricing volatility in smart grids energy market where lot of agents are willing to buy and sell energy. The mechanism we present extends on three time scales; at the first level negotiations provide pareto-optimal solutions where energy bands and tariff bands are defined. We introduce a new tariff scheme which takes place at the two lowest levels to incentive agents to make and respect their forecast consumption a day ahead. We propose the algorithms of prosumer agents to minimise their cost in the context of bounded prices and demand. We show that our mechanism keeps some properties of POU tariff scheme (individual rationality and monotonicity on price w.r.t the consumption) and reduces the price volatility. Finally, we present an experiment evaluation that shows, in the case of our parameters, that the price progresses in a narrow window. This work is developed in collaboration with an industrial partner in building construction. Further works will focus on island of prosumers

(in a building for example) which can be autonomous and disconnected from the grid (in some slot). The goal is to reduce or lower the provider's need and incentives the exchange between prosumer or between islands of prosumers.

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<sup>&</sup>lt;sup>8</sup> http://www.wec-france.org/DocumentsPDF/RECHERCHE/79rapportfinal.pdf