

# Summary Information for Reasoning About Hierarchical Plans

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## Abstract.

Hierarchically structured agent plans are important for efficient planning and acting, and they also serve (among other things) to produce “richer” classical plans, composed not just of a sequence of primitive actions, but also “abstract” ones representing the supplied hierarchies. A crucial step for this and other approaches is deriving precondition and effect “summaries” from a given plan hierarchy. This paper provides mechanisms to do this for more pragmatic and conventional hierarchies than in the past. To this end, we formally define the notion of a precondition and an effect for a hierarchical plan; we present data structures and algorithms for automatically deriving this information; and we analyse the properties of the presented algorithms. We conclude the paper by detailing how our algorithms may be used together with a classical planner in order to obtain abstract plans.

## INTRODUCTION

This paper provides effective techniques for automatically extracting abstract actions and plans from a supplied hierarchical agent plan-library. Hierarchically structured agent plans are appealing for efficient acting and planning as they embody an expert’s domain control knowledge, which can greatly speed up the reasoning process and cater for non-functional requirements. Two popular approaches based on such representations are Hierarchical Task Network (HTN) [7, 10] planning and Belief-Desire-Intention (BDI) [18] agent-oriented programming. While HTN planners “look ahead” over a supplied collection of hierarchical plans to determine whether a task has a viable decomposition, BDI agents interleave this process with acting in the real world, thereby trading off solution quality for faster responsiveness to environmental changes. Despite these differences, HTN and BDI systems are closely related in both syntax and semantics, making it possible to translate between the two representations [19, 20].

While HTN planning and BDI execution are concerned with decomposing hierarchical structures (offline and online, respectively), one may perform other kinds of reasoning with them that do not necessarily require or amount to decompositions. For example, [5] and [21, 22] perform reasoning to coordinate the execution of abstract steps, so as to preempt potential negative interactions or exploit positive ones.

In [6], the authors propose a novel application of such hierarchies to produce “richer” classical plans composed not just of sequences of primitive actions, but also of “abstract” steps. Such abstract plans are particularly appealing in the context of BDI and HTN systems because they respect and re-use the domain control knowledge inherent in such systems, and they provide flexibility and robustness: if a refinement of one abstract step happens to fail, another option may be tried to achieve the step.

A pre-requisite for these kinds of reasoning is the availability of meaningful preconditions and effects for abstract steps (i.e., compound tasks in HTN systems or event-goals in BDI languages). Generally, this information is not supplied explicitly, but embedded within the decompositions of an abstract step. This paper provides techniques for extracting this information automatically. The algorithms we develop are built upon those of [5] and [21, 22], which calculate offline the precondition and effect “summaries” of HTN-like hierarchical structures that define the agents in a multi-agent system, and use these summaries at runtime to coordinate the agents or their plans. The most important difference between these existing techniques and ours is that the former are framed in a propositional language, whereas ours allow for first-order variables. This is fundamental when it comes to practical applicability, as any realistic BDI agent program will make use of variables. A nuance worth mentioning between our work and that of Clement et al. is that the preconditions we synthesise are standard classical precondition formulas (with disjunction), whereas their preconditions are (essentially) two sets of literals: the ones that must hold at the start of *any* successful execution of the entity, and the ones that must hold at the start of *at least one* such execution. Yao et al. [25] extend the above two strands of work to allow for concurrent steps within an agent’s plan, though still not first-order variables.

Perhaps the only work that computes summaries (“external conditions”) of hierarchies specifying first-order variables is [23]. The authors automatically extract a weaker form of summary information (what we call “mentioned” literals) to inform the task selection strategy of the UMCP HTN planner: tasks that can possibly establish or threaten the applicability of other tasks are explored first. They show that even weak summary information can significantly reduce backtracking and increase planning speed. However, the authors only provide insights into their algorithms for computing summaries.

We note that we are only concerned here with how to extract abstract actions (with corresponding preconditions and effects), and eventually abstract plans, from a hierarchical

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know-how structure. Consequently, unlike existing useful and interesting work [11, 8, 2, 1], our approach does not directly involve *guiding a planner* toward finding a suitable primitive plan. We also do not aim to build new “macro” actions from sample primitive solution plans, as done in [3], for example.

Thus, the contributions of this paper are as follows. First, we develop formal definitions for the notions of a precondition and an effect of an (abstract) event-goal. Second, we develop algorithms and data structures for deriving precondition and effect summaries from an event-goal’s hierarchy. Unlike past work, we use a typical BDI agent programming language framework; in doing so, we allow for variables in agent programs—an important requirement in practical systems. Our chosen BDI agent programming language cleanly incorporates the syntax and semantics of HTN planning as a built-in feature, making our work immediately accessible to both communities. Finally, we show how derived event-goal summaries may be used together with a classical planner in order to obtain abstract plans (which can later be further refined, if desired, to meet certain properties [6]).

## THE HIERARCHICAL FRAMEWORK

Our definitions, algorithms, and results are based on the formal machinery provided by the CANPlan [20] language and operational semantics. While designed to capture the essence of BDI agent-oriented languages, it directly relates to other hierarchical representations of procedural knowledge, such as HTN planning [7, 10], both in syntax and semantics.

A CANPlan BDI agent is created by the specification of a *belief base*  $\mathcal{B}$ , i.e., a set of ground atoms, a *plan-library*  $\Pi$ , i.e., a set of plan-rules, and an *action-library*  $\Lambda$ , i.e., a set of action-rules. A plan-rule is of the form  $e(\mathbf{v}) : \psi \leftarrow P$ , where  $e(\mathbf{v})$  is an *event-goal*,  $\mathbf{v}$  is a vector of distinct variables,  $\psi$  is the *context condition*, and  $P$  is a *plan-body* or *program*.<sup>3</sup> The latter is made up of the following components: primitive actions (*act*) that the agent can execute directly; operations to add (+*b*) and remove (−*b*) beliefs; tests for conditions (? $\phi$ ); and event-goal programs (!*e*), which are simply event-goals combined with the label “!”. These components are composed using the sequencing construct  $P_1; P_2$ . While the original definition of a plan-rule also included declarative goals and the ability to specify partially ordered programs [24], we leave out these constructs here and focus only on an AgentSpeak-like [17], typical BDI agent programming language.

There are also additional constructs used by CANPlan internally when attaching semantics to constructs. These are the programs *nil*,  $P_1 \triangleright P_2$ , and  $(\psi_1 : P_1, \dots, \psi_n : P_n)$ . Intuitively, *nil* is the empty program, which indicates that there is nothing left to execute; program  $(\psi_1 : P_1, \dots, \psi_n : P_n)$  represents the plan-rules that are relevant for some event-goal; and program  $P_1 \triangleright P_2$  realises failure recovery: program  $P_1$  should be tried first, failing which  $P_2$  should be tried. The complete language of CAN, then, is described by the grammar

$$P ::= \text{nil} \mid \text{act} \mid ?\phi \mid +b \mid -b \mid !e \mid P_1; P_2 \mid P_1 \triangleright P_2 \mid (\psi_1 : P_1, \dots, \psi_n : P_n).$$

The behaviour of a CANPlan agent is defined by a set of derivation rules in the style of Plotkin’s structural single-step

<sup>3</sup> In [20] an event-goal is of the form  $e(\mathbf{t})$  where  $\mathbf{t}$  is a vector of terms. Here, we replace  $\mathbf{t}$  with  $\mathbf{v}$  and assume WLOG that  $\forall t_i \in \mathbf{t}, \psi \supset (t_i = v_i)$ , where  $v_i \in \mathbf{v}$ .

operational semantics [15]. The *transition relation* on a configuration is defined using one or more derivation rules. Derivation rules have an *antecedent* and a *conclusion*: the antecedent can either be empty, or it can have transitions and auxiliary conditions; the conclusion is a single transition. A *transition*  $C \longrightarrow C'$  within a rule denotes that configuration  $C$  yields configuration  $C'$  in a single execution step, where a configuration is the tuple  $\langle \mathcal{B}, \mathcal{A}, P \rangle$  composed of a belief base  $\mathcal{B}$ , a program  $P$ , and the sequence  $\mathcal{A}$  of actions executed so far. Construct  $C \xrightarrow{\mathbf{t}} C'$  denotes a transition of type  $\mathbf{t}$ , where  $\mathbf{t} \in \{\text{bdi}, \text{plan}\}$ ; when no label is specified on a transition both types apply. Intuitively, *bdi*-type transitions are used for the standard BDI execution cycle, and *plan*-type transitions for (internal) deliberation steps within a *planning* context. By distinguishing between these two types of transitions, certain rules can be disallowed from being used in a planning context, such as those dealing with BDI-style failure handling.

We shall describe three of the CANPlan derivation rules. The rule below states that a configuration  $\langle \mathcal{B}, \mathcal{A}, !e \rangle$  evolves into a configuration  $\langle \mathcal{B}, \mathcal{A}, (\Delta) \rangle$  (with no changes to  $\mathcal{B}$  and  $\mathcal{A}$ ) in one *bdi*- or *plan*-type execution step, with  $(\Delta)$  being the set of all relevant plan-rules for  $e$ , i.e., the ones whose handling event-goal unifies with  $e$ ; *mgu* stands for “most general unifier” [12]. From  $(\Delta)$ , an applicable plan-rule—one whose context condition holds in  $\mathcal{B}$ —is selected by another derivation rule and the associated plan-body scheduled for execution.

$$\frac{\Delta = \{\psi_i \theta : P_i \theta \mid e' : \psi_i \leftarrow P_i \in \Pi \wedge \theta = \text{mgu}(e, e')\}}{\langle \mathcal{B}, \mathcal{A}, !e \rangle \longrightarrow \langle \mathcal{B}, \mathcal{A}, (\Delta) \rangle}$$

The *Plan* construct incorporates HTN planning as a built-in feature of the semantics. The main rule defining the construct states that a configuration  $\langle \mathcal{B}, \mathcal{A}, \text{Plan}(P) \rangle$  evolves into a configuration  $\langle \mathcal{B}', \mathcal{A}', \text{Plan}(P') \rangle$  in one *bdi*-type execution step if the following two conditions hold: (i) configuration  $\langle \mathcal{B}, \mathcal{A}, P \rangle$  yields configuration  $\langle \mathcal{B}', \mathcal{A}', P' \rangle$  in one *plan*-type execution step, and (ii) it is possible to reach a *final* configuration  $\langle \mathcal{B}', \mathcal{A}', \text{nil} \rangle$  from  $\langle \mathcal{B}', \mathcal{A}', P' \rangle$  in a finite number of *plan*-type execution steps. Thus, executing the single *bdi*-type step necessitates zero or more internal “look ahead” steps that check for a successful HTN execution of  $P$ .

Unlike plan-rules, any given action program will have exactly one associated action-rule in the action-library  $\Lambda$ . Like a STRIPS operator, an action-rule  $\text{act} : \psi \leftarrow \Phi^+; \Phi^-$  is such that *act* is a symbol followed by a vector of distinct variables, and all variables free in  $\psi$ ,  $\Phi^+$  (the add list) and  $\Phi^-$  (the delete list) are also free in *act*. We additionally expect any action-rule  $\text{act} : \psi \leftarrow \Phi^-; \Phi^+$  to be *coherent*: that is, for all ground instances  $\text{act}\theta$  of *act*, if  $\psi\theta$  is consistent, then  $\Phi^+\theta \cup \{-b \mid b \in \Phi^-\theta\}$  is consistent. For example, while the rule  $R$  corresponding to an action *move*( $X, Y$ ) with precondition  $\text{at}(X) \wedge \neg \text{at}(Y)$  (or  $X \neq Y$ ) and postcondition  $\neg \text{at}(X) \wedge \text{at}(Y)$  is coherent, the same rule with precondition *true* is not, as there will then be a ground instance of  $R$  such that its precondition is consistent but its postcondition is not: both its add and delete lists contain the same atom.

## ASSUMPTIONS

We shall now introduce some of the definitions used in the rest of the paper and concretise the rest of our assumptions. As usual, we use  $\mathbf{x}$  and  $\mathbf{y}$  to denote vectors of distinct variables,

and  $\mathbf{t}$  to denote a vector of (not necessarily distinct) terms. Moreover, since the language of CANPlan allows variables in programs, we shall frequently make use of the standard notions related to *substitutions* [12].

We assume that the plan-library does not have recursion. Formally, we assume that a *ranking* exists for the plan-library, i.e., that it is always possible to give a child a smaller rank (number) than its parent. We define a ranking as follows.

**Definition 1** (RANKING) A *ranking* for a plan-library  $\Pi$  is a function  $\mathcal{R}_\Pi : E_\Pi \mapsto \mathbb{N}_0$  from event-goal types mentioned in  $\Pi$  to natural numbers, such that for all event-goals  $e_1, e_2 \in E_\Pi$  where  $e_2$  is the same type as some  $e_3 \in \text{children}(e_1, \Pi)$ , we have that  $\mathcal{R}_\Pi(e_1) > \mathcal{R}_\Pi(e_2)$ .<sup>4</sup> ■

In addition, we define the following two related notions: first, given an event-goal type  $e$ ,  $\mathcal{R}_\Pi(e)$  denotes the *rank* of  $e$  in  $\Pi$ ; and second, given any event-goal  $e(\mathbf{t})$  mentioned in  $\Pi$ , we define  $\mathcal{R}_\Pi(e(\mathbf{t})) = \mathcal{R}_\Pi(e(\mathbf{x}))$  (where  $|\mathbf{x}| = |\mathbf{t}|$ ), i.e., the rank of an event-goal is equivalent to the rank of its type. In order that these and other definitions also apply to event-goal *programs*, we sometimes blur the distinction between event-goals  $e$  and event-goal programs  $!e$ .

Finally, we assume that context conditions are written with appropriate care. Specifically, if there is no environmental interference, whenever a plan-rule is applicable it should be possible to successfully execute the associated plan-body without any failure and recovery; this disallows rules such as  $e : \text{true} \leftarrow ?\text{false}$ . Our definition makes use of the notion of a *projection*: given any configuration  $\langle \mathcal{B}, \mathcal{A}, P \rangle$ , we define the *projection* of the first component of the tuple as  $C|_{\mathcal{B}}$ , the second as  $C|_{\mathcal{A}}$ , and the third as  $C|_P$ .

**Definition 2** (COHERENT LIBRARY) A plan-library  $\Pi$  is *coherent* if for all rules  $e : \psi \leftarrow P \in \Pi$ , ground instances  $e\theta$  of  $e$ , and belief bases  $\mathcal{B}$ , whenever  $\mathcal{B} \models \psi\theta\theta'$  (where  $\psi\theta\theta'$  is ground) there is a successful HTN execution  $C_1 \dots C_n$  of  $P\theta\theta'$  (relative to  $\Pi$ ) with  $C_1|_{\mathcal{B}} = \mathcal{B}$ . A *successful HTN execution* of a program  $P$  relative to a plan-library is any finite sequence of configurations  $C_1 \dots C_n$  such that  $C_1|_P = P$ ,  $C_n|_P = \text{nil}$ , and for all  $0 < i < n$ ,  $C_i \xrightarrow{\text{plan}} C_{i+1}$ . ■

Intuitively, the term *HTN execution* simply denotes a BDI execution in which certain BDI-specific derivation rules associated with failure and recovery have not been used.

## SUMMARY INFORMATION

We can now start to define what we mean by preconditions and postconditions/effects of event-goals; some of these definitions are also used later in the algorithms. As a first step we define these notions for our most basic programs.

A basic program is either an *atomic program* or a *primitive program*. Formally, a program  $P$  is an *atomic program* (or simply *atomic*) if  $P = !e \mid \text{act} \mid +b \mid -b \mid ?\phi$ , and  $P$  is a *primitive program* if  $P$  is an atomic program that is not an event-goal program. Then, like the postcondition of a STRIPS

action, the *postcondition* of a primitive program is simply the atoms that will be added to and removed from the belief base upon executing the program. Formally, the *postcondition* of a primitive program  $P$  relative to an action-library  $\Lambda$ , denoted  $\text{post}(P, \Lambda)$ , is the set of literals  $\text{post}(P, \Lambda) =$

$$\begin{cases} \emptyset & \text{if } P = ?\phi, \\ \{b\} & \text{if } P = +b, \\ \{-b\} & \text{if } P = -b, \\ \Phi^+ \theta \cup \{-b \mid b \in \Phi^- \theta\} & \text{if } P = \text{act} \text{ and there exists} \\ & \text{an } \text{act}' : \psi \leftarrow \Phi^+; \Phi^- \in \Lambda \\ & \text{such that } \text{act} = \text{act}'\theta. \end{cases}$$

The postcondition of a test condition is the empty set because executing a test condition does not result in an update to the belief base. The postcondition of an action program is the combination of the add list and delete list of the associated action-rule, after applying the appropriate substitution.

While this notion of a postcondition as applied to a primitive program is necessary for our algorithms later, we do not also need the matching notion of a *precondition* of a primitive program. Such preconditions are already accounted for in context conditions of plan-rules, by virtue of our assumption (Definition 2) that the latter are coherent. What we do require, however, is the notion of a *precondition* as applied to an event-goal. This is defined as any formula such that whenever it holds in some state there is at least one successful HTN execution of the event-goal from that state.

**Definition 3** (PRECONDITION) A formula  $\phi$  is said to be a *precondition* of an event-goal  $!e$  (relative to a plan- and an action-library) if for all ground instances  $!e\theta$  of  $!e$  and belief bases  $\mathcal{B}$ , whenever  $\mathcal{B} \models \phi\theta$ , there exists a successful HTN execution  $C_1 \dots C_n$  of  $!e\theta$ , where  $C_1|_{\mathcal{B}} = \mathcal{B}$ . ■

Unlike the postcondition of a primitive program, the postcondition of an event-goal program—and indeed any arbitrary program  $P$ —is non-deterministic: it depends on what plan-rules are chosen to decompose  $P$ . There are, nonetheless, certain effects that will be brought about irrespective of such choices. We call these *must literals*: literals that hold at the end of *every* successful HTN execution of  $P$ .

**Definition 4** (MUST LITERAL) Let  $P$  be a program and  $l$  a literal where its variables are free in  $P$ . Then,  $l$  is a *must literal* of  $P$  (relative to a plan- and action-library) if for any ground instance  $P\theta$  of  $P$  and successful HTN execution  $C_1 \dots C_n$  of  $P\theta$ , we have that  $C_n|_{\mathcal{B}} \models l\theta$ . ■

A desirable consequence of the two definitions above is that any given set of must literals of an event-goal, like the postcondition of an action, is consistent whenever the event-goal's precondition is consistent.

**Theorem 1** Let  $e$  be an event-goal,  $\phi$  a precondition of  $e$  (relative to a plan-library  $\Pi$  and an action-library  $\Lambda$ ), and  $L^{mt}$  a set of must literals of  $e$  (relative to  $\Pi$  and  $\Lambda$ ). Then, for all ground instances  $e\theta$  of  $e$ , if  $\phi\theta\theta'$  is consistent for some ground substitution  $\theta'$ , then so is  $L^{mt}\theta$ .

**PROOF SKETCH.** We prove this by contradiction. First, note that since  $L^{mt}\theta$  is a set of ground literals (by Definition 4 and because  $e\theta$  is ground), if  $L^{mt}\theta$  is consistent, then for all

<sup>4</sup> We define the function  $\text{children}(\hat{e}, \Pi) = \{e \mid e' : \psi \leftarrow P \in \Pi, \hat{e} \text{ and } e' \text{ are the same type, } P \text{ mentions } !e\}$ , where two event-goals are the *same type* if they have the same predicate symbol and arity. The *type* of an event-goal  $e(\mathbf{t})$  is defined as  $e(\mathbf{x})$ , where  $|\mathbf{x}| = |\mathbf{t}|$ .

literals  $l, l' \in L^{mt}$  it is the case that  $l\theta \neq \bar{l}'\theta$ .<sup>5</sup> If we assume that the theorem does not hold, then there must be a ground instance  $e\theta$  of  $e$  such that  $\phi\theta\theta'$  is consistent for some ground substitution  $\theta'$ , but  $l\theta = \bar{l}'\theta$  for some  $l, l' \in L^{mt}$ .

Since  $\phi\theta\theta'$  is consistent, it is not difficult to show there is a belief base  $\mathcal{B}$  such that  $\mathcal{B} \models \phi\theta\theta'$ . Then, by Definition 3 there is also a successful HTN execution  $C_1 \dots C_n$  of  $e\theta$  with  $C_1|_{\mathcal{B}} = \mathcal{B}$ . Moreover, since  $l, l'$  are must literals of  $e$  and  $e\theta$  is ground, by Definition 4 we know that (i)  $l\theta$  and  $l'\theta$  are also ground, and (ii)  $C_n|_{\mathcal{B}} \models l\theta$  and  $C_n|_{\mathcal{B}} \models l'\theta$ . This leads to a contradiction of our assumption that  $l\theta = \bar{l}'\theta$ .  $\square$

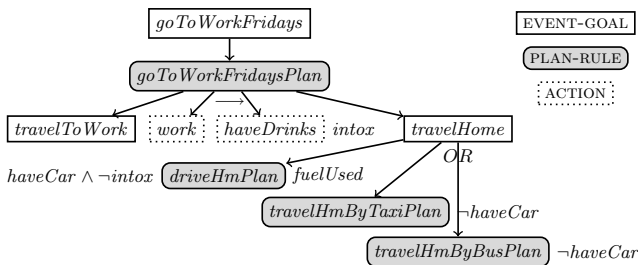
In addition to must literals, there are two related notions. The first, called *may summary conditions* in [5], defines literals that hold at the end of at least one successful HTN execution of the program, and the second, weaker notion defines literals that are simply mentioned in the program or in one of its “descendant” programs; such literals may or may not be brought about when the program executes. It is this second notion, called *mentioned literals*, that we use.

**Definition 5** (MENTIONED LITERAL) If  $P$  is a program, its *mentioned literals* (relative to a plan-library  $\Pi$  and an action-library  $\Lambda$ ), denoted  $mnt(P)$ , is the set  $mnt(P) =$

$$\left\{ \begin{array}{ll} \text{post}(P, \Lambda) & \text{if } P = +b \mid -b \mid \text{act} \mid ?\phi, \\ mnt(P_1) \cup mnt(P_2) & \text{if } P = P_1; P_2, \\ \{l\theta' \mid e' : \psi \leftarrow P' \in \Pi, & \text{if } P = !e. \\ e = e'\theta, l \in mnt(P'), & \\ \theta' \text{ is any substitution} \} & \end{array} \right.$$

■

We use this weaker notion because the stronger notion of a may summary condition in [5] is not suitable for our approach, which reasons about plans that will not be interleaved with one another—i.e., plans that will be scheduled as a sequence. For example, consider the figure below, which shows a plan-library for going to work on Fridays, possibly one belonging to a larger library from an agent-based simulation. The expressions to the left and right sides of actions/plan-rules are their preconditions and postconditions, respectively.



Observe that *fuelUsed* is actually never asserted in the context of the hierarchy shown, because literal  $\neg\text{intox}$  (“not intoxicated”) in the context condition of *driveHmPlan* is contradicted by literal *intox*. However, the algorithms in [5] will still classify *fuelUsed* as a may summary condition of plan *goToWorkFridaysPlan*, because some other plan may have a step asserting  $\neg\text{intox}$ —perhaps a step that involves staying

<sup>5</sup> The *complement* of a literal  $l \in \{a, \neg a\}$ , denoted by  $\bar{l}$ , is  $a$  if  $l = \neg a$ , and  $\neg a$  otherwise.

overnight in a hotel nearby—that can be ordered to occur between *haveDrinks* and *travelHome*.

Since we cannot rely on such steps, we settle for a weaker notion—mentioned literals—than the corresponding definition of a may summary condition. By our definition there can be literals that are mentioned in some plan-body but in fact can never be asserted, because of interactions that preclude the particular plan-body which asserts that literal from being applied. We avoid the approach of disallowing interactions like the one shown above in order to use the stronger notion of a may summary condition because such interactions are natural in BDI and HTN domains: event-goals such as *travelHome* are, intuitively, meant to be self-contained “modules” that can be “plugged” into any relevant part of a hierarchical structure in order to derive all or just some of their capabilities.

Finally, we conclude this section by combining the above definitions of must and mentioned literals to form the definition of the *summary information* of a program.

**Definition 6** (SUMMARY INFORMATION) If  $P$  is a program, its *summary information* (relative to a plan-library and an action-library) is a tuple  $\langle P, \phi, L^{mt}, L^{mn} \rangle$ , where  $\phi$  is a precondition of  $P$  if  $P$  is an event-goal program, and  $\phi = \epsilon$  otherwise;  $L^{mt}$  is a set of must literals of  $P$ ; and  $L^{mn}$  is a set of mentioned literals of  $P$ .  $\blacksquare$

## EXTRACTING SUMMARY INFORMATION

With the formal definitions now in place, in this section we provide algorithms to extract summary information for event-goals in a plan-library. Moreover, we illustrate the algorithms with an example, and analyse their properties.

Basically, we extract summary information from a given plan-library and action-library by propagating up the summary information of lower-level programs, starting from the leaf-level ones in the plan-library, until we eventually obtain the summary information of all the top-level event-goals.

To be able to identify must literals, we need to be able to determine whether a given literal is definitely undone, or *must undone*, and possibly undone, or *may undone* in a program. Informally, a literal  $l$  is *must undone* in a sequence  $P$  of atomic programs if the literal’s negation is a must literal of some atomic program in  $P$ . Formally, then, given a program  $P$  and the set  $\Delta$  of summary information of all atomic programs in  $P$ , a literal  $l$  is *must undone* in  $P$  relative to  $\Delta$ , denoted  $\text{Must-Undone}(l, P, \Delta)$ , if there exists an atomic program  $P'$  in  $P$  and a literal  $l' \in L^{mt}$ , with  $\langle P', \phi, L^{mt}, L^{mn} \rangle \in \Delta$ , such that  $l = \bar{l}'$ , that is,  $l$  is the complement of  $l'$ .

Similarly, we can informally say that a literal  $l$  is *may undone* in a program  $P$  if there is a literal  $l'$  that is a mentioned (or must) literal of some atomic program in  $P$  such that  $l'$  may become the negation of  $l$  after variable substitutions. Formally, given a program  $P$  and the set  $\Delta$  of summary information of all atomic programs in  $P$ , a literal  $l$  is *may undone* in  $P$  relative to  $\Delta$ , denoted  $\text{May-Undone}(l, P, \Delta)$ , if there exists an atomic program  $P'$  in  $P$ , a substitution  $\theta$ , and a literal  $l' \in L^{mn}$ ,<sup>6</sup> with  $\langle P', \phi, L^{mt}, L^{mn} \rangle \in \Delta$ , such that  $l\theta = \bar{l}'\theta$ .

**Algorithm 1.** This is the top-level algorithm for computing the summary information  $\Delta$  of event-goal types occurring

<sup>6</sup> variables occurring in  $l'$  are renamed to those not occurring in  $l$

**Algorithm 1** Summ( $\Pi, \Lambda$ )**Require:** Plan-library  $\Pi$  and action-library  $\Lambda$ .**Ensure:** Set of summary info. of event-goal types in  $\Pi$ .

- 1:  $\Delta \leftarrow \{(P, \epsilon, \text{post}(P, \Lambda), \text{post}(P, \Lambda)) \mid$   
 $P \text{ is a primitive program mentioned in } \Pi\}$
- 2:  $E \leftarrow \{e(\mathbf{x}) \mid e \text{ is an event-goal mentioned in } \Pi\}$
- 3: **for**  $i \leftarrow \min(R)$  **to**  $\max(R)$  **where**  
 $R = \{\mathcal{R}_\Pi(e) \mid e \in E\}$  **do** // Recall  $\mathcal{R}_\Pi(e)$  is the rank of  $e$
- 4:     **for** each  $e \in E$  such that  $\mathcal{R}_\Pi(e) = i$  **do**
- 5:          $\Delta \leftarrow \Delta \cup$   
            $\{\text{SummPlan}(P, \Pi, \Lambda, \Delta) \mid e' : \psi \leftarrow P \in \Pi, e' = e\theta\}$
- 6:          $\Delta \leftarrow \Delta \cup \{\text{SummEvent}(e, \Pi, \Delta)\}$
- 7: **return**  $\Delta \setminus \{u \mid u \in \Delta,$   
 $u \text{ is not the summary information of an event-goal}\}$

**Algorithm 2** SummPlan( $P, \Pi, \Lambda, \Delta_{in}$ )**Require:** Plan-body  $P$ ; plan-library  $\Pi$ ; action-library  $\Lambda$ ; and the set  $\Delta_{in}$  of summary information of primitive programs and event-goal types mentioned in  $P$ .**Ensure:** The summary information of  $P$ .

- 1:  $\Delta \leftarrow \Delta_{in} \cup \{ \langle !e(\mathbf{x}), \phi, L^{mt}, L^{mn} \rangle \theta \mid !e(\mathbf{t}) \text{ occurs in } P,$   
 $\langle e(\mathbf{x}), \phi, L^{mt}, L^{mn} \rangle \in \Delta_{in}, e(\mathbf{t}) = e(\mathbf{x})\theta \}$   
 // We assume variables in  $L^{mn}$  are appropriately renamed
- 2: Let  $P = P_1; P_2; \dots; P_n$  where each  $P_i$  is atomic
- 3:  $L_P^{mt} \leftarrow \{l \mid l \in L^{mt}, \langle P_i, \phi, L^{mt}, L^{mn} \rangle \in \Delta,$   
 $i \in \{1, \dots, n\}, \neg \text{May-Undone}(l, P_{i+1}; \dots; P_n, \Delta)\}$
- 4:  $L_P^{mn} \leftarrow \{l \mid l \in L^{mn} \cup L^{mn}, \langle P_i, \phi, L^{mt}, L^{mn} \rangle \in \Delta,$   
 $i \in \{1, \dots, n\}, \neg \text{Must-Undone}(l, P_{i+1}; \dots; P_n, \Delta)\}$
- 5: **return**  $\langle P, \epsilon, L_P^{mt}, L_P^{mn} \rangle$

**Algorithm 3** SummEvent( $e(\mathbf{x}), \Pi, \Delta$ )**Require:** Event-goal type  $e(\mathbf{x})$ ; plan-library  $\Pi$ ; and the set  $\Delta$  of summary information of plan-bodies of plan-rules  $e' : \psi \leftarrow P \in \Pi$  such that  $e' = e(\mathbf{x})\theta$ .**Ensure:** The summary information of  $e(\mathbf{x})$ .

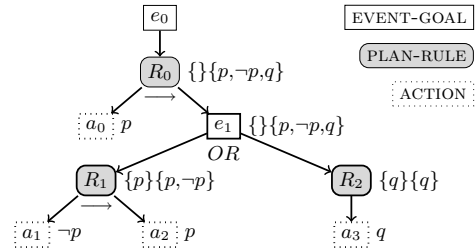
- 1:  $\phi \leftarrow \text{false}, L^{mt} \leftarrow \emptyset, L^{mn} \leftarrow \emptyset,$  and  $S \leftarrow \emptyset$   
 //  $L^{mt}, L^{mn}$  are sets of literals and  $S$  is a set of sets of literals
- 2: **for** each  $e(\mathbf{y}) : \psi \leftarrow P \in \Pi$  such that  $e(\mathbf{x}) = e(\mathbf{y})\theta$  **do**
- 3:      $\phi \leftarrow \phi \vee \psi\theta$   
       // Relevant variables in  $\psi$  and  $L_P^{mt}, L_P^{mn}$  below are renamed
- 4:      $S \leftarrow S \cup \{L_P^{mt}\theta\}$ , where  $\langle P, \epsilon, L_P^{mt}, L_P^{mn} \rangle \in \Delta$
- 5:      $L^{mn} \leftarrow L^{mn} \cup L_P^{mn}\theta$
- 6: **if**  $S \neq \emptyset$  **then** // Obtain the must literals of  $e(\mathbf{x})$
- 7:      $L^{mt} \leftarrow \bigcap S$
- 8:      $L^{mt} \leftarrow \{l \mid l \in L^{mt},$   
       variables occurring in  $l$  also occur in  $e(\mathbf{x})\}$
- 9: **return**  $\langle e(\mathbf{x}), \phi, L^{mt}, L^{mn} \rangle$

in the plan-library. The algorithm works bottom up, by summarising first the leaf-level entities of the plan-library—the primitive programs (line 1)—and then repetitively summarising plan-bodies (Algorithm 2) and event-goals (Algorithm 3) in increasing order of their levels of abstraction (lines 3-6).

**Algorithm 2.** This algorithm summarises the given plan-body  $P$  by referring to the set  $\Delta_{in}$  containing the summary information tuples of programs in  $P$ . First, the algorithm obtains the summary information of each event-goal program in the plan-body from the summary information of the corresponding event-goal types in  $\Delta_{in}$  (line 1). This involves substituting variables occurring in relevant summary information tuples in  $\Delta$  with the corresponding terms occurring in the event-goal program being considered. Second, the algorithm computes the set of must literals ( $L_P^{mt}$ ) and the set of mentioned literals ( $L_P^{mn}$ ) of the given plan-body  $P$ , by determining, from the must and mentioned literals of atomic programs in  $P$ , which literals will definitely hold and which ones will

only possibly hold on successful executions of  $P$  (lines 3 and 4). More precisely, a must literal  $l$  of an atomic program  $P_i$  in  $P = P_1; \dots; P_n$  is classified as a must literal of  $P$  only if  $l$  is not may undone in  $P_{i+1}; \dots; P_n$  (line 3). Otherwise,  $l$  is classified as only a mentioned literal of  $P$ , provided  $l$  is not also must undone in  $P_{i+1}; \dots; P_n$  (line 4). The reason we do not summarise literals that are must undone is to avoid missing must literals in cases where they are possibly undone but then later (definitely) reintroduced, as we illustrate below.

Suppose, on the contrary, that the algorithm *does* summarise mentioned literals that are must undone. Then, given the plan-library below, the algorithm would (hypothetically) compute the summary information denoted by the two sets attached to each node, the one on the left being its set of must literals and the one on the right its set of mentioned literals.



Observe that literal  $p$  asserted by  $a_0$  is not recognised as a must literal of  $R_0$  simply because it is may undone by mentioned literal  $\neg p$  of  $e_1$  (asserted by  $a_1$ ), despite the fact that action  $a_2$  of  $R_1$  also subsequently adds  $p$ . On the other hand, our algorithm does recognise  $p$  as a must literal of  $R_0$  by not including  $\neg p$  in the set of mentioned literals of  $R_1$  (line 4).

**Algorithm 3.** This algorithm summarises the given event-goal type  $e(\mathbf{x})$  by referring to the set  $\Delta$  containing the summary information tuples associated with the plan-bodies of plan-rules handling  $e(\mathbf{x})$ . In lines 2 and 3, the algorithm takes the precondition of the event-goal as the disjunction of the context conditions of all associated plan-rules.<sup>7</sup> Then, the algorithm obtains the must and mentioned literals of the event-goal by respectively taking the intersection of the must literals of associated plan-rules (lines 4 and 7), and the union of the mentioned literals of associated plan-rules (line 5). Applying substitution  $\theta$  in line 4 helps recognise must literals of  $e(\mathbf{x})$ , by ensuring that variables occurring in the summary information of its associated plan-bodies have consistent names with respect to  $e(\mathbf{x})$ .

## An illustrative example

We shall illustrate the three algorithms with the example of a simple agent exploring the surface of Mars. A part of the agent's domain is depicted as a hierarchy in Figure 1. The hierarchy's top-level event-goal is to explore a given soil location  $Y$  from current location  $X$ . This is achieved by plan-rule  $R_0$ , which involves navigating to the location and then doing a soil experiment. Navigation is achieved by rules  $R_1$  and  $R_2$ , which involve moving to the location, possibly after calibrating some of the rover's instruments. Doing a soil experiment involves the two sequential event-goals of getting soil results for  $Y$  and

<sup>7</sup> We do not need to “propagate up” context conditions as we do with plan-bodies' summary information because higher-level context conditions account for lower-level ones due to Definition 2.

transmitting them to the lander. Specifically, the former is refined into actions such as determining moisture content and average soil particle size, and transmitting results involves either establishing a connection with the lander, sending it the results, and then terminating the connection, or if the lander is not within range, navigating to it and uploading the soil results. The table in Figure 1 shows the summary information computed by our algorithms for elements in the figure’s hierarchy. Below, we describe some of the more interesting values in the table.

**Plan-body  $P_7$ .** Must literals  $\neg at(Y)$  and  $at(L)$  of  $P_7$  are derived from those of  $nav(X, Y)$ , after renaming variables  $X$  and  $Y$  to respectively  $Y$  and  $L$  in line 1 of Algorithm 2.

**Plan-body  $P_4$ .** While  $hSS(Y)$  is a must literal of  $P_4$ ’s primitive action  $pickSoil(Y)$ , the literal is must undone by  $P_4$ ’s last primitive action  $dropSoil(Y)$ . Thus,  $hSS(Y)$  is not a must (nor mentioned) literal of  $P_4$ . On the other hand, literal  $\neg hSS(Y)$  is indeed a must literal of  $P_4$ , along with literals  $hMC(Y)$  and  $hPS(Y)$ , both of which are derived from the summary information of event-goal  $analyseSoil(Y)$ .

**Plan-body  $P_0$ .** While  $\neg at(X)$  is a must literal of event-goal  $nav(X, Y)$ , it is only a mentioned literal of  $P_0$  because it is may undone in event-goal  $doSoilExp(Y)$ ; specifically, its mentioned literal  $at(L)$  is such that  $\neg at(X)\theta = \neg at(L)\theta$  for  $\theta = \{X/L\}$ . Similarly, must literal  $at(Y)$  of  $nav(X, Y)$  is also may undone in  $doSoilExp(Y)$ .

**Event-goal  $transmitRes(Y)$ .** Since literal  $rT(Y)$  is a must literal of both of the event-goal’s associated plan-bodies  $P_6$  and  $P_7$ , and  $Y$  also occurs in the event-goal, the literal is classified as a must literal of the event-goal. Recall that this means that for any ground instance  $transmitRes(Y)\theta$  of the event-goal, literal  $rT(Y)\theta$  holds at the end of any successful HTN execution of  $transmitRes(Y)\theta$ .

## Soundness and Completeness

We shall now analyse the properties of the algorithms presented. We show that they are sound, and we then discuss completeness. First, it is not difficult to see that the presented algorithms terminate, and that they run in polynomial time.

**Theorem 2** *Algorithm 1 always terminates, and runs in polynomial time on the number of symbols occurring in  $\Pi \cup \Lambda$ .*

**PROOF SKETCH.** *Since the algorithms presented are non-recursive, the only non-trivial part of the proof concerns the procedure for computing a unification when determining whether  $May\text{-}Undone(l, P, \Delta)$  holds (for a literal  $l$ , program  $P$ , and a set  $\Delta$  of summary information). In [13], one such unification procedure is presented that is linear on the number of symbols occurring in the two literals to be unified.  $\square$*

This result is important when the plan-library changes over time, e.g. because the agent learns from past experience, and summary information needs to be recomputed frequently, or when it needs to be computed right at the start of HTN planning, as done in [23].

The next result states that whenever Algorithm 1 (Summ) classifies a literal as a must literal of an event-goal, this is guaranteed to be the case, and that the algorithm correctly computes its precondition and mentioned literals. More specifically, any computed tuple, which includes one event-goal type

$e$ , formula  $\phi$  and must literals  $L^{mt}$ , respects Definitions 3 and 4. Moreover, there is exactly one tuple associated with  $e$ .

**Theorem 3** *Let  $\Pi$  be a plan-library,  $\Lambda$  be an action-library,  $e$  be an event-goal type mentioned in  $\Pi$ , and let  $\Delta_{out} = \text{Summ}(\Pi, \Lambda)$ . There exists one tuple  $\langle e, \phi, L^{mt}, L^{mn} \rangle \in \Delta_{out}$ , the tuple is the summary information of  $e$ , and  $L^{mn} \subseteq mnt(e)$  (recall  $mnt(e)$  denotes the mentioned literals of  $e$ ).*

**PROOF SKETCH.** *We prove this by induction on  $e$ ’s rank in  $\Pi$ . First, from our ranking function we obtain a new one  $\mathcal{R}_\Pi$  by making event-goal ranks “contiguous” and start from 0.*

*For the base case, take any event-goal  $e$  with  $\mathcal{R}_\Pi(e) = 0$ . According to Definition 1 (Ranking), if  $\mathcal{R}_\Pi(e) = 0$ , then  $children(e, \Pi) = \emptyset$ . Thus, for all rules  $e' : \psi \leftarrow P \in \Pi$  such that  $e = e'\theta$ , no event-goals occur in  $P$ . Let  $P_{all}$  be the set of plan-bodies of all such rules. Then, the two main steps are as follows. First, we show that due to line 1 of procedure  $\text{Summ}(\Pi, \Lambda)$  there is exactly one summary tuple  $\langle P', \epsilon, L_{P'}^{mt}, L_{P'}^{mn} \rangle \in \Delta$  for each primitive program  $P'$  mentioned in each  $P \in P_{all}$ . Second, since  $\text{SummPlan}(P, \Pi, \Lambda, \Delta)$  is called in line 5 for each plan-body  $P \in P_{all}$ , we show that on the completion of this line, there is exactly one summary tuple  $\langle P, \epsilon, L_P^{mt}, L_P^{mn} \rangle \in \Delta$  for each plan-body  $P \in P_{all}$ . A similar argument applies to line 6.*

*For the induction hypothesis, we assume that the theorem holds if  $\mathcal{R}_\Pi(e) \leq k$ , for some  $k \in \mathbb{N}_0$ .*

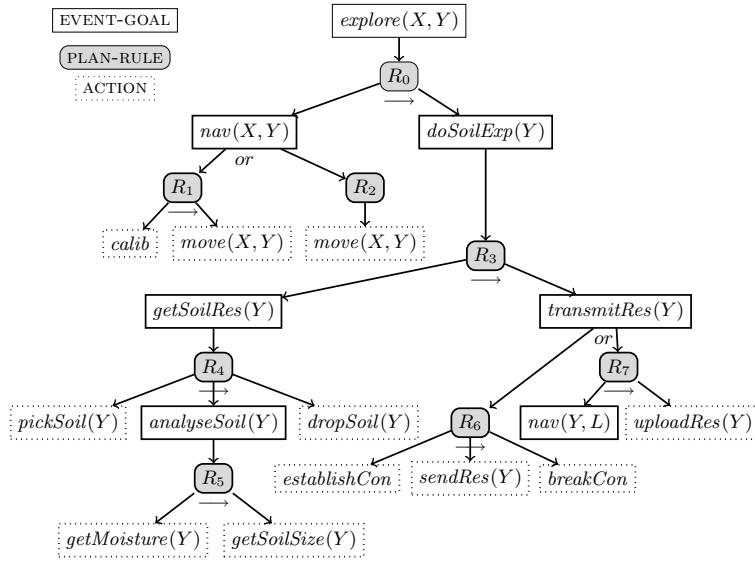
*For the inductive step, we show that the theorem holds for  $\mathcal{R}_\Pi(e) = k + 1$ . The main steps are as follows. Let  $E_{all}$  denote the (non-empty) set of event-goal types mentioned in all plan-bodies  $P \in P_{all}$ , where  $P_{all}$  is as before. By Definition 1, we know that for all  $e' \in E_{all}$ ,  $\mathcal{R}_\Pi(e') < \mathcal{R}_\Pi(e)$ . Then, by the induction hypothesis, it follows that for each  $e' \in E_{all}$ , there is exactly one tuple  $\langle e', \phi_{e'}, L_{e'}^{mt}, L_{e'}^{mn} \rangle \in \Delta$ , and this tuple is the summary information of  $e'$ . Finally, since  $e$  has a higher rank than those of all event-goals  $e' \in E_{all}$ ,  $\text{Summ}(\Pi, \Lambda)$  will only call  $\text{SummEvent}(e, \dots)$  after the above tuples are added to  $\Delta$ , resulting in tuple  $\langle e, \phi, L^{mt}, L^{mn} \rangle$  also being added to  $\Delta$ .  $\square$*

Next, we discuss completeness. The theorem below states that any precondition computed by Algorithm 3 is complete: i.e., given any state from where there is a successful HTN execution of an event-goal, the precondition extracted for the event-goal will hold in that state. This theorem only concerns Algorithm 3 because we can compute preconditions of event-goals without needing to compute preconditions of plans.

**Theorem 4** *Let  $\Pi$  be a plan-library,  $\Lambda$  an action-library,  $e$  an event-goal type mentioned in  $\Pi$ , and let  $\langle e, \phi, L^{mt}, L^{mn} \rangle \in \text{Summ}(\Pi, \Lambda)$ . For all ground instances  $!e\theta$  of  $e$  and belief bases  $\mathcal{B}$  such that there exists a successful HTN execution  $C_1 \dots C_n$  of  $!e\theta$  with  $C_1 |_{\mathcal{B}} = \mathcal{B}$ , it is the case that  $\mathcal{B} \models \phi\theta$ .*

**PROOF SKETCH.** *We prove that if  $!e\theta$  has a successful HTN execution, there is also a plan-rule  $e' : \psi \leftarrow P \in \Pi$  associated with  $e$  such that  $\mathcal{B} \models \psi'\theta$  holds, where  $\psi'$  is an appropriate renaming of variables in  $\psi$ . We then show that  $\psi'$  is a disjunct of  $\phi$ , from which it follows that  $\mathcal{B} \models \phi\theta$ .  $\square$*

There are, however, situations where the algorithms do not detect all *must literals* of an event-goal. The underlying reason



Program	Must Literals	Mentioned Literals
<i>calib</i>	<i>cal</i>	-
<i>move(X, Y)</i>	$\neg at(X), at(Y)$	-
<i>pickSoil(Y)</i>	<i>hSS(Y)</i>	-
<i>dropSoil(Y)</i>	$\neg hSS(Y)$	-
<i>getMoisture(Y)</i>	<i>hMC(Y)</i>	-
<i>getSoilSize(Y)</i>	<i>hPS(Y)</i>	-
<i>establishCon</i>	<i>cE</i>	-
<i>sendRes(Y)</i>	<i>rT(Y)</i>	-
<i>breakCon</i>	$\neg cE$	-
<i>uploadRes(Y)</i>	<i>rT(Y)</i>	-
$P_1$	$\neg at(X), at(Y), cal$	-
$P_2$	$\neg at(X), at(Y)$	-
$P_5$	<i>hMC(Y), hPS(Y)</i>	-
$P_4$	<i>hMC(Y), hPS(Y),</i> $\neg hSS(Y)$	-
$P_6$	<i>rT(Y), \neg cE</i>	-
$P_7$	$\neg at(Y), at(L),$ <i>rT(Y)</i>	<i>cal</i>
$P_3$	<i>rT(Y), hMC(Y),</i> <i>hPS(Y), \neg hSS(Y)</i>	$\neg cE, \neg at(Y), at(L),$ <i>cal</i>
$P_0$	<i>rT(Y), hMC(Y),</i> <i>hPS(Y), \neg hSS(Y)</i>	$\neg cE, at(Y), \neg at(Y),$ <i>at(L), cal, \neg at(X)</i>
<i>nav(X, Y)</i>	$\neg at(X), at(Y)$	<i>cal</i>
<i>analyseSoil(Y)</i>	Same as $P_5$	-
<i>getSoilRes(Y)</i>	Same as $P_4$	-
<i>transmitRes(Y)</i>	<i>rT(Y)</i>	$\neg cE, \neg at(Y), at(L),$ <i>cal</i>
<i>doSoilExp(Y)</i>	Same as $P_3$	Same as $P_3$
<i>explore(X, Y)</i>	Same as $P_0$	Same as $P_0$

**Figure 1:** Must and mentioned literals (right) of atomic programs and plan-bodies in the hierarchy (left). The rightmost column only shows mentioned literals that are not also must literals. Abbreviations in the table are as follows: *cal* = *calibrated*, *hSS* = *haveSoilSample*, *hMC* = *haveMoistureContent*, *hPS* = *haveParticleSize*, *cE* = *connectionEstablished*, *rT* = *resultsTransmitted*, and variable  $L$  = *Lander*. Rule  $R_7$ 's context condition binds  $L$  to the lander's location. Each plan-body  $P_i$  corresponds to rule  $R_i$  in the hierarchy.

for this is that we do not reason about (FOL) precondition formulas; specifically, we do not check entailment, because this is semi-decidable in general [9]. In what follows, we use examples to characterise the four cases in which the algorithms are unable to recognise must literals, and show how some of the cases can be averted.

The first case was depicted in our example about going to work on Fridays: by Definition 4, literal  $\neg haveCar$  is a must literal of *goToWorkFridaysPlan*, but Algorithm 2 classifies it as only a mentioned literal, as it cannot infer that the context condition of rule *driveHmPlan* is contradicted by literal *intox*, and therefore that *driveHmPlan* can never be applied.

The second case is where a literal is a must literal simply because it is entailed by a context condition. For example, take an event-goal *mov(P, T, L)* that is associated with one plan-rule, whose context condition checks whether package  $P$  is in truck  $T$ , i.e., *in(P, T)*, and whose plan-body moves the truck to location  $L$ . Observe that *in(P, T)* is a must literal of *mov(P, T, L)* by definition, but since *in(P, T)* does not occur in the plan-body, Algorithm 2 does not consider the literal. We do not expect this to be an issue in practice, however, because such literals are accounted for by the event-goal's (extracted) precondition.

The third case is where must literals are “hidden” due to the particular variable/constant symbols chosen by the domain writer when encoding literals. For example, given the following two plan-rules for an event-goal that sends an email from  $F$  to  $T$ , literal *sent(T)* is only a mentioned literal of *sendMail(F, T)* according to Algorithm 3 (line 7 in particu-

lar), but a must literal of it by definition:

$$\begin{aligned} sendMail(F, T) : (F \neq T) &\leftarrow +addedSignature ; +sent(T), \\ sendMail(F, T) : (F = T) &\leftarrow +sent(F). \end{aligned}$$

Nonetheless, by changing  $+sent(F)$  to  $+sent(T)$ , which then mentions the same variable symbol as the first plan-body, *sent(T)* is identified by the algorithm as a must literal of *sendMail(F, T)*. In general, such “hidden” must literals can be disclosed by choosing terms with appropriate care.

Finally, while Algorithm 2 “conservatively” classifies any must literal that is may undone as a may literal, it could still be a must literal by definition. For example, given an event-goal *move(X, Y)*, suppose that the following plan-rule is the only one relevant for the event-goal:

$$move(X, Y) : at(X) \wedge \neg at(Y) \leftarrow \neg at(X) ; +at(Y).$$

Then, by Definition 4, both  $\neg at(X)$  and  $at(Y)$  are must literals of the event-goal, but only  $at(Y)$  is its must literal according to Algorithm 2, because it cannot infer that the context condition entails  $X \neq Y$ .<sup>8</sup> While the algorithm does fail to detect some must literals in such domains, this can sometimes be averted by encoding the domain differently. For example, the above rule can be encoded as an action-rule instead, in which case Algorithm 1 (in line 1) will classify  $\neg at(X)$  (and  $at(Y)$ ) as a must literal of *move(X, Y)*, under the assumption that action-rules are coherent.

<sup>8</sup> Note that if the context condition is just  $at(X)$ , then, by definition,  $at(Y)$  would indeed be the only must literal of the event-goal, because it would then be possible for  $X$  and  $Y$  to have the same value, and for  $at(Y)$  to “undo”  $\neg at(X)$ .

## AN APPLICATION TO PLANNING

One application of the algorithms presented is to create abstract planning operators that may be used together with primitive operators and a classical planner in order to obtain abstract (or “hybrid”) plans. While [6] focuses on algorithms for extracting an “ideal” abstract plan from an abstract plan that is supplied, here we give the details regarding how a first abstract plan may be obtained.

To get abstract operators  $\Lambda^a$  from a plan-library  $\Pi$  and an action-library  $\Lambda$ , we take the set  $\Delta = \text{Summ}(\Pi, \Lambda)$  and create an (abstract) operator for every summary information tuple  $\langle e, \phi, L^{mt}, L^{mn} \rangle \in \Delta$ . To this end, we take the operator’s name as  $e$ , appended with its arity and combined with any additional variables occurring in  $\phi$ ; the operator’s precondition as  $\phi$ ; and its postcondition as the set of must literals  $L^{mt}$ .

Since mentioned literals of event-goals are not included in their associated abstract operators, it is crucial that we ascertain whether these literals will cause unavoidable conflicts in an abstract plan found. For example, consider the classical planning problem with initial state  $p$  and goal state  $r$ , and the abstract plan  $e_1 \cdot e_2$  consisting of two event-goals (or abstract operators). Suppose  $e_1$  and  $e_2$  have the following plan-rules:

$$e_1 : \text{true} \leftarrow +p; +q \quad e_1 : \text{true} \leftarrow -p; +q \quad e_2 : p \wedge q \leftarrow +r$$

Notice that the postconditions (must literals) of abstract operators  $e_1$  and  $e_2$  are respectively  $q$  and  $r$ , and that  $e_1 \cdot e_2$  is a classical planning solution for the given planning problem. However, when this plan is executed, if  $e_1$  is decomposed using its second plan-rule, this will cause (mentioned literal)  $\neg p$  to be brought about, thereby invalidating the context condition of  $e_2$  (which requires  $p$ ).

To check for such cases, we present the following simple polynomial-time algorithm. Suppose that  $P = P_1; \dots; P_n$  is the program corresponding to a classical planning solution  $P'_1 \cdot \dots \cdot P'_n$  for some planning problem, where each (ground)  $P_i$  is either an action or event-goal. Then, we say that  $P$  is *correct* relative to  $\Delta$  if for any (ground) literal  $l$  occurring in the precondition of any  $P_i$ , the following condition holds: if  $l$  is not must undone and it is may undone (relative to  $\Delta$ ) in the preceding subplan  $P_1; \dots; P_{i-1}$  by some mentioned literal  $l'$  of a step  $P_k$  in the subplan,<sup>9</sup> then literal  $l$ , or its complement, is also must undone (relative to  $\Delta$ ) in the steps  $P_{k+1}; \dots; P_{i-1}$ . Otherwise,  $P$  is said to be *potentially incorrect*. Interestingly, the situation where  $l$  is must undone in  $P_1; \dots; P_{i-1}$  is not unacceptable because it cannot invalidate the (possibly disjunctive) precondition of  $P_i$ , given that  $P_1; \dots; P_n$  corresponds to a solution for some classical planning problem. The following theorem states that, as expected, a correct program  $P$  will always have at least one successful HTN decomposition.

**Theorem 5** *Let  $\Pi$  be a plan-library,  $\Lambda$  an action-library,  $\Delta = \text{Summ}(\Pi, \Lambda)$ , and  $P$  the program corresponding to a solution for the classical planning problem  $\langle \mathcal{B}, \mathcal{B}_g, A \cup \Lambda^a \rangle$ , where  $\mathcal{B}$  and  $\mathcal{B}_g$  are belief bases representing respectively initial and goal states. Then, if  $P$  is correct (relative to  $\Delta$ ), there*

*is a successful HTN execution  $C_1 \cdot \dots \cdot C_n$  of  $P$  such that  $C_1|_{\mathcal{B}} = \mathcal{B}, C_1|_P = P$  and  $C_n|_{\mathcal{B}} \models \mathcal{B}_g$ .*

**PROOF SKETCH.** *If there is no such successful execution, since  $P = P_1; \dots; P_n$  is a classical planning solution, there must be a mentioned literal of some  $P_i$  that intuitively “conflicts” with a literal occurring in the precondition of some  $P_j$ , with  $j > i$ . The classical planner will not have taken such conflicts into account, but according to the definition of what it means for  $P$  to be correct, such a mentioned literal cannot exist.  $\square$*

If we find that  $P$  is potentially incorrect, we then determine whether it is *definitely incorrect*, i.e., whether there are conflicts that are unavoidable. To this end, we look for a successful HTN decomposition of  $P$ , failing which the plan is discarded and the process repeated with a new abstract plan.

## DISCUSSION & FUTURE WORK

We have presented definitions and sound algorithms for summarising plan hierarchies which, unlike past work, are defined in a typical and well understood BDI agent-oriented programming language. By virtue of its syntax and semantics being inherently tied to HTN planning, our work straightforwardly applies to HTN planners such as SHOP [14]. Our approach is closely related to [5], the main differences being that we support variables in agent programs, and we reason about non-concurrent plans. While these do make a part of our approach incomplete, we have shown how this can sometimes be averted by writing domains with appropriate care. Crucially, we have handled variables “natively”, without grounding them on a finite set of constants. We concluded with one application of our algorithms, showing how they can be used together with a classical planner in order to obtain abstract plans.

We expect that the summaries we compute will be useful in other applications that rely on similar information, such as coordinating the plans of single [21, 22] and multiple [5] agents, and particularly in improving HTN planning efficiency [23]. There is also potential for using such information as guidance when creating agent plans manually [25].

Interestingly, the application we presented mitigates our restriction that plan-libraries cannot be recursive, as the classical planner can, if necessary, repeat an event-goal in an abstract plan. Nonetheless, allowing recursive plan-libraries is still an interesting avenue for future work. Another useful improvement would be to allow partially ordered steps in plan-bodies (i.e., the construct  $P \parallel P'$ ). Given a plan-library  $\Pi$ , one potential approach to that end is to obtain the plan-library  $\Pi'$  consisting of all linear extensions of plan-rules in  $\Pi$ , and then use  $\Pi'$  as the input into Algorithm 1 (Summ). We could use existing, fast algorithms to generate linear extensions [16], or consider simpler plan-rules corresponding to restricted classes of partially ordered sets [4]. Finally, it would be interesting to formally characterise the restricted class of domains in which the presented algorithms are complete.

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<sup>9</sup> We rely here on a slightly extended version of the definition of may undone from before, to have the exact step ( $P_k$ ) and literal ( $l'$ ) responsible for the “undoing”. Moreover, observe that literals  $l$  and  $l'$  are obtained by applying the same substitution that the planner applied to obtain  $P'_i$  and  $P'_k$ , respectively.



## REFERENCES

- [1] R. W. Alford, U. Kuter, and D. Nau. Translating HTNs to PDDL: A small amount of domain knowledge can go a long way. In *Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI-09)*, pages 1629–1634, 2009.
- [2] J. A. Baier, C. Fritz, and S. A. McIlraith. Exploiting procedural domain control knowledge in state-of-the-art planners. In *Proceedings of the International Conference on Automated Planning and Scheduling (ICAPS-07)*, pages 26–33, 2007.
- [3] A. Botea, M. Enzenberger, M. Müller, and J. Schaeffer. Macro-FF: Improving AI planning with automatically learned macro-operators. *Journal of Artificial Intelligence Research (JAIR)*, 24:581–621, 2005.
- [4] V. Bouchitte and M. Habib. The calculation of invariants for ordered sets. In I. Rival, editor, *Algorithms and Order*, volume 255, pages 231–279. Springer Netherlands, 1989.
- [5] B. J. Clement, E. H. Durfee, and A. C. Barrett. Abstract reasoning for planning and coordination. *Journal of Artificial Intelligence Research (JAIR)*, 28:453–515, 2007.
- [6] L. de Silva, S. Sardina, and L. Padgham. First Principles Planning in BDI systems. In *Proceedings of the International Joint Conference on Autonomous Agents and Multiagent Systems (AAMAS-09)*, pages 1105–1112, 2009.
- [7] K. Erol, J. Hendler, and D. S. Nau. HTN planning: Complexity and expressivity. In *Proceedings of the National Conference on Artificial Intelligence (AAAI-94)*, pages 1123–1128, 1994.
- [8] C. Fritz, J. A. Baier, and S. A. McIlraith. ConGolog, Sin Trans: Compiling ConGolog into Basic Action Theories for planning and beyond. In *Proceedings of the International Conference on Principles of Knowledge Representation and Reasoning (KR-08)*, pages 600–610, 2008.
- [9] D. M. Gabbay, C. J. Hogger, and J. A. Robinson, editors. *Handbook of Logic in Artificial Intelligence and Logic Programming*. Oxford University Press, 1994.
- [10] M. Ghallab, D. S. Nau, and P. Traverso. *Automated Planning: Theory and Practice*. Morgan Kaufmann Publishers Inc., 2004.
- [11] S. Kambhampati, A. D. Mali, and B. Srivastava. Hybrid planning for partially hierarchical domains. In *Proceedings of the National Conference on Artificial Intelligence (AAAI-98)*, pages 882–888, 1998.
- [12] J. W. Lloyd. *Foundations of Logic Programming; (2nd Extended Ed.)*. Springer-Verlag New York, Inc., 1987.
- [13] A. Martelli and U. Montanari. An efficient unification algorithm. *ACM Transactions on Programming Languages and Systems*, 4(2):258–282, 1982.
- [14] D. S. Nau, T.-C. Au, O. Ilghami, U. Kuter, J. W. Murdock, D. Wu, and F. Yaman. SHOP2: An HTN planning system. *Journal of Artificial Intelligence Research (JAIR)*, 20:379–404, 2003.
- [15] G. D. Plotkin. A structural approach to operational semantics. Technical Report DAIMI FN-19, Computer Science Department, University of Aarhus, Denmark, 1981.
- [16] G. Pruesse and F. Ruskey. Generating linear extensions fast. *SIAM Journal on Computing*, 23(2):373–386, 1994.
- [17] A. S. Rao. AgentSpeak(L): BDI agents speak out in a logical computable language. In *Proceedings of the European workshop on Modelling Autonomous Agents in a Multi-Agent World : agents breaking away (MAAMAW-96)*, pages 42–55. Springer, 1996.
- [18] A. S. Rao and M. P. Georgeff. BDI-agents: from theory to practice. In *Proceedings of the International Conference on Multiagent Systems (ICMAS-95)*, pages 312–319, 1995.
- [19] S. Sardina, L. de Silva, and L. Padgham. Hierarchical planning in BDI agent programming languages: A formal approach. In *Proceedings of the International Joint Conference on Autonomous Agents and Multiagent Systems (AAMAS-06)*, pages 1001–1008, 2006.
- [20] S. Sardina and L. Padgham. A BDI agent programming language with failure handling, declarative goals, and planning. *Autonomous Agents and Multiagent Systems*, 23(1):18–70, 2011.
- [21] J. Thangarajah, L. Padgham, and M. Winikoff. Detecting and avoiding interference between goals in intelligent agents. In *Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI-03)*, pages 721–726, 2003.
- [22] J. Thangarajah, L. Padgham, and M. Winikoff. Detecting and exploiting positive goal interaction in intelligent agents. In *Proceedings of the International Joint Conference on Autonomous Agents and Multiagent Systems (AAMAS-03)*, pages 401–408, 2003.
- [23] R. Tsuneto, J. Hendler, and D. Nau. Analyzing external conditions to improve the efficiency of HTN planning. In *Proceedings of the National Conference on Artificial Intelligence (AAAI-98)*, pages 913–920, 1998.
- [24] M. Winikoff, L. Padgham, J. Harland, and J. Thangarajah. Declarative and procedural goals in intelligent agent systems. In *Proceedings of the International Conference on Principles of Knowledge Representation and Reasoning (KR-02)*, pages 470–481, 2002.
- [25] Y. Yao, L. de Silva, and B. Logan. Reasoning about the executability of goal-plan trees. In *Engineering Multi-Agent Systems Workshop (EMAS-16)*, pages 181–196, 2016.