

The Two Ontological Faces of Velocity

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Abstract. This article presents a formalization of velocity in the context of a realist and perspectivist upper ontology like BFO. It argues that the term “velocity” can refer to two different entities: a motion-velocity, which is a process profile characterizing a motion process; and an object-velocity, which is a disposition inhering in the moving object. Three different kinds of motion-velocity are presented: left-velocity, right-velocity and bilateral velocity. Motion-velocity could exist without object-velocity, as revealed by a thought experiment presented by Tooley; but in our world, Newton’s first law of inertia implies that every object has both an inertial disposition and a closely related but different disposition that we call “object-velocity.” Those two dispositions are realized by the right-velocity. The left-velocity is a trigger of the inertial disposition, and brings into existence the object-velocity.

Keywords. Velocity, Disposition, Causation, Motion, Inertia, Realism, Perspectivism, BFO

1. Introduction

1.1. Motivation

Most—if not all—material objects around us will be in motion² at some point during their existence. Ontologies should be able to represent the motion of such objects, e.g., marine mammals [1]. However, the theoretical foundations of the ontology of motion have been little investigated in the field of applied ontology. This article will aim at clarifying the ontology of velocity in the context of the ontology of motion [2] of BFO, the Basic Formal Ontology [3].

Physics textbooks’ definitions of velocity are simple: the velocity of an object is the time-derivative of the position³ of this object in space (as represented by a three-dimensional vector); and speed is the magnitude of velocity—and thus, a scalar. For example, physics textbook would say that (3,4,0) m/s is a velocity, with an associated speed of 5 m/s.

Things are less clear from an ontological point of view, though. As a matter of fact, the vector (3,4,0) or the scalar 5 are mathematical entities. When coupled with a unit such as “m/s” (meters per second), they might be viewed as informational entities referring to a velocity or a speed, which are physical entities [5]; but they cannot be considered as identical to those physical entities in a framework that endorses

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² When no further specification is given, “motion” means “motion in the reference frame of the Earth.”

³ For example, the instantaneous velocity of an object according to an axis x is defined as “the derivative of x with respect to t ” [4].

philosophical realism. Thus, we need to clarify what kind of physical entity is a velocity.

In the following, we will work in the framework of Newtonian physics, under the BFO assumption of an absolute, Newtonian space. We will provide a formalization of absolute velocities, which can be seen as a first step towards a formalization of relative velocity in a non-absolute space.

We will present two philosophical schools which interpret velocity either as a characterization of a motion process, or as a property inhering in an object. Instead of rejecting one of those, this article will accept two velocity entities that will be called motion-velocity and object-velocity, which will fit in the realist and perspectivist ontological framework that will be presented now.

1.2. Ontological Framework

Two methodological principles in BFO will be especially relevant here [3]. First, ontological realism [6] assumes that ontologies should formalize the reality as it is described by science, and divides reality into particulars (such as this chair in front of me) and universals (such as the class of all chairs) that explain the similarity between particulars: this chair and that chair are both instances of the universal of chair. Second, perspectivalism maintains that there may be alternative, equally legitimate perspectives on reality.

A manifestation of this perspectivist stance is BFO's acceptance of two broad categories which exist in time in different ways: continuants and occurrents. Continuants are entities extended in three-dimensional space, which are fully present at every instant of time at which they exist, and which preserve their identity over time through change. They include independent continuants, whose existence does not require the existence of other entities—such as a plate or a leaf; and dependent continuants, whose existence does require the existence of other entities—such as the round shape of a plate or the green color of a leaf. By contrast, occurrents are four-dimensional entities which unfold themselves through a period of time. They include e.g., processes, events and changes. Examples of occurrents are a dinner or a motion process. Those two categories of entities are related: continuants are subject to change and participate in occurrents.

In order to formalize entities with specific values, BFO introduces two different kind of universals: determinables and determinates. Determinables are qualities⁴ such as temperature, mass or length. They subsume determinates, which are qualities with specific values such as a temperature of 37°C, a mass of 65 kgs, or a length of 1,80 m. Note that both determinables and determinates are universals: my temperature (which is a particular) may now be an instance of the determinate universal 37°C *temperature*, which is itself a subclass of the determinable universal *Temperature*; tomorrow, it might instantiate 40°C *temperature*, which is another subclass of *Temperature*.

1.3. Reductionism and Primitivism about Velocity

Two philosophical views on the ontological status of velocity have been proposed: reductionism, a view originally articulated by Bertrand Russell [7,8]; and

⁴ Note that this conception can be straightforwardly extended to some other dependent continuants such as dispositions, as will be presented later.

primitivism [9–11]. Reductionists claim that an object’s velocity is entirely grounded in the facts about its position at all times; by contrast, primitivists consider that velocity is a property above and beyond such locations. The classical, physical definition of velocity as the first time-derivative of the position in space is thus a reductionist strategy. In the reductionist view, a velocity would characterize a motion process.

By contrast, in the primitivist view, a velocity characterizes an object, and could be seen as a dependent continuant inhering in this object. Accordingly, there would be a determinable universal *Object-velocity*, and determinates (that is, subclasses) of this determinable such as *Object-velocity_of_(1,-3,7)_m/s*, *Object-velocity_of_(5,2,0)_m/s*, etc. Note that units here are only used to name universals, but do not define the identity of those entities: the universal *Object-velocity_of_10_miles/hour* is identical to the universal *Object-velocity_of_16.0934_kms/hour*, although it could be named either way.

We will see that it makes sense to include both kinds of velocities as two distinct entities in an applied ontology built in a perspectivalist spirit. We will first formalize in a reductionist spirit an entity named “motion-velocity,” and then show how the law of inertia is related to an entity named “object-velocity,” in line with primitivism.

2. Trajectory, Motion Process and Motion-velocity

2.1. Material Object, Motion Process and Trajectories

Before formalizing velocity, we need to define material objects and motion processes. In BFO, material objects are independent continuants. They participate in their *Motion_process*⁵. A motion process is characterized geometrically by its trajectory; but two related concepts of trajectory could be defined (see figure 1). First, the “spatial trajectory” of a motion process of an object **b** that starts at **t₀** and finishes at **t₁** can be defined as the mereological sum of areas in space occupied by **b** at times between **t₀** and **t₁**. Second, the “spatiotemporal trajectory” of this motion process will be defined as the mereological sum of areas in spacetime occupied by an object **b** at times between **t₀** and **t₁**—this is thus a region of spacetime. This second kind of trajectory will be the most useful in our investigation, and will be abbreviated as “trajectory” *simpliciter* in the rest of this article.

⁵ In what follows, universals and their relations will be italicized, and particulars and relations involving at least one particular will be written in boldface.

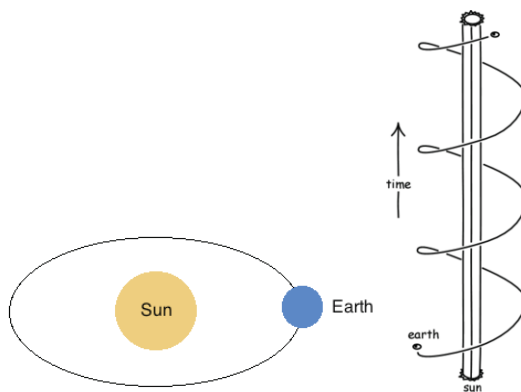


Figure 1. The spatial and spatiotemporal trajectories of the Earth's motion process in the reference frame of the Sun⁶

2.2. Mathematical, Informational and Physical Entities

In the following, several related entities will be distinguished, elaborating on Johansson's work on mathematical and physical vectors [5]. We will introduce mathematical three-dimensional vectors such as $(8,2,-1)$ or $(3,4,0)$, and mathematical scalars such as 4.273 . Those mathematical entities should be distinguished from informational entities composed by a mathematical vector together with a unit such as " $(8,2,-1)$ m" or " $(3,4,0)$ m/s," or by a mathematical scalar with a unit such as " $4,273$ s." Those informational entities can be used to describe physical entities such as the spatial position⁷ of (the center of inertia of) an object $\mathbf{b}$ at some time, its velocity, or a time instant.

This article will not aim at clarifying the ontological status of mathematical entities in BFO, which is a large question exceeding the present work. Instead, it will focus on the ontological status of the velocity itself. As we will see, the term "velocity $v_b(t_0)$ of object $\mathbf{b}$ at time t_0 " can actually refer to two different entities that will be called $\mathbf{p}_{b,t_0}^I$ and $\mathbf{d}_{t_0}^V$. Until we introduce them, we will use the common terminology in physics and speak of a "velocity $v_b(t_0)$ " of an object $\mathbf{b}$ at time t_0 , a "velocity v_0 " or a "velocity of $(3,4,0)$ m/s." We will also introduce the function $r_b(t)$ associating to each time t the representation of the position of the object $\mathbf{b}$ at time t —and we will say that $r_b(t)$ represents the trajectory of $\mathbf{b}$. Figure 2 represents the main entities and relations that are introduced in this paper.

2.3. Constant Velocity

To start clarifying the nature of velocity, let us first consider a motion of $\mathbf{b}$ with a constant velocity v_0 between t_0 to t_1 , and a trajectory given by $r_b(t)$. v_0 is equal to $(r_b(t_1) - r_b(t_0)) / (t_1 - t_0)$. This vector characterizes the four-dimensional geometry of $\mathbf{b}$'s (spacetime) trajectory, and the kinetics of its motion process. Hence a question: how exactly is a velocity related to this motion process?

⁶ Second figure: copyright John D. Norton, used with permission.

⁷ In the remainder of the article, "position of $\mathbf{b}$ " will refer to the position of the center of inertia of $\mathbf{b}$.

In BFO, independent continuants can change across time by losing or getting new qualities, or by having those qualities instantiating different universals at different times. However, since occurrents are extended in time, they cannot change; there are no qualities of occurrents. Thus, a velocity cannot be formalized as a quality of a motion process in BFO.

However, there is something similar between all the motion process that have, say, a constant velocity of 5 m/s: they are all instances of the universal *Motion_process_with_constant_velocity_5m/s*. To facilitate the representation of the similarity between processes (not only for velocities, but also e.g., periodic rates), Barry Smith has introduced the universal *Process_profile*: in a number of cases⁸, if two processes are similar in some respect, “each contains an instantiation of the same [*Process_profile* universal]” [2]. In the present case, we can introduce the subclass of *Process_profile* named *Constant_velocity*, which has as subclass the universal $P^C_{v_0}$, defined such that every particular motion process with a constant speed v_0 has as process profile (**has_process_profile**) an instance of $P^C_{v_0}$.

2.4. Instant Velocity

We can now turn to the representation of instantaneous velocity. First, let us remind that a mathematical function $r(t)$ defined on an interval T has a left-derivative v_{0-} at t_0 if the following mathematical relation (e_1) holds:

$$(e_1) \forall \varepsilon > 0, \exists \delta > 0, \forall t \in T, t_0 - \delta < t < t_0 \rightarrow \left\| \frac{r(t) - r(t_0)}{t - t_0} - v_{0-} \right\| < \varepsilon$$

Similarly, $r(t)$ has a right-derivative v_{0+} at t_0 if the relation (e_2) holds:

$$(e_2) \forall \varepsilon > 0, \exists \delta > 0, \forall t \in T, t_0 < t < t_0 + \delta \rightarrow \left\| \frac{r(t) - r(t_0)}{t - t_0} - v_{0+} \right\| < \varepsilon$$

And $r(t)$ has a derivative (*simpliciter*) v_0 at t_0 iff it has both a left-derivative and a right-derivative at t_0 , which both have the same value v_0 ; this is mathematically equivalent to the following relation (e_3):

$$(e_3) \forall \varepsilon > 0, \exists \delta > 0, \forall t \in T, 0 < |t - t_0| < \delta \rightarrow \left\| \frac{r(t) - r(t_0)}{t - t_0} - v_0 \right\| < \varepsilon$$

Such definitions can be straightforwardly adapted if $r(t)$, v_{0-} , v_{0+} , v_0 and t_0 refer to physical vectors and scalars (that is, mathematical vectors and scalars with a unit). Suppose that $r_b(t)$ is a function giving the spatial position of $\mathbf{b}$ across time and $\mathbf{mp}_b$ is the motion process of $\mathbf{b}$, so that $\mathbf{b}$ participates in $\mathbf{mp}_b$. Then if $r_b(t)$ has a left-derivative v_{0-} (resp. right-derivative v_{0+} or derivative *simpliciter* v_0) at t_0 , we will say that $\mathbf{b}$ has a left-velocity (respectively right-velocity, or bilateral velocity) at t_0 with value v_{0-} (resp. v_{0+} or v_0). Such a left-velocity (resp. right-velocity or bilateral velocity) will be formalized as an instance $\mathbf{p}^I_{\mathbf{b},t_0}$ (resp. $\mathbf{p}^{I+}_{\mathbf{b},t_0}$ or $\mathbf{p}^I_{\mathbf{b},t_0}$) of the process profile $P^I_{v_{0-}}$ (resp. $P^{I+}_{v_{0+}}$ or $P^I_{v_0}$), subclass of *Left_velocity* (resp. *Right_velocity* or

⁸ As pointed by Smith [2], “Not every dimension of comparison between processes corresponds to a determinable process profile universal in the sense here intended. [...] We can compare processes also for example in terms of whether they involve the same participants, or take place in the same spatial regions. Process profiles enter into the picture only where it is something (thus some occurrent entity) in the processes themselves that serves as *fundamentum comparationis*.”

Bilateral_velocity), subclass of *Process_Profile*, such that $\mathbf{mp}_b$ **has_profile** $\mathbf{p}_{b,t_0}^L$ (resp. $\mathbf{mp}_b$ **has_profile** $\mathbf{p}_{b,t_0}^{I+}$ or $\mathbf{mp}_b$ **has_profile** $\mathbf{p}_{b,t_0}^I$).

In particular, given equations (e₁), (e₂) and (e₃), if $\mathbf{b}$ has at $\mathbf{t}_0$ both a left-velocity and a right-velocity with the same value v_0 , then $\mathbf{b}$ has a bilateral-velocity with value v_0 . Note that (e₁), (e₂) or (e₃) cannot be expressed in description logic: they can only serve as an elucidation of what it means for a motion process $\mathbf{mp}_b$, whose trajectory is described by the function $r(t)$ to have a process profile which is an instance of $P_{v_0}^L$, $P_{v_0}^{I+}$ or $P_{v_0}^I$.

We will introduce *Instant_velocity* as the partition of *Left_velocity*, *Right_velocity* and *Bilateral_velocity*; as well as the relation **instant_velocity_at**, which relates an instance of *Instant_velocity* with the time associated to this velocity. For example, $\mathbf{p}_{b,t_0}^L$, $\mathbf{p}_{b,t_0}^I$ or $\mathbf{p}_{b,t_0}^{I+}$ are all **instant_velocity_at** $\mathbf{t}_0$. We will also introduce *Motion_velocity*, subclass of *Process_profile* that encompasses both *Instant_velocity* and *Constant_velocity*. Therefore, all motion-velocities are occurrents.

Finally, a detailed ontology of the relations between process profiles has not been developed yet and is out of scope of the present article, but we will introduce the relation **has_profile_part** to express the fact that $\mathbf{p}_{b,t_0}^I$ is composed by both $\mathbf{p}_{b,t_0}^L$ and $\mathbf{p}_{b,t_0}^{I+}$:

$$\begin{array}{l} \mathbf{p}_{b,t_0}^I \text{ has_profile_part } \mathbf{p}_{b,t_0}^L \\ \mathbf{p}_{b,t_0}^I \text{ has_profile_part } \mathbf{p}_{b,t_0}^{I+} \end{array}$$

2.5. Towards Velocity as a Dependent Continuant

2.5.1. Insufficiency of the Motion-velocity Account

We have thus given a “motion-velocity account” of velocity. If left alone, it would suffer from two general problems: first, it is partly incompatible with the vocabulary of classical physics—this is the “problem of exoticism”; second, it may lead to practical difficulties—this is the “problem of practicality.”

The problem of exoticism has a first component insofar as classical physics commonly speaks of the velocity of an object: the “instantaneous velocity of a car” ([13], p. 69), the “velocity of a ship” (*idem*, p. 73), “a particle with velocity +v” (*idem*, p. 80), etc. However, the motion-velocity account does not assign directly any velocity to an object: saying that an object has a velocity v_0 at $\mathbf{t}_0$ is here a shorthand meaning that the object participates in a motion process that has as process profile an instance of $P_{v_0}^I$ that is **instant_velocity_at** $\mathbf{t}_0$.

The problem of exoticism has also a second, related component insofar as classical physics commonly speaks of “variation of velocity” (*idem*, p. 86), defining e.g., acceleration as “the rate of change of velocity with time” (*idem*, p. 85). However, an instant-velocity is an occurrent, and therefore does not evolve in time. Saying that the velocity of an object $\mathbf{b}$ evolved from value v_0 at $\mathbf{t}_0$ to value v_1 at $\mathbf{t}_1$ should here be understood as saying that $\mathbf{b}$ ’s motion process has two process profiles $\mathbf{p}_0$ and $\mathbf{p}_1$, respectively instances of $P_{v_0}^I$ and $P_{v_1}^I$ [2], such that $\mathbf{p}_0$ **instant_velocity_at** $\mathbf{t}_0$ and $\mathbf{p}_1$ **instant_velocity_at** $\mathbf{t}_1$.

This leads to the second issue, the problem of practicality: the motion-velocity account makes it difficult to track the evolution of velocity of an object, as it does not provide a physical continuant to which we could assign the value v_0 at $\mathbf{t}_0$ and v_1 at $\mathbf{t}_1$.

2.5.2. Introducing the Object-velocity Account

The problems of exoticism and practicality may not be fatal to the motion-velocity account, but we may hope avoiding them by introducing another kind of velocity⁹ $\mathbf{d}^V$ as a dependent continuant inhering in $\mathbf{b}$, that would instantiate at each time $\mathbf{t}$ a distinct determinate of velocity. For example, if $\mathbf{b}$ has a velocity $v_0 = (2, 4, -6)$ m/s at $\mathbf{t}_0$ and $v_1 = (1, 5, 7)$ m/s at $\mathbf{t}_1$, $\mathbf{d}^V$ would instantiate the universal $D^V_{v_0}$ at $\mathbf{t}_0$ and the universal $D^V_{v_1}$ at $\mathbf{t}_1$, where $D^V_{v_0}$ would be a universal of velocity with value $(2, 4, -6)$ m/s and $D^V_{v_1}$ would be a universal of velocity with value $(1, 5, 7)$ m/s.

First, this would fit with classical physics' usual vocabulary: we could speak directly of the velocity $\mathbf{d}^V$ of an object $\mathbf{b}$, and say that it evolves with time. Second, we could track the evolution of $\mathbf{b}$'s velocity value by tracking the changes of $\mathbf{d}^V$ in time.

This suggestion to complement the motion-velocity account is in line with BFO's spirit, as formulated by Smith ([2], p. 478, fn. 33): "[...] we could view speed in BFO terms as a (non-rigid) quality of the moving object [...]. We believe that a view along these lines for process measurement data in general can and should be developed, since processes of each different type can occur only if there are corresponding types of qualities and dispositions on the side of the continuants which are their participants. Thus we see a view of this sort as a supplement to an account along the lines presented in the text."

We will now show that there are indeed good philosophical reasons to introduce another velocity entity as a dependent continuant inhering in the object, which will be called "object-velocity"; this account will complement the "motion-velocity" account presented so far.

3. Object-velocity and the Inertial Disposition

3.1. The Causal Role of Velocity

An important question in the debate between reductionism and primitivism concerns the causal role of velocity. Lange [14] argues that there is a sense in which velocity has a causal (and therefore explanatory) role on the future trajectory: an object $\mathbf{b}$ at a position $\mathbf{r}(\mathbf{t}_0)$ at $\mathbf{t}_0$ will be close to position $\mathbf{r}(\mathbf{t}_0) + \mathbf{v}_0 \Delta t$ at time $\mathbf{t}_0 + \Delta t$ *because* it had at $\mathbf{t}_0$ a velocity $\mathbf{v}_0$. More precisely, Newton's second law of motion involves the equation $\mathbf{F} = m \cdot d^2 \mathbf{r} / dt^2$, which can be read as stating that the trajectory during the interval $]\mathbf{t}_0, \mathbf{t}_0 + \Delta t[$ will be causally influenced by the Newtonian forces on the object in the interval $]\mathbf{t}_0, \mathbf{t}_0 + \Delta t[$; but since it is a second order differential equation, any solution also requires to state the initial conditions $\mathbf{r}(\mathbf{t}_0)$ and $\mathbf{v}(\mathbf{t}_0)$. Thus, in the general case, not only the forces but also the initial velocity of the object have a causal role on the evolution of its position.

However, bilateral velocity could not have such a causal role: as a matter of fact, it is grounded not only on facts happening before $\mathbf{t}_0$ (namely $\mathbf{b}$'s positions in a neighborhood before $\mathbf{t}_0$, which also ground $\mathbf{b}$'s left-velocity at $\mathbf{t}_0$), but also on facts happening after $\mathbf{t}_0$ (namely $\mathbf{b}$'s positions in a neighborhood after $\mathbf{t}_0$ written $]\mathbf{t}_0, \mathbf{t}_0 + \Delta t'[$ which also ground $\mathbf{b}$'s right-velocity at $\mathbf{t}_0$); and facts during an interval $]\mathbf{t}_0, \mathbf{t}_0 + \Delta t'[$ cannot causally explain the trajectory during the interval $]\mathbf{t}_0, \mathbf{t}_0 + \Delta t[$, as some of the former facts happen after some

⁹ This velocity is written " $\mathbf{d}^V$ " because we will formalize it later as a disposition inhering in the object.

of the latter facts (see [14] for a meticulous demonstration of this point), and a cause needs to precede its effect. On the other hand, left-velocity, which is grounded only on facts before t_0 , could explain causally the trajectory after t_0 . However, we will now see that we could imagine a possible world in which left-velocity would not have such a causal role.

3.2. Tooley Worlds

Following Tooley [9], imagine a fictional world in which objects' positions are not guided by Newton's law of motions, and in which an object's location at one time puts no constraint on where it might be at any later time. In such Tooley worlds, the position of an object at each time might for example be totally random, or it might be decided by an omnipotent God. In either case, it might happen—either by chance, or by God's will—that during an extended period of time $]t_0 - \Delta t, t_0 + \Delta t[$, an object b_1 's trajectory would form a smooth curve, in which case it would be differentiable at t_0 . This motion process has a left-velocity at t_0 , but this left-velocity does not have any causal role in explaining the trajectory during the interval $]t_0, t_0 + \Delta t[$: the trajectory is due to chance or God's will, not to the object having a given motion-velocity at t_0 .

Several philosophical views have established close ties between causation and dispositions [15][16]. Accordingly, we will now see how Tooley's thought experiment reveals that any massive object in our world has an inertial disposition that objects do not have in a Tooley world.

3.3. The Inertial Disposition

Disposition has been formalized by Röhl and Jansen [21] as a type of *Dependent_continuant* that *inheres_in* a *Material_entity*, and is realized (*has_realization*) via a process. The realization process has the material entity as a participant, and the disposition is triggered by (*has_trigger_D*) some event or process. Finally, the disposition *has_base* some quality (or more generally, some “aggregate of dependent continuants”), and the material entity is the *bearer_of* the disposition. For example, a glass is fragile: fragility can be formalized as a disposition of the glass to break (the breaking process is a realization of the disposition) when it undergoes some tension (the tension process is a trigger of the disposition). The glass is the bearer of this disposition, and the base of this disposition is a particular molecular structure of the glass.

A dispositional account of velocity has been suggested by several authors in one form or another (see [14] for a review, and [17]). MacLaurin [18] spoke of velocity as a “power” and said “the velocity of motion is always measured by the space that would be described by that motion continued uniformly for a given time.” Walton [19] spoke of a “[t]endency forward in the body.” Such dispositions are linked to Newton's first law, the law of inertia, which states that an object b in an inertial reference frame has a constant velocity unless acted upon by external forces. This law of inertia implies the existence of several dispositions inhering in b , two¹⁰ of which—named d^C and d^I —will be described now.

¹⁰ We will not take position here on whether both dispositions are different but closely related entities, or are the same entity.

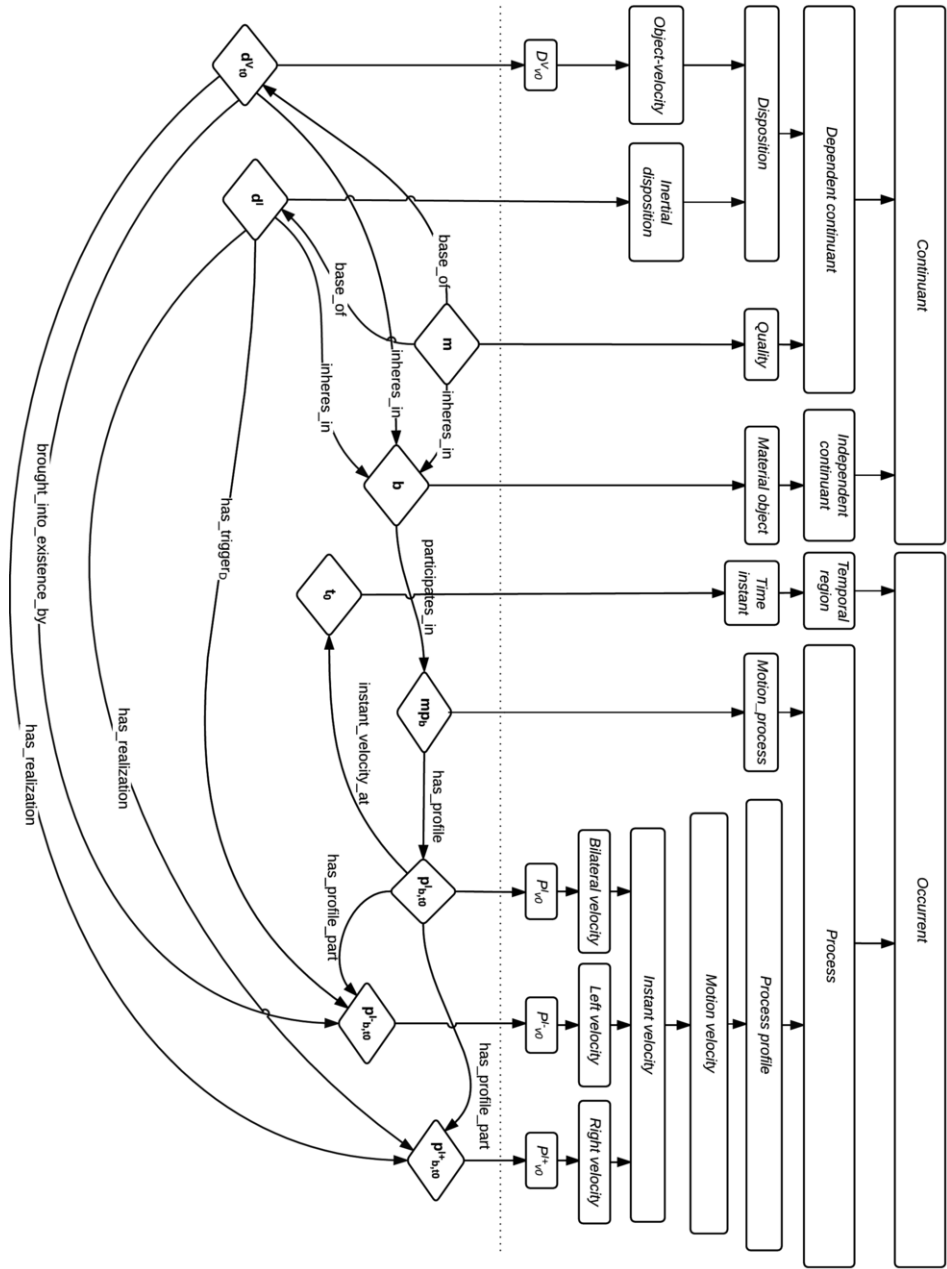


Figure 2. Entities and relations between particulars (the dotted line separates particulars from universals; relations between a particular and a universal are *instance_of*; relations between two universals are *is_a*)

$\mathbf{d}^C$ is the disposition, whenever $\mathbf{b}$ arrives with a left-velocity with value v_0 at a time t_0 , to continue its motion with a constant velocity profile with value v_0 on any interval $[t_0, t_1]$. Note that $\mathbf{d}^C$ will rarely, if ever, be realized: at least some external forces (such as gravitational forces) will be acting on the object, in which case its velocity may be at least slightly different from v_0 at some point on any interval $[t_0, t_1]$ —and such external forces can be interpreted as additional dispositions [20].

$\mathbf{d}^I$ is the disposition, whenever $\mathbf{b}$ arrives with a left-velocity with value v_0 at a time t_0 , to continue its motion with a right-velocity with value v_0 at t_0 . This disposition will be realized at all times when it exists, except when there is a resultant force with an infinite magnitude acting on $\mathbf{b}$ at t_0 , as given e.g., by a Dirac delta function (but note that the physical existence of such forces is questionable); therefore, $\mathbf{d}^I$ will be more relevant for our analysis than $\mathbf{d}^C$, and the rest of the article will deal exclusively with $\mathbf{d}^I$, which will be classified as an instance of *Inertial_disposition*.

At any time, $\mathbf{d}^I$ is triggered by the left-velocity of $\mathbf{b}$; and when realized, its realization is a right-velocity of $\mathbf{b}$ which has the same value as its left-velocity at this time. This means that if no force with infinite magnitude is acting on $\mathbf{b}$, $\mathbf{d}^I$ is triggered at t_0 by the left-velocity $\mathbf{p}^L_{\mathbf{b}, t_0}$ and is realized by the right-velocity $\mathbf{p}^{I+}_{\mathbf{b}, t_0}$; and it is triggered at t_1 by the left-velocity $\mathbf{p}^L_{\mathbf{b}, t_1}$ and is realized by the right-velocity $\mathbf{p}^{I+}_{\mathbf{b}, t_1}$. The base of $\mathbf{d}^I$ (and $\mathbf{d}^C$) is $\mathbf{b}$'s mass $\mathbf{m}$, as “mass is a quantitative measure of inertia” [22]. Even if the causal base of $\mathbf{d}^I$ remains unchanged during $\mathbf{b}$'s existence, various triggers at various time instants will bring about various realizations.

In a Tooley world, an object has no such inertial disposition, as its left-velocity at t_0 does not constrain its right-velocity at t_0 . Thus, the fact that an object has a motion-velocity does not conceptually imply that it also has an inertial disposition; but the laws of nature (namely, Newton's first law) are such that any object has such an inertial disposition in our world. In philosophical vocabulary, this translates by saying that massive objects in our world all have an inertial disposition in virtue of nomic necessity (that is, the laws of nature make it necessary)—rather than in virtue of logical or metaphysical necessity.

This translates in Röhl and Jansen's [21] formalization of disposition universals with the following five axioms:

- (A₁) *Inertial_disposition is_a Disposition*
- (A₂) *Inertial_disposition inheres_in Material_object*
- (A₃) *Inertial_disposition has_base Mass*
- (A₄) *Inertial_disposition has_realization Right_velocity*
- (A₅) *Inertial_disposition has_trigger_D Left_velocity*

We will now introduce a closely related disposition called “object-velocity,” and explain its link with the inertial disposition; for this, we will need to extend Röhl & Jansen's model by introducing the idea that a process can bring into existence a disposition.

3.4. Object-velocity at a Time

Let us consider again the case of fragility. A fragile object $\mathbf{fb}$ has a disposition $\mathbf{d}^F$ to break if it undergoes some stress, because of its molecular structure $\mathbf{ms}$: this disposition exists at every time at which $\mathbf{fb}$ exists. Imagine now that $\mathbf{fb}$ undergoes some tension in a process $\mathbf{s}$ lasting until t_0 , such that it has not broken yet at t_0 , but will break with

certainty just after t_0 —the tension is already stretching its molecular structure, and this will inevitably lead to its breaking. We can then say that $\mathbf{fb}$ just got a new disposition $\mathbf{d}_{t_0}^S$ which exists only at t_0 , namely a disposition to break, that will be realized for sure. Although $\mathbf{d}^F$ has been triggered by $\mathbf{s}$, $\mathbf{d}_{t_0}^S$ has been brought into existence by $\mathbf{s}$. As a matter of fact, had $\mathbf{s}$ not occur, then $\mathbf{d}_{t_0}^S$ would not have existed; on the other hand, the existence of $\mathbf{d}^F$ (that is, the existence of the object's fragility) does not depend on the occurrence of $\mathbf{s}$: it would have existed even if $\mathbf{s}$ would not have occurred, and only its realization at t_0 depends on the occurrence of $\mathbf{s}$. Therefore, we say that $\mathbf{s}$ brings $\mathbf{d}_{t_0}^S$ into existence¹¹, and we introduce accordingly a new relation: **$\mathbf{d}_{t_0}^S$ brought_into_existence_by $\mathbf{s}$** ¹². We could say that $\mathbf{d}^F$ is a formalization of the fragility of the object (which exists at every time of the life of the object), whereas $\mathbf{d}_{t_0}^S$ is a formalization of a consequence of this fragility at t_0 . Note that $\mathbf{d}_{t_0}^S$ exists because of both $\mathbf{ms}$ and $\mathbf{s}$; however, the fragility $\mathbf{d}^F$ exists only because of $\mathbf{ms}$.

Let us now apply this model to the case at hand. As we explained, the inertial disposition $\mathbf{d}^I$ is the disposition of a massive object to continue its motion at any time $\mathbf{t}$ with a right-velocity that has the same magnitude as the left-velocity with which it arrived at $\mathbf{t}$. However, for any given t_0 , once $\mathbf{b}$ has arrived with a left-velocity with value v_0 at time t_0 , it gets a new disposition called $\mathbf{d}_{t_0}^V$. $\mathbf{d}_{t_0}^V$ is the disposition of $\mathbf{b}$ to continue its motion with a right-velocity with value v_0 at t_0 : that is, both $\mathbf{d}^I$ and $\mathbf{d}_{t_0}^V$ are realized by $\mathbf{p}_{b,t_0}^+$. But whereas the left-velocity $\mathbf{p}_{b,t_0}^-$ is a trigger of $\mathbf{d}^I$ (which exists at any time during the life of the object), it brings into existence $\mathbf{d}_{t_0}^V$ (which exists only at t_0).

To summarize the parallel between the velocity case and the fragility example, $\mathbf{d}_{t_0}^V$ is to $\mathbf{d}_{t_0}^S$ what $\mathbf{d}^I$ is to $\mathbf{d}^F$: both $\mathbf{d}_{t_0}^V$ and $\mathbf{d}_{t_0}^S$ exist because of some process (**$\mathbf{d}_{t_0}^V$ brought_into_existence_by $\mathbf{p}_{b,t_0}^-$** and **$\mathbf{d}_{t_0}^S$ brought_into_existence_by $\mathbf{s}$**), whereas the inertia disposition $\mathbf{d}^I$ and the fragility $\mathbf{d}^F$ exist independently of those processes but are triggered by them (**$\mathbf{d}^I$ has_trigger_D $\mathbf{p}_{b,t_0}^-$** , and **$\mathbf{d}^F$ has_trigger_D $\mathbf{s}$**).

To clarify further, we can re-use Walton's notion of "tendency forward in a body." On one hand, $\mathbf{d}^I$ is a tendency forward in the body that will be realized *only* if the object arrives with a certain left-velocity: if it arrives at t_0 with a left-velocity with value v_0 , it will continue after t_0 with a right-velocity with value v_0 . On the other hand, $\mathbf{d}_{t_0}^V$ is an unconditional tendency forward in a body, in the sense that it will be realized no matter what, as soon as it exists; and it exists because the object had a left-velocity $\mathbf{p}_{b,t_0}^-$. We could also say that $\mathbf{d}^I$ is an expression of the law of inertia in $\mathbf{b}$, whereas $\mathbf{d}_{t_0}^V$ is a consequence of the law of inertia in $\mathbf{b}$.

The universal *Object-velocity* satisfies four axioms obtained by replacing *Inertial_disposition* by *Object-velocity* in (A_1) , (A_2) , (A_3) and (A_4) . However, *Object-velocity* does not satisfy an axiom adapted from (A_5) : "*Inertial_disposition* has_trigger_D *Left_velocity*"; instead, it satisfies:

(A_5') *Object-velocity brought_into_existence_by Left_velocity*

¹¹ The process bringing the disposition into existence is in many respects similar to the "existential condition" that was introduced in [20], as both entities bring into existence a disposition rather than causing its realization (see [15] for the related distinction between alpha-conditions and beta-conditions). However, they have different ontological natures: the former is a process, whereas the latter is a condition.

¹² As $\mathbf{d}_{t_0}^S$ will be realized for sure, any process in which $\mathbf{fb}$ participates at t_0 will act as a trigger—that is, $\mathbf{d}_{t_0}^S$ does not need a specific trigger to be realized.

3.5. Transtemporal object-velocity

The problem of exoticism mentioned earlier in 2.6.1 had two components: classical physics commonly speaks of the velocity of an object, and velocity can vary. The formalization of an object-velocity $\mathbf{d}^V_{t_0}$ solves the first component, as $\mathbf{d}^V_{t_0}$ inheres in $\mathbf{b}$. For this formalization to solve the second part of the problem of exoticism, we need to introduce an object-velocity $\mathbf{d}^V$ inhering in $\mathbf{b}$ at every instant at which $\mathbf{b}$ exists, such that for every time $\mathbf{t}$ at which $\mathbf{b}$ exists, $\mathbf{d}^V_t$ is a temporal part¹³ of $\mathbf{d}^V$ [26]. This entity $\mathbf{d}^V$ would also solve the problem of practicality: the object-velocity is a physical continuant evolving in time, to which we can assign the value v_0 at $\mathbf{t}_0$ and v_1 at $\mathbf{t}_1$ by formalizing that it instantiates a universal $D^V_{v_0}$ at $\mathbf{t}_0$ and a universal $D^V_{v_1}$ at $\mathbf{t}_1$.

One could object that each $\mathbf{d}^V_t$ is brought into existence at every time $\mathbf{t}$ by the left-velocity $\mathbf{p}^L_{\mathbf{b},\mathbf{t}}$ —and this could cast doubt on the existence of a continuant $\mathbf{d}^V$ existing at various times that would have as temporal parts the various $\mathbf{d}^V_t$. However, it is not absurd to think that a particular could exist transtemporally by having instantaneous temporal parts constituted by various particulars: a material wave in a liquid or a waterfall [27] may for example be such entities. We make therefore the assumption that this disposition $\mathbf{d}^V$ does exist—an assumption that calls for more detailed investigations of the transtemporal identity of dispositions.

4. Conclusion

4.1. Summary

Velocity is Janus-faced: what we commonly call “velocity” may refer to either motion-velocity on the occurrent side or object-velocity on the continuant side. Physics’ vocabulary sometimes suggests that velocity is captured by motion-velocity, when it defines a velocity as the time-derivative of the object’s position; but it sometimes suggests that it is better captured by object-velocity, when it assigns the velocity to an object rather than to a motion process.

In our world, those entities are systematically associated given the laws of nature: any massive object has an object-velocity (an entity closely related to its inertial disposition) that is brought into existence by its left-velocity, and can be realized in a right-velocity with the same value. However, motion-velocity and object-velocity are conceptually independent, as shown by Tooley’s thought experiment. Motion-velocity is a non-causal entity, whereas object-velocity and the inertial disposition both have a causal dimension.

The introduction of an object-velocity $\mathbf{d}^V_{t_0}$ at every time $\mathbf{t}_0$ solves the first component of the problem of exoticism, by formalizing velocity as a continuant inhering in the object. Furthermore, the introduction of the continuously existing object-velocity $\mathbf{d}^V$, which has $\mathbf{d}^V_t$ as temporal part at any time $\mathbf{t}$ at which it exists, solves the second component of the problem of exoticism as well as the problem of practicality.

¹³ Note that when we speak here of a “temporal part of x ,” both x and its temporal part are continuants.

4.2. Extensions

The dispositions $\mathbf{d}^I$ and $\mathbf{d}^V$ have various realizations at various instants in time. Therefore, their representation would involve ternary relationships such as **realized_in**($\mathbf{d}^I, \mathbf{p}^{I+}_{b,t}, \mathbf{t}$). Such relationships cannot be directly represented in description logic and thus in OWL. A review of different methods to transform such relations into binary relations is proposed in [12], which also suggests a new methodology called the temporally qualified continuant (TQC) approach, that might be compatible with the present formalization¹⁴.

This point is also related to a more general question pertaining to the formalization of dispositions. As pointed by Röhl and Jansen [21], some dispositions may have different kinds of triggers—they are “multi-trigger”—or different kinds of realizations—they are “multi-track”; but to our knowledge, no formalization has been offered so far for such kinds of dispositions. As $\mathbf{d}^I$ can have various triggers (namely, left-velocities with potentially different values at different instants), it is a multi-trigger disposition; and as $\mathbf{d}^I$ and $\mathbf{d}^V$ can have various realizations (namely, right-velocities with potentially different values at different instants), they are both multi-track dispositions. Thus, a methodology like the TQC approach mentioned earlier might be a tool to formalize other multi-trigger and multi-track dispositions.

A next step in the development of a dispositional ontology of Newtonian mechanics would be to unify the present formalization with the formalization of forces as dispositions [20]—which would imply in particular to clarify the ontological status of acceleration. Also, we have been working here under the assumption of an absolute Newtonian space, but the present account should be seen as a first step before turning to relative velocities in a non-absolute space: as a relative velocity is indexed by some reference frame, a disposition that would formalize this relative velocity would also need to be indexed by a reference frame. Another question is how such an account would relate with Galton and Mizoguchi’s account [27] (see also [28]) that endorses qualities of processes—and in which velocities could be formalized as qualities of motion processes.

Finally, as velocity characterizes a position rate of change, a question is whether other forms of rate of change such as blood pressure rate of change or glycaemia rate of change should be formalized by a dual continuant and occurrent model; and whether dispositions would play a similar role as they do here, relating causally process profiles describing those rates of changes.

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¹⁴ If the TQC approach was adopted to the problem at hand, $\mathbf{d}^V_{t_0}$ might be interpreted as identical to the temporally qualified continuant $\mathbf{d}^V@t_0$ (see the notation in [12]), but it would certainly not be the same entity as $\mathbf{d}^I@t_0$: as explained earlier, the latter is triggered by $\mathbf{p}^{I-}_{b,t_0}$, whereas the former is brought into existence by $\mathbf{p}^{I-}_{b,t_0}$.

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