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Bearing capacity of rectangular footings on two-layer clay

Capacité porteuse des semelles de fondation rectangulaires sur deux couches argileuses

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ABSTRACT
While bearing capacity of footings has been a widely researched area, only recently have rigorous solutions to square and rectangular footings been attempted. A finite element analysis of square and rectangular footings over two-layer clay foundation soil is presented. Bearing capacity results are shown for a limited range of parameters. While the bearing capacity is distinctly affected by both the ratio of the strengths of the two layers and the depth of the weak layer, the shape factors are only dependent on the depth ratio.

RÉSUMÉ
Bien que de nombreuses recherches aient été conduites sur la capacité porteuse des semelles de fondation, c'est seulement récemment que des solutions exactes pour semelles carrées et rectangulaires ont été investiguées. Une analyse aux éléments finis, de ces semelles carrées et rectangulaires, sur un sol de fondation composé de deux couches argileuses, est présentée. Les résultats sont analysés en termes de capacité porteuse pour un nombre limité de paramètres. Alors que la capacité porteuse est clairement influencée par le rapport de capacité des deux couches, ainsi que par la profondeur de la couche la plus faible, les facteurs de formes sont uniquement dépendants du rapport des profondeurs.

1 INTRODUCTION

Design of shallow footings requires both the settlement and bearing capacity calculations to assure the servicibility and safety of structures. This presentation focuses on the latter. Multilayer soils are commonly encountered in practice, and the specific foundation soil considered here is a relatively weak clay overlaid by a stronger clay layer.

The early consideration of a strip footing over a two-layer clay is owed to Button (1953), and the investigations since then all focused on the two-dimensional problems (strip footings), although the methods of analysis varied. For instance, Button (1953) and Reddy and Srinivasan (1967) used the limit equilibrium method with a cylindrical failure surface, whereas Meyerhof and Hanna (1978) used a semi-empirical technique that is somewhat more difficult to interpret, as it was based on small-scale tests. Numerical methods (finite element, finite difference) were utilized by Burd and Frydman (1997). Michalowski (1992, 2002) utilized the kinematic theorem of limit analysis to arrive at the bearing capacity of two-layer clay with distinctly different compressive strength, and Merifield et al. (1999) employed a numerical technique to calculate both lower and upper bounds to limit loads based on the static and kinematic theorems of limit analysis.

All approaches to bearing capacity over two-layer soils, so far, have included plane-strain analyses, appropriate for long (or strip) footings. This paper employs the finite element method to assess the true three-dimensional problem of soil collapse under square and rectangular footings over two-layer clays.

Recent three-dimensional analyses considering the three-dimensional problem of collapse of footings are briefly described in the next section, followed by the description of the method used in this paper. The results for a limited range of parameters are presented. The results for a special case of a long (strip) footing are compared to those calculated by others using different techniques. Then, for a special case when the strength of the two layers is identical (uniform soil), the results for square and rectangular footings are compared to those obtained by the limit analysis approach. The paper is completed with brief conclusions.

2 FORMULATION

A rigorous limit analysis approach (upper bound) to solving for bearing capacity of a two-layer foundation soil was described by Michalowski and Shi (1995) for strip footings. The three-dimensional nature of the collapse of the soil under square and rectangular footings is traditionally addressed by introducing 'shape factors' to a formula that was originally devised for strip footings (Buisman 1940, Terzaghi 1943). A recent approach to a true 3-D problem using the kinematical approach of limit analysis was presented by Michalowski (2001), who considered a three-dimensional failure mechanism illustrated in Figure 1.

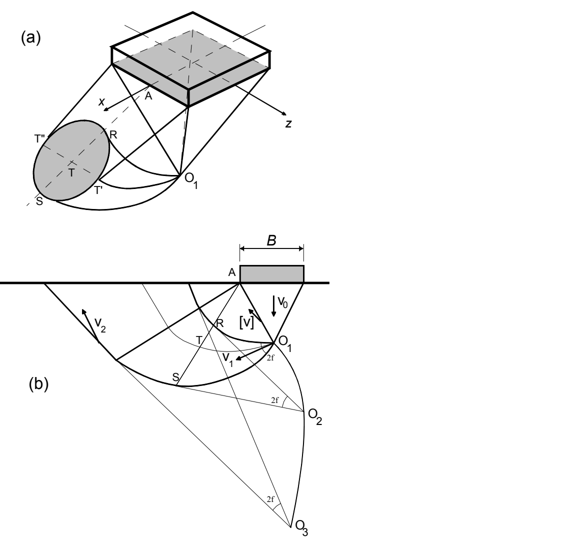

Figure 1. Collapse mechanism of frictional soil under a square footing.

The mechanism consists of an inverted pyramid immediately under the footing, and four curvilinear cones extending in four mutually perpendicular directions from the apex of the pyramid (only a part of one cone is shown in Fig. 1(a) to preserve the clarity of the geometrical features). Utilizing the kinematic theorem of limit analysis, the computations were performed using an optimization technique that led to the least upper bound for the bearing capacity. It was found that a better (lower) upper bound could be obtained if the curvilinear cone was replaced with a series of linear cone segments. This counterintuitive result was due to restrictions that dilatancy imposes on the 3-D mechanism, i.e., the piece-wise linear cone allowed more 'geometrical flexibility' during optimization than the nonlinear cone did.

A numerical approach based on finite element analysis was used very recently (Zhu and Michalowski 2005) to indicate that the upper bound approach based on the mechanism suggested (Fig. 1) leads to considerable overestimation of the bearing capacity (square and rectangular footings) for dilatant soils, but it is an acceptable mechanism for incompressible soils.

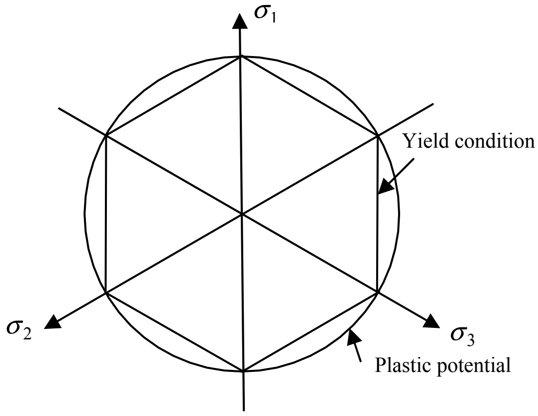

Figure 2. Yield condition and plastic potential on the octahedral plane.

The calculations of the bearing capacity over a two-layer clay presented here are based on the finite element analysis of an elasto-plastic soil with the strength characterized by the Tresca yield condition, but with a smooth plastic potential (circular on the octahedral plane, Fig. 2). The flow rule predicts incompressible deformation but it is not associated with the yield condition. The bearing capacity is independent of elastic properties, and isotropic linear elasticity was assumed with Young's modulus E = 20 MPa and Poisson's ratio ν = 0.3. The undrained shear strength of the upper clay layer was assumed to be c_1 = 20 kPa, and the lower layer's strength was determined from given ratio c_1/c_2.

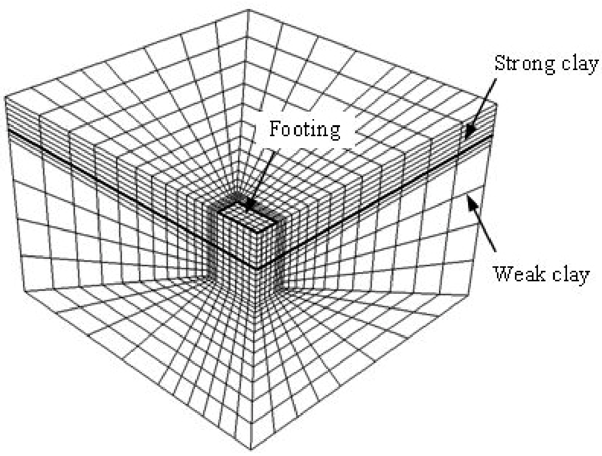

Figure 3. Finite element mesh for rectangular footing L/B = 2.0.

The soil beneath the rough footing was discretized into finite elements, and an example mesh is shown in Fig. 3 (footing length-to-width ratio L/B = 2.0).

This particular mesh has 4512 20-node brick elements, with 20101 nodes. The relative size of the model was: length 7.5B, width 7.5B, and depth 5B. Only ¼ of the footing was simulated due to double symmetry. The soil under the ¼ footing is discretized into 24 elements, and the mesh is refined near the footing edge to capture the significant displacement gradients. No horizontal displacement was allowed on any vertical surfaces of the model, and no displacement was allowed at the bottom boundary.

Finite element system ABAQUS was used to build the model and to carry out the computations. A single calculation using a state-of-the-art UNIX work station (at the time of computations, 2004) took about 14 hours.

3 RESULTS

The pattern of clay deformation for a rectangular footing is illustrated in Fig. 4.

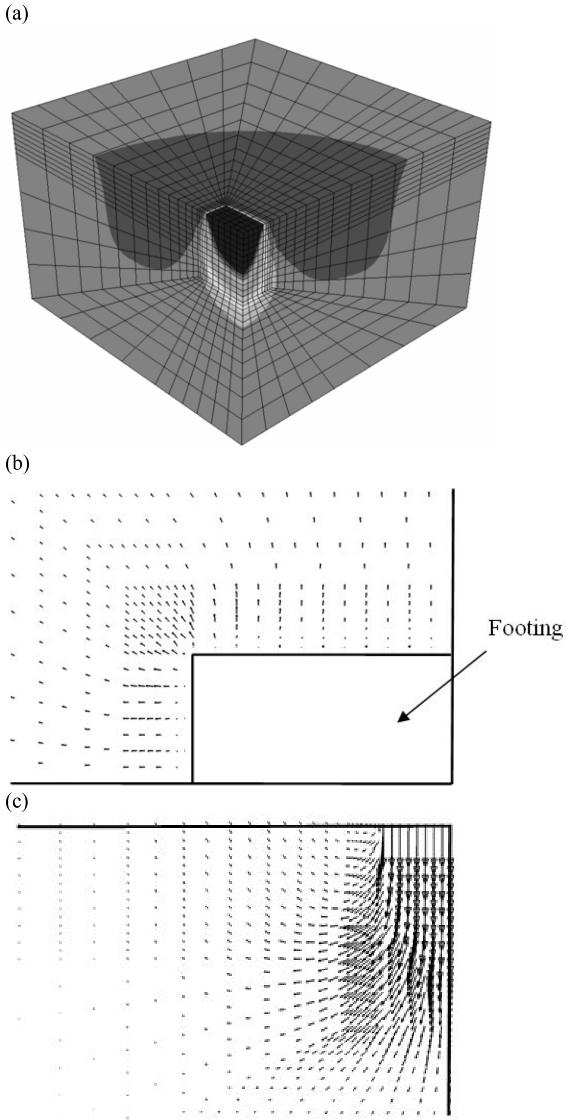

Figure 4. Deformation pattern under a rectangular footing: (a) intensity of the vertical displacements, (b) projections of total displacement vectors on the soil surface, and (c) total displacement vectors on the vertical cross section.

The largest magnitude of vertical displacements occur immediately under the footing (dark gray), and in the localized areas in close proximity to the flanks of the footing. The displacement field does not appear to have any (strong or weak) discontinuity at the interface of the weak and strong clay.

Consistent with the traditional notation, the average bearing pressure p is written here as

$$p = c_1 N_c^* \tag{1}$$

where c_1 is the cohesion of the upper layer and N_c* is the bearing capacity factor for a rectangular footing. Both layers of clay are considered in the analysis as incompressible. Consequently, the work of the weight of the portion of clay moving downward during collapse is exactly opposite to the work of clay moving upward, the net work being zero, hence $N_\gamma* = 0$, and one can also show that $N_q* = 1$ (N_q*, N_γ* being bearing capacity coefficients accounting for overburden pressure and soil weight, respectively).

Table 1. Bearing capacity coefficient N_c*.

c_1/c_2	H/B 0.50		0.75		1.00	
	L/B 1.0	2.0	1.0	2.0	1.0	2.0
5	3.456	2.970	4.317	3.699	5.096	4.359
3	4.284	3.754	5.108	4.486	5.590	5.094
2	5.002	4.463	5.583	5.095	5.622	5.484
1	5.624	5.499	5.624	5.499	5.624	5.499

Coefficient N_c* was calculated for square and strip footings, and rectangular ones with aspect ratio of L/B = 2, and for 3 combinations of the cohesion in the upper (first) and bottom layer c_1/c_2: 5, 3 and 2. Calculations were also carried out for uniform clay ($c_1/c_2 = 1$). All combinations were repeated for three depths of the lower (weaker) layer: H/B: 0.5, 0.75 and 1.0. The results are presented in Table 1.

Table 2. Comparison of coefficient N_c for a strip footing.

H/B	c_1/c_2	FEM (1)	Lower bound (2)	Upper bound (3)	Upper bound (4)
0.50	5	2.18	2.16	2.44	2.57
	3	2.91	2.84	3.16	3.17
	2	3.65	3.52	3.89	3.80
	1	5.20	4.94	5.32	5.14
0.75	5	2.71	2.64	2.98	3.19
	3	3.43	3.36	3.72	3.75
	2	4.11	4.00	4.37	4.29
	1	5.20	4.94	5.32	5.14
1.00	5	3.21	3.10	3.54	3.76
	3	3.93	3.89	4.24	4.29
	2	4.53	4.44	4.82	4.74
	1	5.20	4.94	5.32	5.14

(1) This paper's results; (2) and (3) Merifield et al. (1999);
(4) Michalowski (2002).

Calculations were carried out also for a strip footing, so that the effectiveness of the method could be compared to existing results from the limit analysis approach. The results for a strip footing are shown in Table 2, along with those by Merifield et al. (1999) and Michalowski (2002). Since some of these results in Table 2 were given in the literature with two digits past the decimal point, other results were truncated to match this format. As expected, FEM results in Table 2 yield a bearing capacity in between the two bounds from the limit analysis calculations.

Keeping with the traditional approach of presenting the bearing capacity of rectangular footings, we introduce 'shape factor' s_c*, so that the bearing capacity of a rectangular footing can be expressed through the bearing capacity factor of a strip footing N_c

$$p = c_1 N_c^* = c_1 s_c^* N_c \tag{2}$$

The shape factor was then calculated simply as ratio N_c*/N_c, with both coefficients taken from FEM calculations, and the results are presented in Table 3.

Table 3. Shape factor s_c*.

c_1/c_2	H/B 0.50		0.75		1.00	
	L/B 1.0	2.0	1.0	2.0	1.0	2.0
5	1.580	1.357	1.556	1.365	1.584	1.355
3	1.469	1.287	1.487	1.306	1.422	1.296
2	1.368	1.220	1.357	1.239	1.239	1.209
1	1.080	1.056	1.080	1.056	1.080	1.056

While the shape factor is very much dependent on the ratio of the strengths of the two layers, there is only a small influence of the depth of the weak layer. This is because the influence of the weak layer depth on the bearing capacity of the strip footing is comparable to that on square and rectangular footings. This is clearly illustrated in Figure 5.

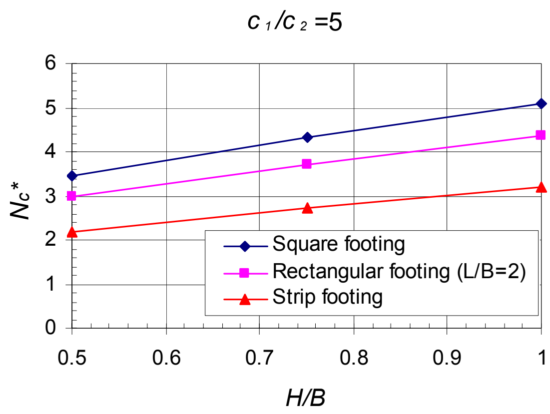

Figure 5. Bearing capacity factor as function of the depth of the weak layer.

A comparison of the bearing capacity factor for the square and rectangular footings obtained here using FEM to those from a rigorous kinematic limit analysis is presented in Table 4. This comparison is only for a uniform clay, as no 3-D limit analysis results for two-layer clay are available. Earlier limit analysis calculations overestimate the FEM bearing capacity estimates by about 10%.

Table 5. Comparison of N_c* from FEM and kinematic limit analysis (uniform clay).

FEM		Michalowski 2002	
L/B 1.0	2.0	1.0	2.0
5.624	5.499	6.561	6.060

Future efforts will concentrate on calculations of N_c* for a wide range of parameters, and on presenting design recommendations.

4 CONCLUSIONS

Finite element analysis is an effective technique for considering limit state problems. Methods that have been used earlier, in particular, the kinematic limit analysis approach, have been successful in considering plane-strain problems, but they become quite elaborate when applied in 3-D analysis. Although the numerical limit analysis approach is capable of solving 3-D problems effectively, it still only yields bounds to the solution that is obtained directly from FEM.

The bearing capacity of clay is reduced if a weaker layer of clay is present below a stronger crust. The limit load is affected by both the depth of the weaker layer and the ratio of the strengths of the two layers. However, the shape factor appears to be only weakly dependent on the depth, whereas it varies distinctly with a change in the strength ratio of the two layers.

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