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Application of non-coaxial plasticity models in geotechnical analysis

Application de modèles de plasticité non-coaxiaux pour l'analyse géotechnique

H.S. Yu, Y. Yang & X. Yuan
Nottingham Centre for Geomechanics, School of Civil Engineering, University of Nottingham, UK

ABSTRACT
This paper is concerned with the application of two non-coaxial plasticity models in soil modelling and geotechnical analysis. These non-coaxial models have been developed based on early work of Spencer (1982), Harris (1993) and Rudnicki & Rice (1975) among others. Simple shear behavior and footing settlement problems are analysed by using the finite element program ABAQUS incorporating these two models in its material subroutines. Numerical results suggest that non-coaxial models tend to give a softer response.

RÉSUMÉ
Cette publication se rapporte à l'application de modèles de plasticité non-coaxiaux en géotechnique. Ces modèles non-coaxiaux ont été développés sur la base de travaux publiés, entre autres, par Spencer (1982), Harris (1993) et Rudnicki & Rice (1975). Le comportement de contrainte simple ainsi que les problemes de tassement de la base sont analysés par le biais du programme aux elements finis, ABAQUS, dans lequel ces deux modèles ont été incoporés. Les résultats numériques suggèrent que les modèles non-coaxiaux tendent à prédire une réponse plus progressive.

1 INTRODUCTION

The non-coaxiality between principal stresses and principal plastic strain rates in granular material behaviour has been well recognized in the geotechnical community. On the experimental perspective, Roscoe et al (1967) found that the principal axes of strain rates and stresses are not coincident during the early stage of shearing in simple shear tests on sand. Figure 1 shows the experimental results reported by Roscoe (1970). Other researchers have also observed that the direction of the principal stress deviates from that of the principal strain increment under rotations of the principal stresses (Drescher & de Jong, 1972; Oda & Konishi, 1974; Ishihara & Towhata, 1983). On micromechanics perspective, principal plastic strain rates are also found not to be coincident with principal stresses (Christoffersen et al., 1981). All of these evidences indicate the use of conventional plasticity characterized with coaxiality is not justified for problems involving strong stress rotations.

Several theories have been developed to represent the non-coaxiality for granular material behaviour. Notable examples of these theories are double-shearing theory (Spencer, 1964, 1982; Harris, 1993), and the theory proposed by Rudnicki & Rice (1975). However, these theories have so far been mainly employed to study shear bands of granular materials, and they are yet to be used to solve boundary value problems. The objective of this paper is to investigate how the non-coaxial models influence the solution of boundary value problems. Two non-coaxial plasticity theories proposed by Harris (1993) and Rudnicki & Rice (1975) are considered and elasto-perfect plasticity is employed in this paper. Investigations are performed on simple shear of sands and footing settlements, which involve strong stress rotation.

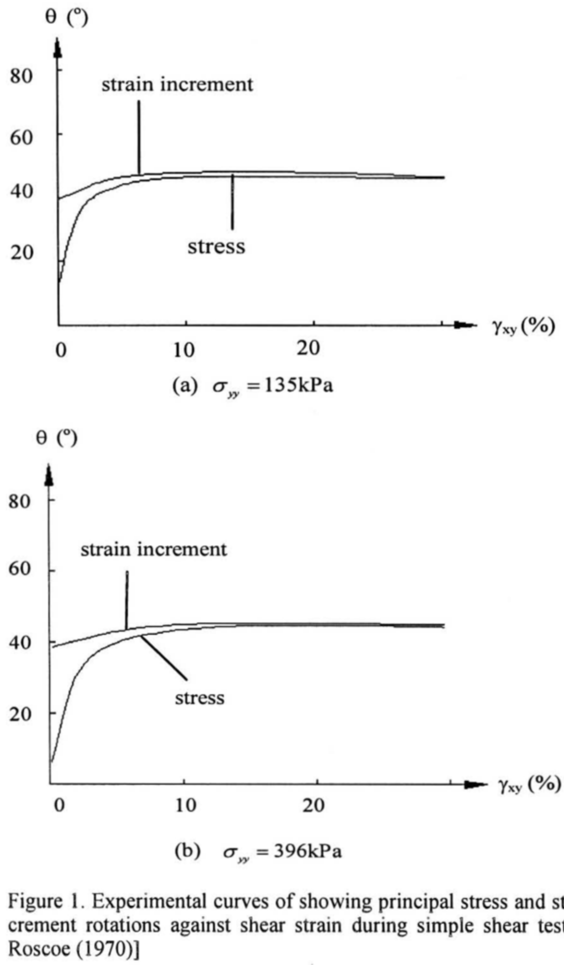

Figure 1. Experimental curves of showing principal stress and strain increment rotations against shear strain during simple shear test [After Roscoe (1970)]

2 HARRIS' NON-COAXIAL PLASTICITY THEORY

In a non-coaxial plasticity theory, the plastic strain rate may be assumed to include a coaxial ($\dot{\boldsymbol{\varepsilon}}^{pc}$) and a non-coaxial parts ($\dot{\boldsymbol{\varepsilon}}^{pn}$):

$$\dot{\boldsymbol{\varepsilon}}^{p} = \dot{\boldsymbol{\varepsilon}}^{pc} + \dot{\boldsymbol{\varepsilon}}^{pn} \tag{1}$$

According to Spencer (1982), Harris (1993) and Yu and Yuan (2004), the plastic strain rate in plane strain may be defined as,

$$\dot{\boldsymbol{\varepsilon}}^{p} = \dot{\lambda}\frac{\partial g}{\partial \boldsymbol{\sigma}} + \alpha \dot{\boldsymbol{t}} \tag{2}$$

where the first term on the right hand side represents the coaxial plastic strain rate and the second term represents the non-coaxial plastic strain rate. g represents the plastic potential. α is a non-coaxial model constant. When α is zero, equation (2) reduces to the conventional, coaxial stress-strain relationship. The vector $\boldsymbol{t}$ is related to the direction of principal stresses,

$$\boldsymbol{t} = \begin{bmatrix} \cos 2\theta_\sigma & -\cos 2\theta_\sigma & 2\sin 2\theta_\sigma \end{bmatrix}^T \tag{3}$$

where θ_σ is the angle between the minor principal compressive stress direction and x-axis, defined as,

$$\cos 2\theta_\sigma = \frac{\sigma_{xx} - \sigma_{yy}}{\left[\left(\sigma_{xx} - \sigma_{yy}\right)^2 + 4\sigma_{xy}^2\right]^{\frac{1}{2}}}, \quad \sin 2\theta_\sigma = \frac{2\sigma_{xy}}{\left[\left(\sigma_{xx} - \sigma_{yy}\right)^2 + 4\sigma_{xy}^2\right]^{\frac{1}{2}}} \tag{4}$$

The above formulations also indicate that there is only non-coaxial plastic deviatoric strains and non-coaxial plastic volumetric strains are zero. Using the consistency condition, Yu and Yuan (2004) obtain the following relationship between the stress rate and strain rate:

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{D}^{ep}\dot{\boldsymbol{\varepsilon}} \tag{5a}$$

$$\boldsymbol{D}^{ep} = \left(\tilde{\boldsymbol{D}}^{e} - \frac{\tilde{\boldsymbol{D}}^{e}\dfrac{\partial g}{\partial \boldsymbol{\sigma}} \cdot \left(\dfrac{\partial f}{\partial \boldsymbol{\sigma}}\right)^T \cdot \tilde{\boldsymbol{D}}^{e}}{\left(\dfrac{\partial f}{\partial \boldsymbol{\sigma}}\right)^T \cdot \tilde{\boldsymbol{D}}^{e} \cdot \dfrac{\partial g}{\partial \boldsymbol{\sigma}}} \right) \tag{5b}$$

where $\tilde{\boldsymbol{D}}^{e}$ is different from the original elastic matrix, and it incorporates the angle of principal stresses. f represents the yield surface and Mohr-Coulomb yield surface in plane strain condition is used in this paper. Equation (5) has been implemented into ABAQUS as a subroutine.

3 RUDNICKI AND RICE'S NON-COAXIAL THEORY

Following Rudnicki & Rice (1975), the non-coaxial plastic strain rate is assumed to be given as

$$\dot{\boldsymbol{\varepsilon}}^{pn} = \frac{1}{h_1}\boldsymbol{n} \tag{6}$$

$$\boldsymbol{n} = \dot{\boldsymbol{s}} - \frac{(\boldsymbol{s} : \dot{\boldsymbol{s}})}{2\tau^2}\boldsymbol{s} \tag{7}$$

where $\boldsymbol{s}$ denotes the deviatoric stress tensor and $\tau = \sqrt{0.5(\boldsymbol{s} : \boldsymbol{s})}$. h_1 represents the non-coaxial plastic modulus and is assumed to be a constant. In the above formulations, when $\dot{\boldsymbol{s}}$ and $\boldsymbol{s}$ are in the same direction, $\boldsymbol{n}$ is zero and the non-coaxial plastic strain rates vanish. Similar to Harris' non-coaxial theory, there are only deviatoric plastic non-coaxial strains and the volumetric plastic strain is entirely coaxial. According to Papamichos and Vardoulakis (1995) and Yang and Yu (2004), the relationship between total strain rates and stress rates can be obtained as,

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{D}^{ep}\dot{\boldsymbol{\varepsilon}} \tag{8a}$$

$$\boldsymbol{D}^{ep} = \left(\boldsymbol{D}^{e} - \frac{\boldsymbol{D}^{e}\dfrac{\partial g}{\partial \boldsymbol{\sigma}}\left(\dfrac{\partial f}{\partial \boldsymbol{\sigma}}\right)^T \boldsymbol{D}^{e}}{\left(\dfrac{\partial f}{\partial \boldsymbol{\sigma}}\right)^T \boldsymbol{D}^{e}\left(\dfrac{\partial g}{\partial \boldsymbol{\sigma}}\right)} - h_1\boldsymbol{D}^{e}\boldsymbol{N} \right) \tag{8b}$$

In equation (8), the first two terms on the right hand side represent the stiffness matrix in conventional elasto-plasticity theory, and $\boldsymbol{N}$ in the last term represents the stiffness matrix contributed by non-coaxial effect. Details can be found in Yang and Yu (2004). The Drucker-Prager yield surface is used in this paper. The above formulations are implemented into ABAQUS as a subroutine.

4 PREDICTION OF SIMPLE SHEAR BEHAVIOUR

Simple shear tests involve a strong principal stress rotation and are suitable for validating non-coaxial soil models. The simple shear sample is simulated by using an 8-noded element with reduced integration. The sample is subject to a constant vertical pressure and shear displacement is applied until failure of the sample. The investigations are performed under various conditions such as different initial static lateral pressure coefficients K_0, different flow rules and hardening rules. Full details can be found in Yang and Yu (2004).

Typical results of simulations by using Harris' and Rudnicki & Rice's theories are presented in Figures 2-5, which illustrate the evolutions of shear stress normalized by vertical stress and the evolutions of orientations of major principal stress and plastic strain rate. For comparisons, the predictions with coaxial models are also shown in these figures. In these predictions, K_0 are 0.4 and 3.0, respectively. In Harris' Mohr-Coulomb model, the frictional angle is 35 degrees and the dilation angle is 5 degrees. In Rudnicki & Rice's Drucker-Prager model, the ultimate shear stress ratio is 0.7, and zero plastic volumetric strain is assumed.

Both of the models show that when K_0 is 0.4, non-coaxial models give softer responses than coaxial models. In addition, the orientation of major plastic strain rate is ahead of that for the major principal stress. These results are consistent with the experiment results by Roscoe et al (1967).

On the other hand, when K_0 is 3.0, the responses exhibit softening behavior and the use of non-coaxial models hinder the stress ratio to reach its ultimate state in the regime of softening. In addition, the orientation of major plastic strain rate is behind that for the major principal stress. In most predictions, the quantities predicted using non-coaxial models approach those using coaxial models at large shear strains.

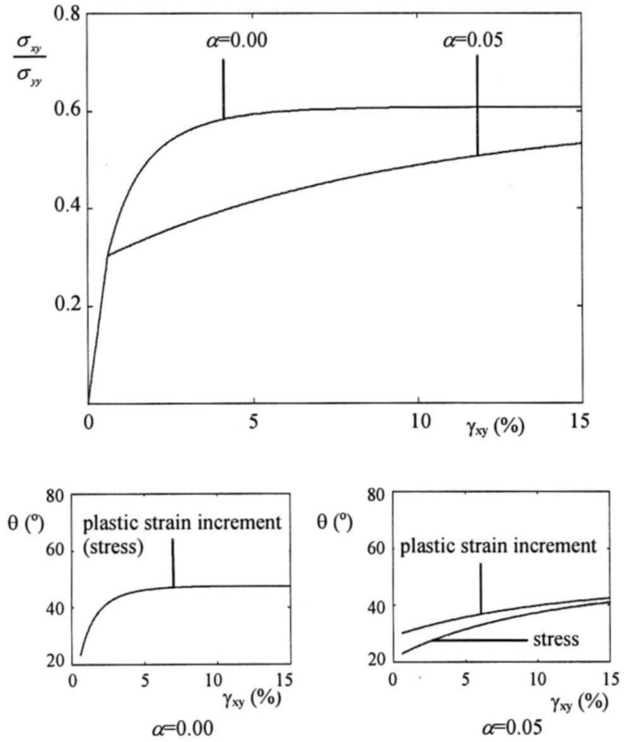

Figure 2. The evolutions of shear stress ratio and orientation of major principal stress and plastic strain rates in a simple shear of sand with K_0 equal to 0.4 using Harris' theory

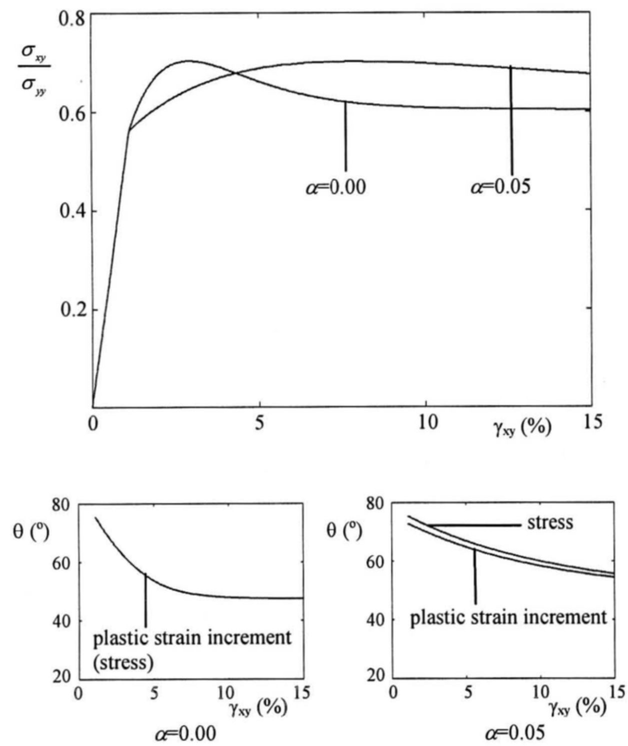

Figure 3. The evolutions of shear stress ratio and orientation of major principal stress and plastic strain rates in a simple shear of sand with K_0 equal to 3.0 using Harris' theory

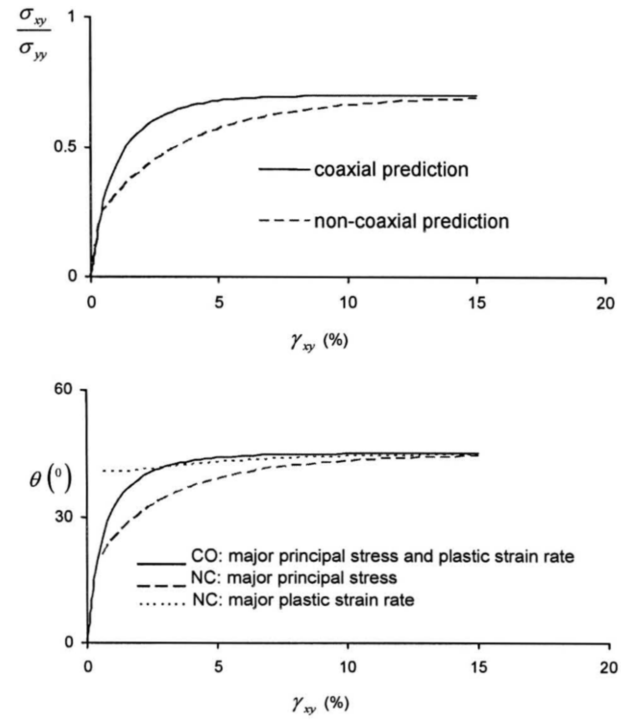

Figure 4. The evolutions of shear stress ratio and orientation of major principal stress and plastic strain rates in a simple shear of sand with K_0 equal to 0.4 using Rudnicki & Rice's theory

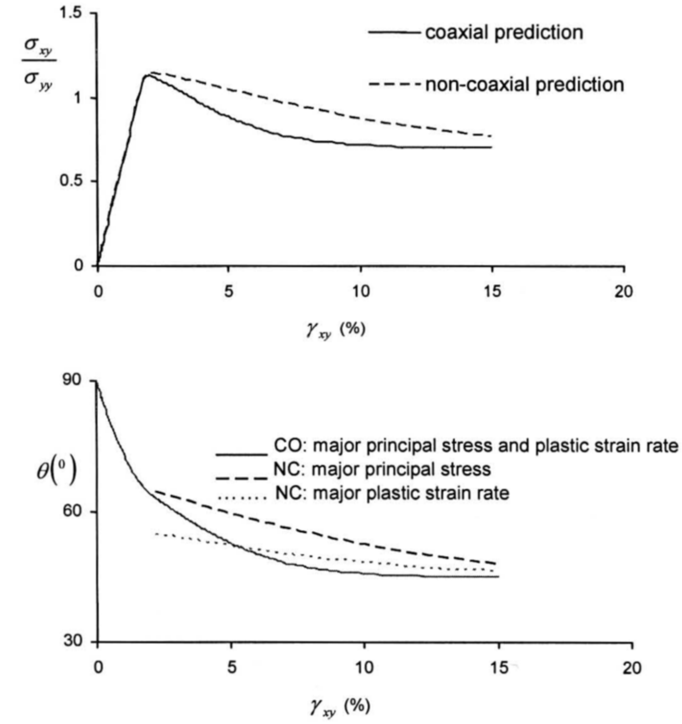

Figure 5. The evolutions of shear stress ratio and orientation of major principal stress and plastic strain rates in a simple shear of sand with K_0 equal to 3.0 using Rudnicki & Rice's theory

5 FOOTING SETTLEMENT PREDICTIONS

A rigid rough footing in soils is considered here. In Harris' Mohr-Coulomb model, the friction angle is 30 degrees and the dilation angle is 20 degrees, and the cohesion c is 69 kPa. Different α values are considered including α of zero, representing coaxial behavior. In addition, the bearing capacity obtained from Prandtl's theory is used for comparison. In Rudnicki & Rice's Drucker-Prager model, the ulitmate shear strength is 0.5 and an associated flow rule is used. A preloading (surcharge) p_0 of 100 kPa is applied on the surface of sand mass before the footing is loaded. Different values of h_1/G together with the coaxial model are employed, where G denotes the elastic modulus. Moreover, the predictions using the Drucker-Prager model provided in ABAQUS are performed under the exactly same conditions as in the authors' developed model to verify the author's predictions. The load-settlement curves predicted using these two methods are shown in Figures 6 and 7, where the pressure applied is normalized with c or p_0 and the settlement is normalized with footing width B.

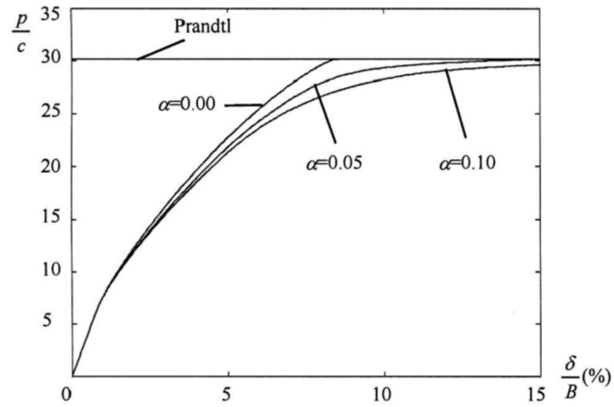

Figure 6. Footing settlement predictions under uniform loading using Harris' non-coaxial model with different values of α

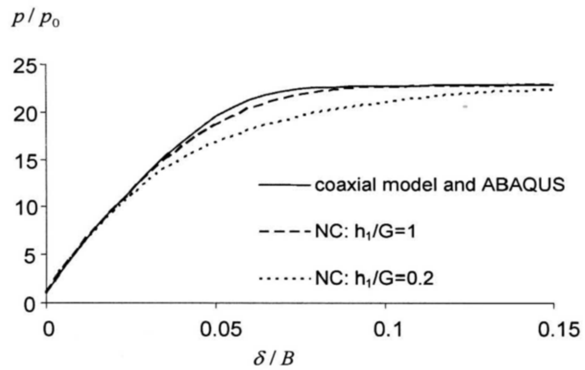

Figure 7. Footing settlement predictions under uniform loading using Rudnicki & Rice's non-coaxial model with different values of h_1/G

These two figures show that the non-coaxial models give larger settlements than coaxial models. This is caused by the rotation of principal stresses in soil elements underneath the footing. However, both the coaxial and non-coaxial models give similar bearing capacities for the case of vertical loading. For the coaxial and non-coaxial Mohr-Coulomb models, the predicted bearing capacity for all the cases is the same as the analytical solution from Prandtl's theory. For the Drucker-Prager models, the predicted bearing capacities from the authors' own computer procedures are the same as that from the existing Drucker-Prager model in the standard ABAQUS. This confirms the correctness of the numerical procedure used by the authors to implement the non-coaxial models within ABAQUS.

6 SUMMARY AND CONCLUSIONS

This paper is concerned with the application of non-coaxial plasticity models in soil modelling and geotechnical analysis. The investigations are focussed on simple shear of sands and footing settlement predictions, both of which involve principal stress rotations. The predictions are made using both the Mohr-Coulomb and Drucker-Prager yield surfaces with associated and non-associated flow rules. The results show that non-coaxial models generally give a softer response than coaxial models. In other words, for a given load the predicted deformatioin using a non-coaxial model is larger than that from a conventional, coaxial model. This has a signficant implication in geotechnical design involving granular soils where desgin is usually controlled by deformation rather than limit loads.

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