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Numerical implementation of a constitutive model for soil creep

Application numérique d'un modèle comportement pour la fluage de terre

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ABSTRACT

A differential form of 1-D creep model, proposed by Bjerrum in 1967, which was later modified for transient loading conditions is extended to multi-dimensional (2-D and 3-D) state of stress and strain by incorporating concepts of visco-plasticity (Vermeer et al., 1998). The devised creep models take into account both volumetric creep strain and deviatoric creep strain, and creep deformation of soil is defined by several material parameters. The model is incorporated as a plane strain element subroutine in a non-linear, time-incrementing finite element program. The model enables the solution of complicated foundation-soil interaction problems involving creep of soil. Numerical analyses are conducted using published experimental data and parametric studies are conducted to evaluate the sensitivity of different input parameters of the model.

RÉSUMÉ

Une forme différentielle du modèle en fluage mono-dimensionnel proposé par Bjerrum en 1967, qui a été ensuit modifié pour le régime transitoire est étendu aux états multi-dimensions (2-D et 3-D) de contrainte et de déformation en incorperant les concepts de visco-plasticité (Vermeer et al. 1998). Ces modèles en fluage tiens compte tous les aspects de la défomation en fluage de terre définit par plusieurs paramètres de matériaux. Ce modèle est incorporé comme un sons-programme de la déformation plane avec un programme éléments finis non-linéaire augmentation de temps. Ce modéle permettra des solutions dans la domaine de temps pour les problèms en fluage du terre liés aux foundation-terre interactions. Les résultats numériques ont été obtenns en utilisent les donées expérimentaux déja publiés. Finalment, une analyse paramétrique est faite afin d' identifier les parameters de ce modèle les plus sensitifs.

1 INTRODUCTION

In many soils primary compression (consolidation settlement) is always followed by a certain amount of creep settlement, also known as secondary compression. In general engineering practice, secondary compression (for instance during a period of 10 or 30 years) is assumed to be a small percentage of primary compression; thus, secondary compression is significant only when large primary settlements are experienced. Hence large primary settlement of roads, river embankments, dams or buildings, which have been constructed on soft soils, is usually followed by substantial creep settlement in later years.

Contrary to the above process, potentially treacherous situations may arise where a small primary settlement is followed by a large creep settlement, especially in over-consolidated clays. When the structure is founded on initially over consolidated clays it shows relatively small primary settlement. Then, as a consequence of the loading, a state of normal consolidation may be reached and significant creep may follow without giving an advance warning of large primary compression.

Apart from the usually considered foundation-related problems, creep plays an important role in slope stability also; gradual geometric changes due to creep and associated reduction of strength due to smoothening of soil particles may then lead to slope slides. The different problems that relate to creep of soil have made it necessary to develop a stress-strain relationship that takes creep in two- and three-dimensional situations into account.

Literature considers the development of time dependent models of one-dimensional soil compression and Bjerrum (1967) suggested a comprehensive 1-D creep model based on the behaviour of over-consolidated Norwegian clays with a sedimentation history of 3000 years. This 1-D model is based on an expression for creep strain rate for constant effective stress, and was later modified for transient loading conditions by Vermeer et al. (1998).

In the present work, this 1-D creep model is extended to multi-dimensional state of stress and strain by incorporating concepts of visco-plasticity. A non-linear, time incrementing finite element program, along with iterative corrections within each time step, was developed by the first author (Puswewala et al. 1992). Certain modifications were done in the latter main program to incorporate the present model as an element subroutine for plane strain condition. Numerical analyses are conducted using published experimental data and parametric studies are conducted to evaluate the sensitivity of different input parameters of the model.

2 THE 1-D CREEP MODEL

In his Rankine Lecture, Bjerrum (1967) described the compression of clays exhibiting creep under constant effective stress. Based on the work done by Bjerrum on soil creep, Garlanger (1972), proposed a creep equation for loading above preconsolidation pressure, σ'_c, of the form:

$$\Delta e = C_r \log\frac{\sigma'_c}{\sigma'_o} + C_c \log\frac{\sigma'_f}{\sigma'_c} + C_\alpha \log\frac{t_c + t}{t_c} \tag{1}$$

where Δe is the change in void ratio, C_r is the slope on an e versus $\log \sigma'$ diagram of the compression line from σ'_o to σ'_c, C_c is the slope of the instant line, C_α is the slope of the e versus log *time* curve, and t_c is the time for end of primary consolidation.

Above expression was later modified by Butterfield (1979) to fit into the framework of critical state soil mechanics. He introduced the logarithmic strain concept and the total volumetric total strain, ε_v, was decomposed into volumetric elastic strain, ε_v^e, and volumetric creep strain, ε_v^{cr}. Soil parameters C_r, C_c, and C_α are replaced with Modified Clam-Clay Model parameters $\lambda^*, \kappa^*, \mu^*$, and primary consolidation time, t_c, is replaced with the time factor τ_c.

$$\begin{aligned}\varepsilon_v &= \varepsilon_v^e + \varepsilon_v^{cr} \\ &= \kappa^* \ln\frac{\sigma'}{\sigma'_o} + \left(\lambda^* - \kappa^*\right)\ln\frac{\sigma'_{pc}}{\sigma'_{po}} + \mu^* \ln\frac{\tau_c + t'}{\tau_c}\end{aligned} \tag{2}$$

where σ'_{pc} and σ'_{po} represent preconsolidation pressures corresponding to before-loading state and end-of-consolidation state respectively.

By adopting Bjerrum's concept (Bjerrum, 1967) that the secondary compression increases the preconsolidation pressure, by eliminating σ'_{pc} and τ_c from the equation (2), Vermeer et al. (1998) proposed a differential form of expression that accounts for creep of soft soil, of the form:

$$\begin{aligned}\dot{\varepsilon}_v &= \dot{\varepsilon}_v^e + \dot{\varepsilon}_v^c \\ &= \kappa^* \frac{\dot{\sigma}'}{\sigma'} + \frac{\mu^*}{\tau}\left(\frac{\sigma'}{\sigma'_p}\right)^{\frac{\lambda^* - \kappa^*}{\mu^*}} \\ &\text{where } \sigma'_p = \sigma'_{po} \exp\left(\frac{\varepsilon_v^c}{\lambda^* - \kappa^*}\right)\end{aligned} \tag{3}$$

3 GENERALIZATION OF 1-D MODEL TO 3-D

The expression for volumetric creep strain rate $\left(\dot{\varepsilon}_v^c\right)$ in equation (3) is extended for a general state of stresses and strains. Well-known stress invariant quantities are adopted for pressure p and deviatoric stress q as $p = \sigma_{oct}$ and $q = 3\tau_{oct}/\sqrt{2}$, with σ_{oct} and τ_{oct} being the octahedral normal stress and octahedral shear stress, respectively. In terms of principal stresses, pressure p and deviatoric stress q can be expressed as:

$$\begin{aligned}p &= \frac{1}{3}\left(\sigma_1 + \sigma_3 + \sigma_3\right) \ and \\ q &= \sqrt{\frac{\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2}{2}}\end{aligned} \tag{4}$$

where σ_1, σ_2 and σ_3 are principal stresses. The extension of equations (4) to a state of general stress and strain yields the following:

$$p = \frac{1}{3}\left(\sigma_{ii}\right) \ and \ q = \sqrt{\frac{3\,(s_{ij}.s_{ij})}{2}} \tag{5}$$

where summation is implied over indices i and j, ($i, j = 1, 2, 3$) and s_{ij} denotes the deviatoric stress tensor in terms of multi-dimensional states of stress quantities as follows:

$$s_{ij} = \sigma_{ij} - \tfrac{1}{3}\sigma_{ii}\delta_{ij} \quad \text{with} \quad \delta_{ij} = \begin{cases}1 \ if \ i = j \\ 0 \ if \ i \neq j\end{cases} \tag{6}$$

Now invariants p and q, incorporating the concept of Modified Clam-Clay Model (Roscoe and Burland, 1968), can be used to define a new stress measure named equivalent pressure p^{eq}, which has the dimension of pressure:

$$p^{eq} = p + \frac{q^2}{M^2 p} \tag{7}$$

Together with p and q in equation (5) this leads to:

$$p^{eq} = \tfrac{1}{3}\sigma_{ii} + \frac{9\, s_{ij}.s_{ij}}{2M^2\sigma_{ii}} \tag{8}$$

The soil parameter M represents the slope of the critical state line and can be computed by using critical state friction angle $\left(\varphi_{cs}\right)$ as follows:

$$M = \frac{6\sin\phi_{cs}}{3 - \sin\phi_{cs}} \tag{9}$$

By considering volumetric creep strain rate part $\left(\dot{\varepsilon}_v^c\right)$ in the equation (3) and introducing the new equivalent pressure p^{eq} and the apparent equivalent preconsolidation pressure p_p^{eq} the expression for volumetric creep strain rate can be written as:

$$\begin{aligned}\dot{\varepsilon}_v^c &= -\frac{\mu^*}{\tau}\left(\frac{p^{eq}}{p_p^{eq}}\right)^{\frac{\lambda^* - \kappa^*}{\mu^*}} \\ &\text{where } p_p^{eq} = p_{po}^{eq} \exp\left(\frac{-\varepsilon_v^c(t)}{\lambda^* - \kappa^*}\right)\end{aligned} \tag{10}$$

where $\varepsilon_v^c(t)$ denotes the accumulated volumetric creep up to the current time t. The value of initial equivalent preconsolidation pressure p_{po}^{eq} can be computed by using initial preconsolidation pressure σ_{po} (Vermeer et al., 1998).

4 FORMULATION OF THE SOIL CREEP PROBLEM

The basic concept used here is that the total strain vector consists of an elastic strain component and a creep strain component, i.e.

$$\boldsymbol{\varepsilon}(t) = \boldsymbol{\varepsilon}^e(t) + \boldsymbol{\varepsilon}^c(t) \tag{11}$$

where $\boldsymbol{\varepsilon}(t)$ is the current total strain vector, $\boldsymbol{\varepsilon}^e(t)$ the current elastic strain vector, $\boldsymbol{\varepsilon}^c(t)$ the current creep strain vector, and t denotes the current time. The elastic strain component can be related to the stress vector $\boldsymbol{\sigma}(t)$ through the use of a constitutive matrix $\mathbf{D}$ as:

$$\boldsymbol{\varepsilon}^e(t) = [\mathbf{D}]^{-1}\boldsymbol{\sigma}(t) \tag{12}$$

In the present analysis $\mathbf{D}$ is composed of E (Young's Modulus) and ν (Poisson's ratio), which implies that the elastic strain is related to stress according to isotropic linear elasticity. In order to introduce general creep strain, one can adopt the view that creep strain is simply a time-dependent plastic strain. Then it is logical to assume a flow rule for the creep rate component and equivalent pressure p^{eq} is introduced as the plastic potential function (Vermeer et al., 1998); this yields:

$$\dot{\underline{\boldsymbol{\varepsilon}}}(t) = [\mathbf{D}]^{-1}\dot{\underline{\boldsymbol{\sigma}}}(t) + \lambda\frac{\partial p^{eq}}{\partial\underline{\boldsymbol{\sigma}}(t)} \tag{13}$$

where the plastic multiplier λ can be eliminated from the equation (11) by adopting the concept that the creep strain rate is proportional to partial derivative of the plastic potential function with respect to the corresponding stress component, i.e.

$$\lambda = \frac{\dot{\boldsymbol{\varepsilon}}_v^c}{\boldsymbol{\alpha}} \tag{14}$$

where λ can be evaluated for multi-dimensional situation as:

$$\begin{aligned}\dot{\boldsymbol{\varepsilon}}_v^c &= \dot{\varepsilon}_{11}^c + \dot{\varepsilon}_{22}^c + \dot{\varepsilon}_{33}^c \ and \\ \boldsymbol{\alpha} &= \left(\frac{\partial p^{eq}}{\partial\sigma_{11}} + \frac{\partial p^{eq}}{\partial\sigma_{22}} + \frac{\partial p^{eq}}{\partial\sigma_{33}}\right)\end{aligned} \tag{15}$$

Equations (11)-(15) yield:

$$\underline{\dot{\boldsymbol{\varepsilon}}}^c(t) = \frac{\dot{\boldsymbol{\varepsilon}}_v^c}{\alpha}\frac{\partial p^{eq}}{\partial\underline{\boldsymbol{\sigma}}(t)} \tag{16}$$

By combining equations (10) and (16), the following is obtained:

$$\dot{\underline{\boldsymbol{\varepsilon}}}^{c}(t) = -\frac{1}{\alpha}\frac{\mu^{*}}{\tau}\left(\frac{p^{eq}}{p_{p}^{eq}}\right)^{\frac{\lambda^{*}-\kappa^{*}}{\mu^{*}}}\frac{\partial p^{eq}}{\partial \underline{\boldsymbol{\sigma}}(t)} \tag{17}$$

where $p_p^{eq} = p_{po}^{eq} \exp\left(\frac{-\varepsilon_v^c(t)}{\lambda^* - \kappa^*}\right)$

It should be noted that the subscript 'o' is used in the equations to denote initial conditions and that $\varepsilon_v^c = 0$ for time $t=0$.

5 FINITE ELEMENT ALGORITHM FOR CREEP MODEL

Modifications were made to the finite element algorithm developed by Puswewala et al. (1992) in order to incorporate the developed creep model as a plane strain element subroutine. At any point within a material domain of volume V and surface S, discretized by finite elements, the displacement vector field will be denoted by $\mathbf{u}$, strain vector by $\boldsymbol{\varepsilon}$, and stress vector $\boldsymbol{\sigma}$. From the principle of virtual work, equilibrium of the material domain at the time t_k, which is reached after the accumulation of k time steps starting from $t=0$, can be expressed as:

$$\int_v \mathbf{B}^T \boldsymbol{\sigma}_k dv + \mathbf{f}_k = \mathbf{0} \tag{18}$$

where f_k is the known force vector consists of body forces and surface traction forces. Vector $\mathbf{B}$, $\mathbf{u}$, $\boldsymbol{\varepsilon}$ and the nodal displacement vector $\mathbf{a}$ hold the following relationships, i.e.

$$\boldsymbol{\varepsilon} = \mathbf{Lu}, \quad \mathbf{u} = \mathbf{Na} \quad \text{and} \quad \boldsymbol{\varepsilon} = \mathbf{LNa} = \mathbf{Ba} \tag{19}$$

where $\mathbf{N}$ is the shape function matrix and $\mathbf{L}$ is a differential operator matrix. It is necessary to evaluate $\mathbf{a}_{k+1}$ and $\boldsymbol{\sigma}_{k+1}$ at the end of the next time interval Δt_k, provided $\boldsymbol{\varepsilon}_k, \mathbf{a}_k$ and $\boldsymbol{\sigma}_k$ at the time t_k are known. In order to evaluate the accumulated volumetric creep strain (ε_v^c) at the time t_k, equations (11) and (12) are used since $\boldsymbol{\varepsilon}_k, [\mathbf{D}]^{-1}$ and $\boldsymbol{\sigma}_k$ are known quantities at the time t_k. Together with equations (11), (12) and (19), the expression for the difference between the stress vectors $\boldsymbol{\sigma}_{k+1}$ and $\boldsymbol{\sigma}_k$ can be obtained as:

$$\boldsymbol{\psi}_{k+1} = \boldsymbol{\sigma}_{k+1} - \boldsymbol{\sigma}_k - \mathbf{DB}\{\mathbf{a}_{k+1} - \mathbf{a}_k\} + \mathbf{D}\Delta t_k \boldsymbol{\beta}(\boldsymbol{\sigma}_{k+\theta}) = \mathbf{0} \tag{20}$$

In the above, the following relationship has been used:

$$\boldsymbol{\varepsilon}_{k+1}^c - \boldsymbol{\varepsilon}_k^c = \boldsymbol{\beta}(\boldsymbol{\sigma}_{k+\theta})\Delta t_k \tag{21}$$

where $\boldsymbol{\beta}$ denotes the strain rate vector given by equation (17), and:

$$\boldsymbol{\sigma}_{k+\theta} = (1-\theta)\boldsymbol{\sigma}_k + \theta\boldsymbol{\sigma}_{k+1} \quad ,(0 \le \theta \le 1) \tag{22}$$

For $\theta \ge 1/2$ (i.e. to have a unconditionally stable scheme), Newton-Raphson procedure is used to iterate within the time interval Δt_k for the unknowns $\boldsymbol{\sigma}_{k+1}$ and $\mathbf{a}_{k+1}$. The iterate number is denoted by a superscript numeral. After successive iterations, the iteration cycle n can be reached, while the convergence criterion may not yet be satisfied. At this point, equations (18) and (20) can be written using the appropriate current stress and displacement values, but these expressions would now not reduce to zero since convergence has not yet been achieved. Using the curtailed Taylor expansion on the latter expressions the following two equations are obtained to yield the unknown incremental corrections $\Delta\boldsymbol{\sigma}_{k+1}^n$ and $\Delta\mathbf{a}_{k+1}^n$ upon solution (this is the $(n+1)^{th}$ iterate):

$$\begin{aligned}\Delta\boldsymbol{\sigma}_{k+1}^n = \overline{\mathbf{D}}^n [\ & \mathbf{B}(\Delta\mathbf{a}_{k+1}^n + \mathbf{a}_{k+1}^n - \mathbf{a}_k) \\ & - \mathbf{D}^{-1}(\boldsymbol{\sigma}_{k+1}^n - \boldsymbol{\sigma}_k) - \Delta t_k \boldsymbol{\beta}(\boldsymbol{\sigma}_{k+\theta}^n)\ ]\end{aligned} \tag{23}$$

$$\begin{aligned}\int_v \mathbf{B}^T \overline{\mathbf{D}}^n \mathbf{B}\Delta\mathbf{a}_{k+1}^n dv = \int_v \ & \mathbf{B}^T \overline{\mathbf{D}}^n \{\ \mathbf{D}^{-1}(\boldsymbol{\sigma}_{k+1}^n - \boldsymbol{\sigma}_k) - \mathbf{B}(\mathbf{a}_{k+1}^n - \mathbf{a}_k) \\ & + \Delta t_k \boldsymbol{\beta}(\boldsymbol{\sigma}_{k+\theta}^n)\ \} dv - \int_v \mathbf{B}^T \boldsymbol{\sigma}_{k+1}^n dv\end{aligned} \tag{24}$$

where $\boldsymbol{\sigma}_{k+\theta}^n$ is obtained from equation (22) by replacing $\boldsymbol{\sigma}_{k+1}$ with $\boldsymbol{\sigma}_{k+1}^n$, and,

$$\overline{\mathbf{D}}^n = \left[\mathbf{D}^{-1} + \Delta t_k \mathbf{S}^n \theta\right]^{-1} \tag{25}$$

where matrix $\mathbf{S}^n$ given by

$$\mathbf{S}^n = \left(\frac{\partial \boldsymbol{\beta}}{\partial \underline{\boldsymbol{\sigma}}}\right) \tag{26}$$

is evaluated for $\boldsymbol{\sigma}_{k+\theta}^n$. In the above equation (24) the right hand side does not include provisions for increasing body forces. If convergence occurs at the above $(n+1)^{th}$ iterate, it is set $\boldsymbol{\sigma}_{k+1} = \boldsymbol{\sigma}_{k+1}^{n+1}$ (note that $\boldsymbol{\sigma}_{k+1}^{n+1} = \boldsymbol{\sigma}_{k+1}^n + \Delta\boldsymbol{\sigma}_{k+1}^n$), and similarly $\mathbf{a}_{k+1} = \mathbf{a}_{k+1}^{n+1}$, and proceeded to the next time step; otherwise iteration is continued.

As per the constitutive model in equation (17), the solution process is initiated by obtaining the instantaneous elastic deformation and stress distribution at the time of application of the initial load ($t=0$). These displacement and stress vectors are used to obtain the incremental displacement and stress vectors during the first time interval Δt_o by solving equations (23) and (24). The solution process is repeated for each time interval, until the termination of analysis.

6 NUMERICAL ANALYSIS AND DISCUSSION

Numerical analyses were conducted using published experimental data; parametric studies were conducted to investigate the sensitivity of different material parameters of the model. It was found that sensitivity of some of the parameters was so high that the model did not work properly except for a limited range of numerical values for such parameters. For example, when applied pressure exceeded the preconsolidation pressure, numerical problems were encountered. The power term in equation (3) is a real number and also a relatively large number. When a term is raised to powers in real numbers rather than integers, the term had to be set as positive, irrespective of the sign achieved by the computations. Due to problems like this, the parametric study could be carried out only for limited ranges of magnitudes of some parameters.

Numerical analyses were carried out by a finite element mesh composed of 8-node serendipity elements representing a 8m deep layer of soft soil underlain by a hard surface like rock. The simulation was done under plane strain condition, with the top of the surface subjected to a uniform load of 120 kPa applied over a length of 8m. The material properties of the soil were selected similar to Haney Clay investigated by Vaid and Campanella (1977), and are as given in Table 1:

Table 1: Material properties

E = 6000 kPa	$\nu = 0.25$	$\sigma_{po} = 400$ kPa	$\phi_{cs} = 32.1$
$\kappa^* = 0.016$	$\lambda^* = 0.105$	$\mu^* = 0.004$	$\tau = 1$ day

The settlement of the loaded surface with time is plotted in Figure 1.

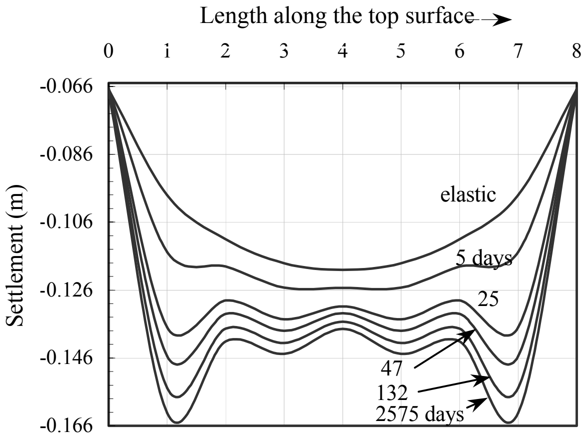

Figure 1. Total settlement behavior of the top surface with time

By taking L as the ratio of distance to the point from the center of the loaded area to the total load width (8m), the total settlement with time is shown in Figure 2 for three selected points on the loaded surface.

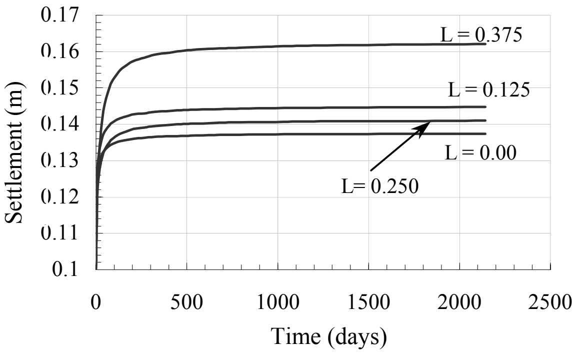

Figure 2. Total settlement versus time

Results of some parametric investigations are presented here. The applied load was varied within the permissible range as allowed by the numerical performance of the model, and the settlement with time of the point with L = 0.375 is shown in Figure 3.

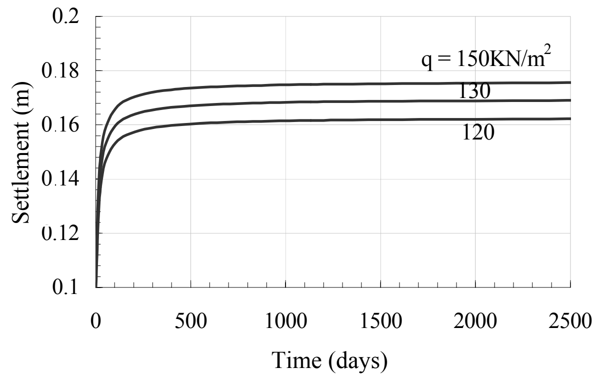

Figure 3. Settlement behavior for L = 0.375 with loads

The effect of Poisson's ratio (ν) on the settlement is shown in the Figure 4, where the settlement with time at the point with L = 0.375 under the surface load of 120 kN/m^2 is shown for three different values of ν.

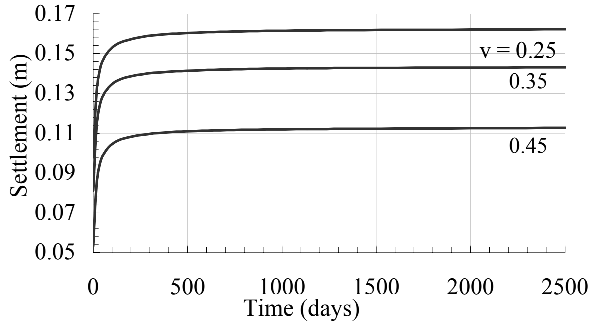

Figure 4. Settlement behavior for L = 0.375 with Poisson's ratio

Figures 2, 3 and 4 indicate that the creep model yields on attenuating form of continuing settlement. The numerical results show that the vertical normal stress in the surface soil elements at the axis of symmetry reach the value of applied stress after about 2000 days, and the rates of settlement become very small after about 500 days. These numbers are of course dependent on the material parameters selected for the investigation.

7 CONCLUSION

The modified creep model of Bjerrum (1967) was generalized to multi-dimensions and implemented in a time-incrementing, iterative, non-linear finite element code. Analyses were conducted by applying a flexible uniform load on the soil layer. The sensitivity of various parameters on the numerical performance of the model was investigated. It is found that some of the material parameters of the model can be varied only within limited ranges due to numerical complications inherent in the model. The model yields an attenuating type of creep settlement for the range of parameters used.

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REFERENCES

Bjerrum, L. 1967. Engineering geology of Norwegian normally consolidated marine clays as related to settlements of buildings. Seventh Rankine Lecture. *Geotechnique* 17: 81-118.

Butterfield, R. 1979. A natural compression law for soils (an advance on e-log p′). *Geotechnique* 29: 469-480.

Garlanger, J.E. 1972. The consolidation of soils exhibiting creep under constant effective stress. *Geotechnique* 22: 71-78.

Puswewala, U.G.A., Rajapakse, R.K.N.D., Domaschuk, L. and Lach, R.P. 1992. Finite element modelling of pressuremeter tests and footings on frozen soils. *International Journal for Numerical and Analytical Methods in Geomechanics. Vol. 16, 351-375*

Roscoe, K.H. and Burland, J.B., 1968. On the generalized stress-strain behavior of 'wet' clay. In: *Engineering Plasticity*, Ed. J. Heyman & F. A. Leckie, pp. 535-609. Cambridge Univ. Press. London.

Vaid, Y. and Campanella, R.G. 1977. Time-dependent behavior of undisturbed clay. *ASCE journal of the Geotechnical Engineering Division,* 103(GT7): 693-709.

Vermeer, P.A., Stolle, D.F.E. and Bonnier, P.G. 1998. From the classical theory of secondary compression to modern creep analysis. *Proceedings of the 9th International Conference on Computer Methods and Advances in Geomechanics.* Wuhan, China, Vol. 4: 2469-2478.