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Some applications of the Sydney Soil Model

Quelques applications du modèle des sols de Sydney

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ABSTRACT
The recently proposed Sydney Soil Model is employed to simulate the behaviour of samples of a sensitive natural clay in undrained triaxial shearing tests. In this elastoplastic model the behaviour of soil is divided into two parts: that at a reference state and that attributed to the influence of soil structure. It is demonstrated that the model provides a good description of the behaviour of sensitive soils possessing a pronounced structure.

RÉSUMÉ
Le modèle récemment proposé de sol de Sydney est utilisé pour simuler le comportement des échantillons d'un argile normal sensible undrained dedans les essais de cisaillement à trois axes. Dans ce modèle élastoplastique le comportement du sol est divisé en deux parts: cela à un état de référence et cela attribué à l'influence de la structure de sol. On le démontre que le modèle fournit une bonne description du comportement des sols sensibles possédant une structure prononcée.

1 INTRODUCTION

Recently, there have been important developments in formulating constitutive models incorporating the influence of soil structure, *e.g.*, Manzari and Dafalias (1997) and Li (2002). In a model proposed recently by the authors, the Sydney Soil Model (SSM), (Liu and Carter, 2004, 2005), the behaviour of structured soils including both clays and sands can be represented in a single, consistent theoretical framework. In this paper, SSM is employed to simulate the behaviour of samples of a natural clay during undrained triaxial shearing tests.

2 MODEL CONCEPTS

For brevity, only the main concepts behind the Sydney Soil Model are described here and the key constitutive equations are summarised. In SSM the strain parameters are the same as those commonly adopted in soil mechanics (*e.g.*, Muir Wood, 1990). Mean effective stress p' and a general shear stress $q^\wedge$ are defined as follows:

$$p' = \frac{1}{3}(\sigma'_1 + \sigma'_2 + \sigma'_3) \quad , \tag{1}$$

$$q^\wedge = p'\eta^\wedge \quad , \tag{2}$$

where σ'_1, σ'_2 and σ'_3 are the principal stresses, and $\eta^\wedge$ is the general shear stress ratio given by:

$$\eta^\wedge = \frac{3\left[f_2 + \sqrt{f_2(f_2 + 27 - 15s^*)}\right]}{f_2 + \sqrt{f_2(f_2 + 27 - 15s^*)} + 4(27 - 15s^*)} \quad . \tag{3}$$

The stress quantity f_2 is defined as:

$$f_2 = \frac{(\sigma'_1 + \sigma'_2 + \sigma'_3)\left[(\sigma'_1 + \sigma'_2 + \sigma'_3)^2 - s^*\left(\sigma'^2_1 + \sigma'^2_2 + \sigma'^2_3\right)\right]}{\sigma'_1\sigma'_2\sigma'_3} - 27 + 9s^* \tag{4}$$

The parameter s* is an intrinsic soil parameter which defines the shape of the final failure surface of the soil in the π plane.

Virgin yielding occurs when the stress state engages the structural yield surface, the size and shape of which in stress space are affected by stress history and the natural structure of the soil. In SSM plastic straining also occurs for stress excursions inside this yield surface, a phenomenon termed "sub-yielding". Some soils may also exhibit what is termed "first loading" behaviour inside the structural yield surface, and this aspect of soil behaviour is described in greater detail below.

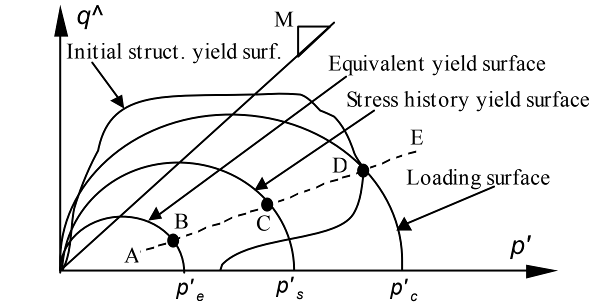

Figure 1 Various surfaces in the stress space

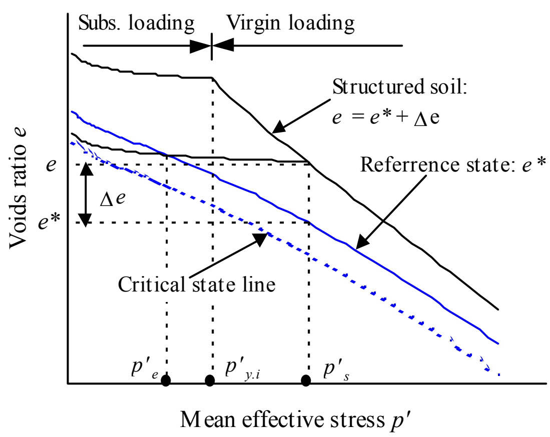

Figure 2 Compression behaviour of soil

The formulation of SSM is based on the plastic volumetric deformation of soil during virgin yielding. The corresponding plastic deviatoric deformation is determined from an assumed flow rule. Plastic deformation during sub-yielding is related to that during virgin yielding by means of a mapping quantity.

Based on soil properties at critical states of deformation and the assumption of plastic-volumetric-deformation-dependent hardening, soil behaviour at a hypothesised reference state is derived and this behaviour is regarded as the intrinsic soil behaviour. Virgin isotropic compression behaviour of a given soil is required as input information for the model, and the compression behaviour is divided into two parts: the behaviour of the soil at the reference state (the intrinsic soil behaviour) and the difference in behaviour between soil at the given state and the reference state. This difference in behaviour is attributed to the influence of soil structure. A general form of the volumetric deformation contributed by soil structure is obtained under the assumption that both hardening and destructuring of soil are dependent on plastic volumetric deformation, which is further modified by taking into account the effect of shearing on the volumetric deformation.

2.1 *Virgin yielding, sub-yielding and first loading*

Division of the elastoplastic soil behaviour into each of these categories depends on the relationship between the current effective stress state and the yield surface applicable to the soil. Various surfaces in stress space used to define soil behaviour are presented in Figure 1. The volumetric relationships are illustrated schematically in Figure 2.

The virgin yielding boundary of a soil is created by two factors, stress history and the structure of the soil. The yield surface associated with stress history is assumed to be elliptical in p' - $q\hat{}$ stress space (Fig. 1). This surface is described by:

$$f = q\hat{}^{2} - \mathrm{M}^{2} p'\left(p'_{s} - p'\right) = 0 \quad , \tag{5}$$

where M is the aspect ratio of the yield surface, which is dependent on soil structure, and p'_s is the size of the yield surface associated with stress history.

Following a suggestion of Hashiguchi (1980), the concept of a loading surface is introduced and is defined as that surface on which the current stress state always remains. The loading surface is also assumed to be elliptical with the same aspect ratio M as the yield surface determined by stress history (Fig. 1). The size of the loading surface is denoted by p'_c.

For natural soil, a structural yield surface may exist due to the arrangement and bonding of soil constituents (Fig. 1). The shape and size of the surface are generally dependent on the geological processes that formed the soil. In SSM the initial structural yield surface may possess a shape different from the yield surface associated with stress history, *i.e.*, equation (5). The initial structural yield surface is defined in general terms as:

$$f_{s,i}\left(p', q\hat{}\right) = 0 \quad . \tag{6}$$

The equivalent yield surface is also elliptical with its size defined by p'_e. It is the yield surface for the same soil at the same stress state if it were in the reference state, *i.e.*, if it had no structure. In its most general form, SSM allows the aspect ratio of the elliptical surfaces, M, to vary during loading according to:

$$\mathrm{M} = \frac{\mathrm{M}^{*}}{1 + \mu \ln\left(p'_{s} / p'_{e}\right)} \quad , \tag{7}$$

where μ is a soil parameter.

The relationships between the structural yield surface, the stress history yield surface and the loading surface require careful specification. With reference to Figure 1 it is assumed that the soil in question possesses a structure which has imposed on it an initial structural yield surface. Previous loading of this soil has also imposed on it a stress history yield surface which in this space is elliptical. For the stress path A to E, identification of the various types of behaviour is as follows. For loading along the stress path AC, the soil experiences stress levels below previous maximum values, and so the behaviour involves sub-yielding. When the loading continues from C to D, the soil begins to experience stress levels that exceed any experienced during its previous loading history, and so $p'_c = p'_s = p'_{c,max}$. First loading may occur between C and D, depending on the type of soil, as explained below. Virgin yielding commences at point D when the structural yield surface is first engaged. For loading beyond D, the boundary of the region in stress space in which subsequent yielding may occur is defined by the initial structural yield surface and the portion of the loading surface that now lies outside the initial structural yield surface. At this point $p'_c = p'_s = p'_{c,max}$ and $p'_{c,max}$ is now the maximum size of the stress history yield surface the soil has ever experienced.

If the stress state and therefore the loading surface retreat inside the virgin yielding boundary, *i.e.*, the soil is unloaded, sub-yielding will occur. Reloading inside the virgin yield surface will also be associated with sub-yielding.

The term "first loading" is introduced to describe the behaviour of a particular soil type during sub-yielding, as the loading surface expands for the first time within the initial structural yield surface. It is observed that only some soils exhibit "first loading" behaviour and consequently all soils may be divided into two types: *viz.*, clay-type soils and sand-type soils. Clay-type soils do not exhibit "first loading" but sand-type soils do, and for this latter case the following behaviour is identified:

$$\begin{cases} \textit{virgin yielding : stress on the boundary. } \& \ dp'_s \neq 0 \\ \textit{first loadg : stress inside the boundary. } \& \ p'_{c,\max} = p'_c \ \& \ dp'_c > 0 \\ \textit{subsequent loading : for all the other situations} \end{cases} \tag{8}$$

2.2 *CSL reference states*

The mechanical properties of soil at critical states of deformation are used to derive soil behaviour at a reference state, referred to as the CSL reference state. The critical state line in $p' - e$ space may be expressed in its general form in terms of both elastic and plastic components, *i.e*,

$$e = e^{*}_{cs} - E^{e}\left(p'\right) - E^{p}\left(p'\right) \quad , \tag{9}$$

where e^*_{cs} is a soil parameter defining the position of the critical state line, and $E^e(p')$ and $E^p(p')$ are the components of the variation in the voids ratio associated with elastic and plastic deformation respectively. Explicit forms of E^e and E^p must be determined for individual soils, usually from compression data.

It is assumed that soil of a given mineralogy is at a CSL reference state if it satisfies the following conditions: (a) the size of the yield surface is uniquely dependent on the plastic volumetric deformation, and (b) the soil can reach a critical state of deformation by a way of continuous deformation through CSL reference states. Based on these assumptions, it can be shown that for soils at a CSL reference state the voids ratio is given by:

$$e^{*} = e^{*}_{cs} - E^{e}\left(p'\right) - E^{p}\left(p'_{s} / N\right) \quad . \tag{10}$$

For SSM, it is assumed that $N = 2$, because the stress history yield surface is elliptical and because of the flow rule adopted in the model. Equation (10), with $N = 2$, describes the virgin compression behaviour of soil at the CSL reference state. The position of this line is an intrinsic soil property.

2.3 *Virgin yielding*

2.3.1 *Volumetric strain*

Based on the proposed behaviour at the CSL reference state, the virgin isotropic compression behaviour of a structured soil may be described in terms of e^*, the voids ratio of the soil at the reference state, and Δe, the difference in voids ratio between a structured soil and the soil at the reference state (Fig. 2), so that:

$$e = e^* + \Delta e\ . \tag{11}$$

Substituting equation (10) and (11) into (9) provides:

$$e = e^*_{cs} - E^e(p') - E^p(0.5p'_s) + \Delta E(p'_s)\ . \tag{12}$$

$\Delta E(p'_s)$ is the additional voids ratio sustained by soil structure during isotropic virgin compression. On the assumption that elastic deformation is independent of soil structure, the additional voids ratio must be associated with only plastic deformation and should therefore be expressed in terms of the size of the yield surface p'_s, not the current value of p'.

Under the assumption that hardening and destructuring of structured soil not at a CSL reference state is also dependent on volumetric deformation, the total volumetric strain increment for a structured soil is obtained. By introducing the effect of shearing on destructuring the following equation for plastic volumetric strain is obtained:

$$d\varepsilon_v^p = \frac{dE^p(0.5p'_s)}{dp'_s}\frac{dp'_s}{(1+e)} - \frac{\langle \Delta e - c\rangle}{\Delta E(p'_s) - c}\frac{d[\Delta E(p'_s)]}{dp'_s}\times \frac{\langle dp'_s\rangle}{(1+e)} + \frac{\gamma \Delta e}{(1+e)}\left(\frac{\eta^\wedge}{M^*}\right)^n\left\langle\frac{d\eta^\wedge}{M^* - \eta^\wedge}\right\rangle\ , \tag{13}$$

where

$$\langle a\rangle = \begin{cases} a & \text{if } a > 0 \\ 0 & \text{if } a \le 0\end{cases} \text{ and } c = \begin{cases} \lim\limits_{p'_s \to \infty} \Delta E(p'_s) & \text{if the limit is finite,} \\ 0 & \text{otherwise.}\end{cases} \tag{14}$$

The first term in equation (13) is associated with volumetric-dependent hardening, similar to the Modified Cam Clay model. The second and the third terms are associated with destructuring. The second term occurs only when the yield surface expands. The third term occurs during virgin yielding when the current stress ratio increases or when softening occurs.

2.3.2 *Elastic strain*

The elastic deformation of soil is described by Hooke's law and the elastic volumetric strain is given by:

$$d\varepsilon_v^e = \left[\frac{dE^e(p')}{dp'}\right]\frac{dp'}{(1+e)}\ . \tag{15}$$

The deviatoric component of elastic strain can be determined by specifying either Poisson's ratio v^* or the shear modulus G^*, properties assumed to be independent of soil structure.

2.3.3 *Plastic deviatoric strain*

Plastic volumetric and deviatoric strain increments are related via a flow rule, so that:

$$d\varepsilon_d^p = \left(\frac{1}{1+e}\right)\frac{\frac{3}{2}\sqrt[3]{\frac{\eta^\wedge}{M^*}}}{\left|1 - \frac{\eta^\wedge}{M^*}\right| + \frac{\omega\eta^\wedge}{M^*}\left|1 - \sqrt{\frac{p'_e}{p'_s}}\right|}\times d\varepsilon_v^p\ , \tag{16}$$

where ω is a model parameter, and p'_e is the size of the equivalent yield surface. For soil at the reference state, the plastic volumetric deformation is uniquely dependent on the size of the current yield surface, so that p'_e may be determined from:

$$e_{cs}^* - E^e(p') - E^p(0.5p'_s) - e = 0\ . \tag{17}$$

2.4 *Sub-yielding and first loading*

Several simplifying assumptions are made to model behaviour during sub-yielding. These are: (a) no change in the virgin yielding boundary occurs during sub-yielding; (b) the effects of stress history on soil behaviour during sub-yielding are simplified. Details can be found in the paper by Liu and Carter (2004).

3 MODEL PARAMETERS

SSM is defined in terms of eight material parameters, and the specification of three curves, one surface, and the soil type. The eight parameters are the critical state friction angle φ_{cm} and the parameter s^* (used to describe the shape of the critical state failure surface in the π plane), Poisson's ratio ν^* (or the shear modulus G^*), parameter m for describing soil behaviour during subsequent yielding, parameters n and r defining shear destructuring, parameter μ defining the variation of the aspect ratio of the stress history yield surface, and parameter ω used in the flow rule. The three curves are the critical state line in $e - p'$ space, the elastic volumetric deformation function of the soil $E^e(p')$, and the additional voids ratio sustained by the structure of the soil $\Delta E(p'_s)$. The initial structural yield surface is also required. The soil must also be classified as either sand or clay type soil, as this determines whether first loading will occur.

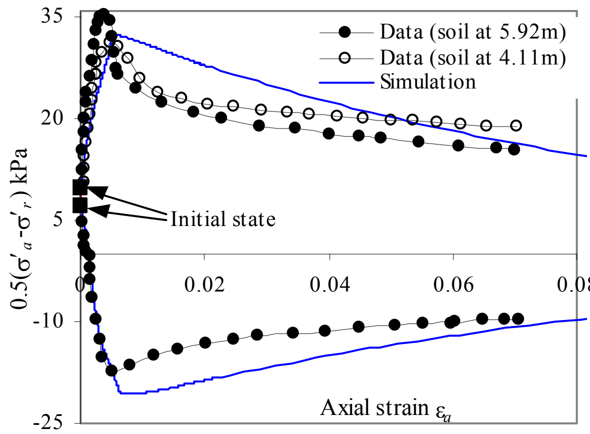

Figure 4 Sress-strain behaviour of Emmerstad clay

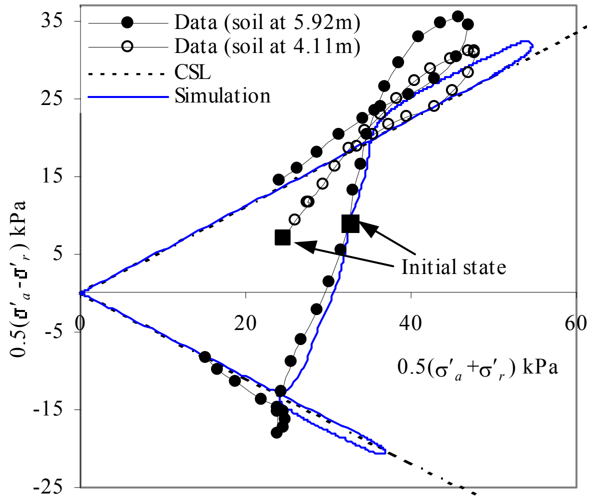

Figure 3 Stress paths of sensitive natural Emmerstad clay

4 MODEL APPLICATION

The behaviour of a natural sensitive Norwegian marine clay, Emmerstad clay, is simulated and the simulated response is compared with data from undrained triaxial tests performed by Lacasse *et al.* (as reported by Burland, 1990). Values of the model parameters used in these simulations are listed in Table 1. It is assumed that the CSL reference state for clay is identical to a reconstituted state. Based on the equation proposed by Burland (1990), the ICL* obtained for this clay is:

$$e^* = 0.879 - 0.07 \ln p' + 0.00016 (\ln p')^3 \quad (18)$$

Based on the work by Liu and Carter (1999, 2000), the additional voids ratio is given by:

$$\Delta E(p'_s) = 0.35 + 0.26 \left({}^{98}\!/_{p'_s} \right)^{0.4} \quad (19)$$

The elastic volumetric deformation is described by,

$$E^e(p') = 0.006 \ln p' \quad (20)$$

Table 1 Model parameters for Emmerstad clay

φ_{cm}	s^*	ν^*	m	N	ω	μ	γ
34°	1.117	0.25	2	0.25	1	0	0.005

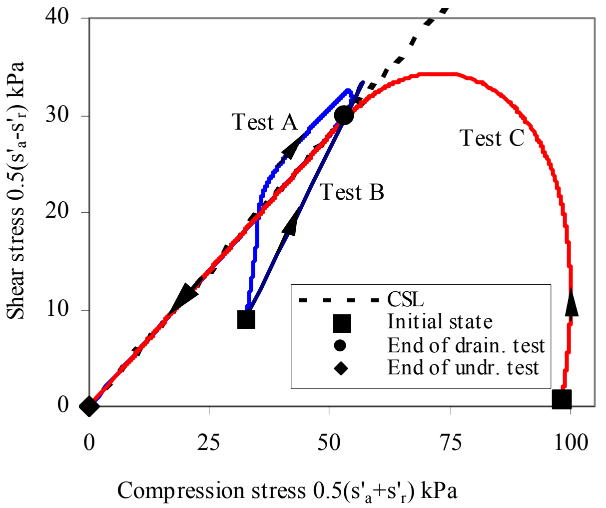

Figure 5 Simulated stress paths for Emmerstad clay in both drained and undrained tests

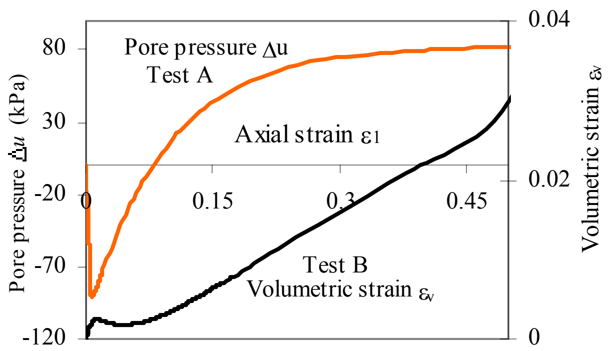

Figure 6 Volumetric deformation and pore pressure

A comparison of the simulations and the experimental data for undrained triaxial shearing is shown in Figures 3 and 4, where the soil specimens experienced triaxial compression and extension tests from initial anisotropic stress states. Overall, it is seen that the proposed model gives a highly satisfactorily description of the behaviour of the natural soft Emmerstad clay.

An interesting feature of the behaviour of Emmerstad clay is observed (Burland, 1990). When sheared undrained from the in situ stress state, the clay in compression initially responds almost elastically, followed by plastic deformation as the stress path travels upward and to the right (indicating generation of negative pore pressure), and then changes direction and travels downwards along the critical state line (indicating positive pore pressure). Finally the resistance of the clay to shearing is reduced to almost to zero, like the liquefaction behaviour of loose sand. This feature has been captured successfully by SSM.

To understand further the mechanism of this type of soil behaviour, two simulations were undertaken: a drained triaxial compression from the in situ stress state (Test B) in which the soil hardens steadily until it reaches the yield surface and then softens to a critical state; and undrained virgin compression from an isotropic stress state with $p' = p'_{s,i}$ (Test C). These simulations together with the simulation for the undrained compression from an anisotropic stress state (Test A) are shown in Figs 5 and 6.

Comparing Tests A and B, it may be seen the generation of pore pressure is closely linked to the type of volumetric deformation (Fig. 6). When softening occurs the soil exhibits volumetric expansion (corresponding to the production of negative pore pressure), but subsequently volumetric compression gradually increases (corresponding to the production of positive pore pressure). Unlike a reconstituted soil, the volumetric deformation of structured soil is dependent on two mechanisms: hardening and destructuring, and consequently the undrained stress paths can be very complicated.

Unlike a reconstituted soil, the peak strength of a structured soil is determined by the structure (the initial structural yield surface) and the stress path. Both over-consolidated and normally consolidated soils can have peak strengths which may not depend on the initial stress and voids ratio. This is probably the main reason that some natural clays can have a very high value of sensitivity index. Indeed, Burland (1990) observed that Emmerstad clay has a extremely high sensitivity index, which varies from 60 to infinity. However, the final strength of the soil is the critical state strength which is independent of soil structure.

5 CONCLUSIONS

The Sydney Soil Model was used to simulate the behaviour of a natural sensitive clay under undrained triaxial shearing. It has been shown that the model can provide good simulations of the behaviour of structured soils. Furthermore, it has been reported (Liu and Carter, 2004, 2005) to predict successfully the behaviour of the sand as well as clay in both compression and extension from anisotropic as well as isotropic initial stress states.

ACKNOWLEDGEMENTS

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REFERENCES

Burland J.B. 1990. On the compressibility and shear strength of natural soils, *Géotechnique*, 40(3), 329-378.

Hashiguchi K. 1980. Constitutive equations of elastoplastic materials with elastic-plastic translation, *Journal of Applied Mechanics*, ASME, 47, 266-272.

Li X.S. 2002. A sand model with state-dependent dilatancy, *Géotechnique*, 52(3), 173-186.

Liu M.D. and Carter J.P. 1999. Virgin compression of structured soils, *Géotechnique*, 49(1), 43-57.

Liu M.D. and Carter J.P. 2000. Modelling the destructuring of soils during virgin compression, *Géotechnique*, 50(4), 479-483.

Liu M.D. and Carter J.P. 2003. The volumetric deformation of natural clays, *International Journal of Geomechanics*, ASCE, 3(3/4), 236-252.

Liu M.D. and Carter J.P. 2004. Evaluation of the Sydney Soil Model, *Advances in Geotechnical Engineering: The Skempton Conference*, Thomas Telford, London, 1, 498-509.

Liu M.D. and Carter J.P. 2005. The Effect of Sample Preparation Methods on Sand Behaviour Simulated by Sydney Soil Model, *Proceedings 11th Conference of the International Association for Computer Methods and Advances in Geomechanics*, Torino, In press

Manzari M.T. and Dafalias Y.F. 1997. A critical state two-surface plasticity model for sands, *Géotechnique*, 47(2), 255-272.

Muir-Wood D. 1990. *Soil Behaviour and Critical State Soil Mechanics*, Cambridge University Press.