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Application of packing models on geophysical property of sediments

Modéles de materiau mixté pour propriétés géophysiques des sédiments

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ABSTRACT
Fines content plays an important role in the properties of two-size particle mixtures, for example sand-gravel, sand-silt and sand-clay. The determination of the porosity or void ratio based on the fines content is useful for characterization of the properties of such mixtures. The application of petrophysical and geotechnical models for binary packing materials are discussed in this study. The geotechnical model was used to predict the geophysical properties of sandstones with different amounts of fines. This study suggests a good potential to use this model for interpreting geophysical properties of sediments.

RÉSUMÉ
Le pourcentage de fines joue un rôle important dans les propriétés d'un matériau formé de particules de deux tailles différentes, comme par exemple les sables graveleux, les sables silteux et les sables argileux. La détermination de la porosité ou de l'indice des vides, basée sur le pourcentage de fines, est utile pour caractériser les propriétés de ces types de formations. Les applications des modèles pétrophysiques et géotechniques pour l'ensemble des matériaux sont présentées dans cette étude. Le modèle géotechnique a été utilisé afin de prévoir les propriétés géophysiques des grès avec différentes quantités d'argile. Cette étude montre le potentiel d'utilisation de ce modèle pour interpréter les données géophysiques des forages.

1 INTRODUCTION

The packing and porosity of two-size mixtures such as sand-gravel, sand-silt and sand-clay have been studied by hydrogeologists interested in hydraulic properties (Koltermann and Gorelick 1995), by geologists considering oil reservoir properties (Clarke, R.H. 1979), by geophysicists concerned with the relationship between acoustic property and porosity of rocks (Marion et al. 1992; Knackstedt et al. 2003), by geotechnical engineers interested in the static and cyclic shear strength and the liquefaction potential of the soil (Fragaszy et al. 1992; Thevanayagam 1998). For natural materials, the packing of the particles is complex. Only binary packings will be discussed in this study (Figure 1). The binary packing refers to an ideal packing where the diameter ratio (large/small) is large, and the very small spheres do not disturb the original packing of the large spheres and vice-versa (Marion et al. 1992). At low fines content, the smaller particles fit within and fill the voids between the large particles; at high fines content, the larger particles are dispersed in the fine-grained matrix and are more or less surrounded by fines (Figure 3). Petrophysical models of porosity and geotechnical models of void ratio for binary packings have been proposed, but these models have not been interrelated. In the present study, the two models are discussed, and the geotechnical model is used to predict the acoustic properties of sandstones.

2 PETROPHYSICAL MODELS FOR POROSITY OF SEDIMENT MIXTURES

In the sand-shale model proposed by Marion et al. (1992), there is a transitional volume fraction of the fines (c) which is equal to the porosity of the sand component (n_c). When $c<n_c$, the clay fits within the pore space of the sand and the porosity of the mixture decreases with increasing clay volume. The porosity is the coarse grain porosity multiplied by the fines porosity at $c=n_c$. When the clay volume fraction becomes greater than the sand porosity, the porosity of the mixture increases with increasing clay volume (Figure 2). The porosity for the mixture can be calculated from Equations (1) to (3) in Table 1.

The fractional packing model by Koltermann and Gorelick (1995) was based on the sand-shale model by Marion et al. (1992), and added the influence of non-ideal packings on the porosity. The porosity for the mixture was corrected based on the relative proportions of coarse and fine materials. When $c<n_c$, some fine grains are distributed in the pore space of the coarse-grained components, the remaining fine grains displace the coarse grains. When the volume fraction of fines is equal to the porosity of the coarse-grained component, the minimum porosity is reached, non-ideal packing prevents the fines from completely filling the pore space of the coarse particles. When the volume fraction of fines is greater than the porosity of the coarse-grained component, the coarse particles are dispersed in the fines matrix. The corrected porosities can be obtained from Equations (4) to (6) in Table 1.

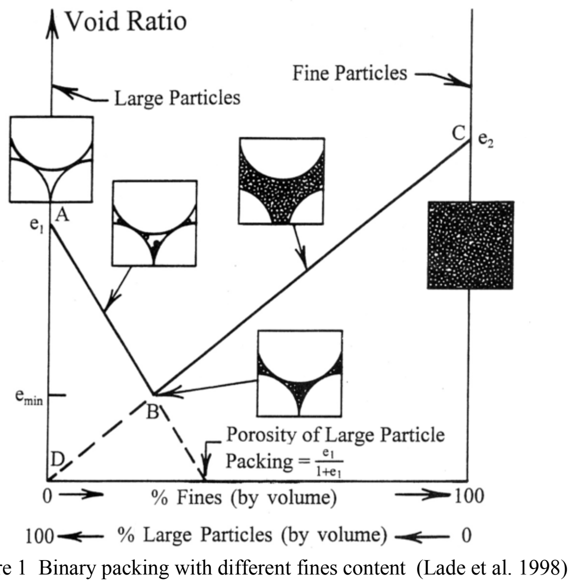


Figure 1 Binary packing with different fines content (Lade et al. 1998)

This model has been used by geologists and geophysicists to determine the porosity of mixtures and to predict the hydraulic conductivity of binary packing mixtures. When the porosity for clean coarse material and clean fine material and the volume fraction of fines are known at the same confining pressures, the porosity for mixtures with different fines volumes can be predicted.

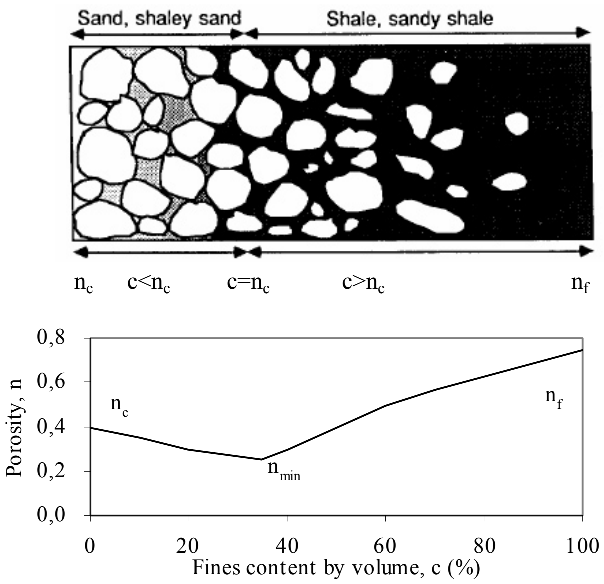


Figure 2 Ideal packing model (modified from Marion et al. 1992 and Koltermann and Gorelick, 1995)

Table 1 Porosity at different fines content by volume (Koltermann and Gorelick, 1995)

Porosity	Equation	Volume fraction	Equation No.
n	$n = n_c - c(1-n_f)$	c<n$_c$	(1)
	$n_{\min} = n_c n_f$	c=n$_c$	(2)
	$n = cn_f$	c>n$_c$	(3)
n$_{cor}$	$n_{cor} = n_c - cy(1-n_f) + (1-y)cn_f$ $y = c((y_{\min}-1)/n_c)+1$	c<n$_c$	(4)
	$n_{\min-cor} = n_c(1-y_{\min}) + n_c n_f$	c=n$_c$	(5)
	$n_{cor} = n_c(1-y) + cn_f$ $y = (c-1)((y_{\min}-1)/n_c)+1$	c>n$_c$	(6)

where

- c Volume fraction of the fine-grained component based on total volume
- n Porosity of the mixture
- n_c Porosity of the coarse-grained component
- n_f Porosity of the fine-grained component
- n_{min} Minimum porosity of the mixture
- y Coefficient which reflects the relative proportions of coarse and fine materials (y_{min}<y<1)
- y_{min} Value of y occurring when $n=n_{min}$, $y_{\min} = 1 + n_f - n_{\min}/n_c$

3 GEOTECHNICAL MODELS FOR VOID RATIO OF SAND-SILT MIXTURES

A geotechnical model using the intergrain state concept to describe binary packings was proposed to characterize the behaviour of sand-silt mixtures. In this model, the void ratio characterizes the behaviour of silty soils (Thevanayagam and Mohan 2000; Thevanayagam et al. 2002), and a transitional fines content (TFC) indicates if the mixture behaves fundamentally as a sand or as a fine material.

In this geotechnical packing model (Figure 3), when the fines content is low, the fine particles are regarded as void and the fines are assumed to not contribute to the shear resistance. When the fines content is high, the sand grains are regarded as void and the sand particles are assumed to not contribute to the shear resistance. Thevanayagam and Martin (2002) introduced the equivalent intergrain void ratio or corrected void ratio as to include the additional contribution of the fines or the sand to the strength of the soil matrix.

(a)

(b)

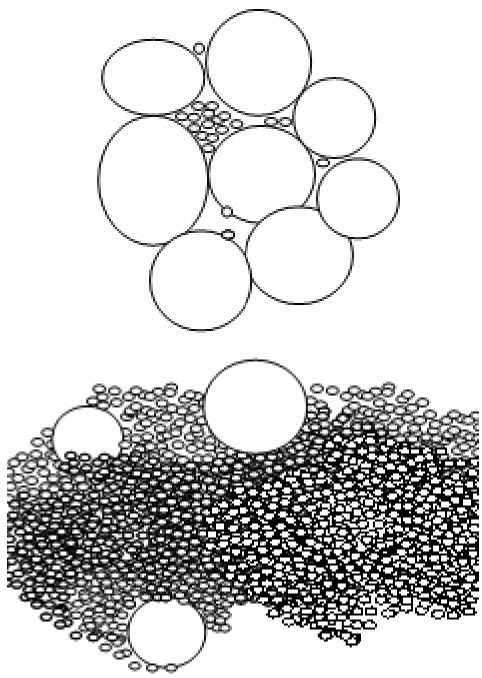


Figure 3 Sand with low (a) and high fines content (b)

The parameters of the model are shown in Table 2.

Table 2 Void ratio at low and high fines content (Thevanayagam and Martin 2002)

Void ratio	Equation	Fines content	Equation No.
e$_s$	$e_s = (e+fc)/(1-fc)$	fc<TFC	(7)
e$_f$	$e_f = e/fc$	fc> TFC	(8)
(e$_s$)$_{cor}$	$(e_s)_{cor} = (e+(1-b)fc)/(1-(1-b)fc)$ 0<b<1	fc< TFC	(9)
(e$_f$)$_{cor}$	$(e_f)_{cor} = e/(fc+(1-fc)/(R_d)^m)$ $R_d = D_{50}/d_{50}$, 0 < m < 1	fc> TFC	(10)

Where

- fc Fines content by weight, in decimal here
- TFC Transitional fines content
- e Void ratio of sand-silt mixtures
- R_d Size disparity, $R_d = D_{50}/d_{50}$, D_{50} and d_{50}, mean grain sizes of the coarser and finer grains,where 50% of the particles are finer than D_{50} or d_{50}
- m and b Factor used to include the contribution of sand to the fines and fines to the sand to the shear resistance of the matrix
- e_s Intergranular void ratio
- $(e_s)_{cor}$ Corrected intergranular void ratio
- e_f Interfine void ratio
- $(e_f)_{cor}$ Corrected interfine void ratio

The intergrain state concept was used to characterize static and cyclic strength and the liquefaction potential of silty soils (Thevanayagam et al. 2002; Xenaki and Athanasopoulos 2003; Tao et al. 2004; Yang 2004). Research results (Yang 2004) show that for sand-silt mixtures, the peak stress ratio and yield strength ratio, among the other parameters, can be characterized by the same methods as for the host clean sand if the fines content is less than the TFC, and the parameters can be evaluated by the same methods as for the host silt if the fines content is higher than the TFC. The behaviour of silty soils depends more on whether the fines content is above or below the TFC rather than on the actual fines content. The TFC can be determined from index data or from the results of triaxial tests (Yang 2005)

4 VELOCITY-POROSITY RELATIONSHIPS

Compressional or shear wave velocity is an important parameter for the analysis of rock properties. Several relationships between velocity and porosity have been proposed based on various assumptions, but few can be applied reliably to sandstones containing different amount of clay sized material. The relationships can be improved if the clay content is included. The previously published relationships are listed in Table 3.

Table 3 Literature review on velocity-porosity relationships

Results	Applicability	Reference
1/Vp=n/Vw+(1-n)/Vm	Sedimentary rock	Wyllie et al. 1956
Vp=$(1-n)^2$Vm+nVw	Sedimentary rock, n<0.37	Raymer et al. 1980
Vp=6.08-8.06n Vs=4.06-6.28n	Clean sandstone P=40 kPa	Han 1986
Vp=5.59-6.93n-2.18c Vs=3.52-4.91n-1.89c	Sandstones P=40 kPa	Han and Morgan 1986
Vp=4.45-7.62n	Shale	Vernik, 1997
Velocity predicted by a clay-sand mixture model, where pore geometry can explain most of the scatter in the porosity-velocity relationship	Sandstone, siltstone and limestone	Xu and White 1995
Velocity estimated based on clay content only	Unconsolidated sediment	Pratson et al. 2003
Velocity estimated from numerical method with morphology based on overlapping spheres of a mineral phase	Clean consolidated sandstone	Knackstedt et al. 2003

Where
Vp Compressional wave velocity
n Porosity
c Volume fraction of clay
Vs Shear wave velocity
Vm Velocity of the matrix
Vw Water velocity
P Confining pressure

The relationships between velocity and void ratio are studied by the geotechnical model in Section 5, which shows another way to predict the acoustic properties of sandstones.

5 APPLICATION OF THE GEOTECHNICAL MODEL ON THE GEOPHYSICAL DATA

The principles for the petrophysical model and the geotechnical model are similar: both are based on the assumption of ideal packings and use a corrected porosity or void ratio. Since n=e/(1+e), porosity and void ratio can be interchanged. The porosities of the clean coarse-grained and the clean fine-grained sediments and the volume fraction of the fines for the mixture have to be known before using the fractional packing model by Koltermann and Gorelick (1995). In the geotechnical model, only the void ratio of the mixture and the fines content should be known. The other parameters, such as peak stress ratio, number of cycles to liquefaction can be predicted. This model has been used for sand-silt mixtures to characterize the static and cyclic behaviour of the mixtures. Whether it can be used to predict geophysical properties of the mixtures is not well known.

In this study, the geotechnical model was used to interpret the acoustic properties of sandstones. The data were obtained from Han et al. (1986) and klimentos and McCann (1990).

The relationship between clay content and porosity at a confining pressure of 30 Mpa is shown in Figure 4. The data are from different samples including building stone, tight gas sandstone, gulf sandstone, P-sandstone and clean sandstones, and no trend similar to that in Figure 2 can be observed. Neither was it possible to obtain the transitional fines content from index data or the other laboratory results. The TFC for sand-silt, clay-sand, sand-gravel has been observed to be between 20-45% (Polito 1999; Yang 2004; Marion et al. 1992; Evans and Zhou 1995). The clay content by volume for the sandstones used in the present study is less than 51% and there are not enough data available at high fines content (Han et al. 1986). It was assumed that the transitional clay content for the sandstones is around 40%. Furthermore, data for the sandstones with clay content higher than 40% was not included in this study. The clay content by volume was used in the analysis considering the difference of the specific gravity of the grains is not much. The data were analysed using both void ratio and the corrected intergranular void ratio. Two data points for clean sandstone gave unexpected values and were not included.

Figures 5 to 10 present the results of the analysis as a function of the traditional void ratio and the corrected void ratio:
Figure 5: Compressional wave velocity
Figure 6: Shear wave velocity
Figure 7: Bulk modulus
Figure 8: Shear modulus
Figure 9: Compressional modulus
Figure 10: Compressional wave velocity (more data)

Figures 5 to 9 apply for a confining pressure of 30 MPa, Figure 10 for a confining pressure of 40 MPa. Regression analysis was done with a linear and an exponential function. Using the corrected void ratio, all the sandstone samples with different clay contents fall essentially on the same curve, both for compressional and shear wave velocity at confining pressures of 30 MPa and 40 MPa. The results of the best fit regressions as a function of the corrected void ratio are listed in Table 4. Especially for the acoustic properties, the correlation coefficient is high and the root mean square error is low. The uniqueness of the behaviour is somewhat less evident for the three different moduli, but the relationship is still quite convincing. The exponential relationships between compressional wave velocity, shear wave velocity, bulk modulus, shear modulus, compressional modulus and the corrected void ratio are better than those from linear relationships. The regression equations can be used to predict the acoustic properties of the sandstones when the corrected void ratio is known.

The above analysis fits to the data from Han et al. (1986) at the other confining pressures too, and the results are not given in this paper.

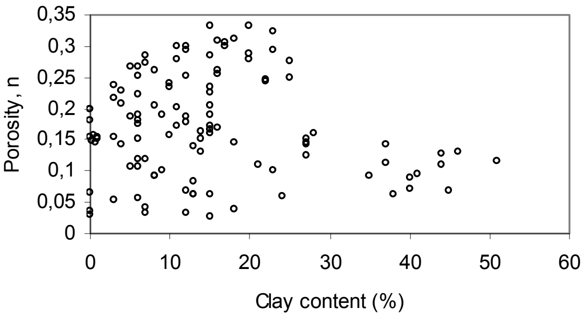
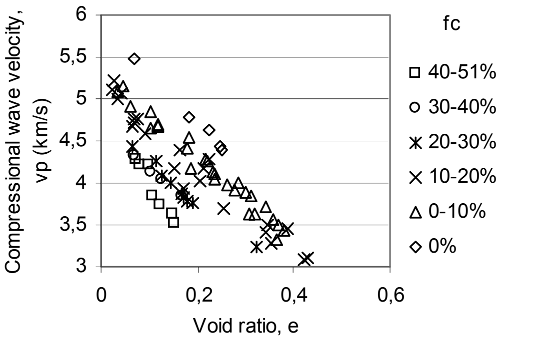


Figure 4 Porosity versus clay content in sandstones (Han et al. 1986 and klimentos and McCann 1990).

(a)



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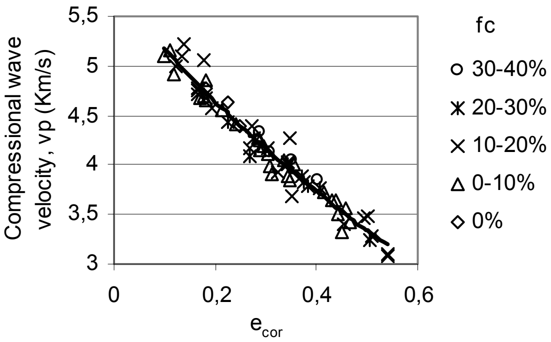
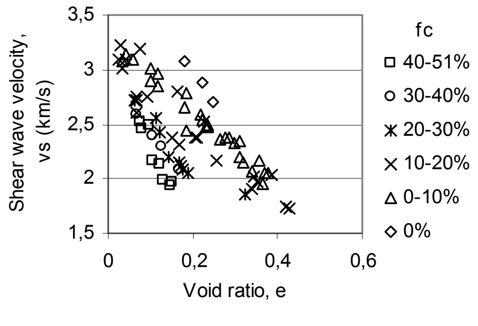


Figure 5 Compressional wave velocity versus void ratio (a) and corrected void ratio (b) for different fines content fc, confining pressure 30 MPa (data from Han et al. 1986)

(a)



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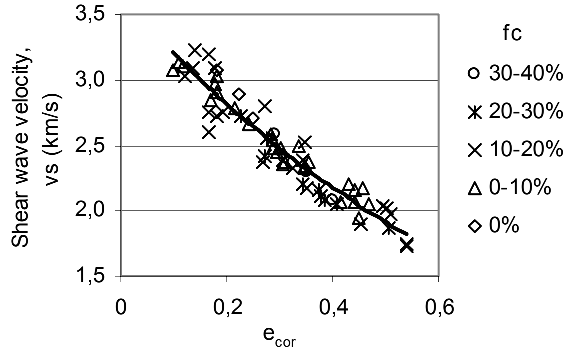
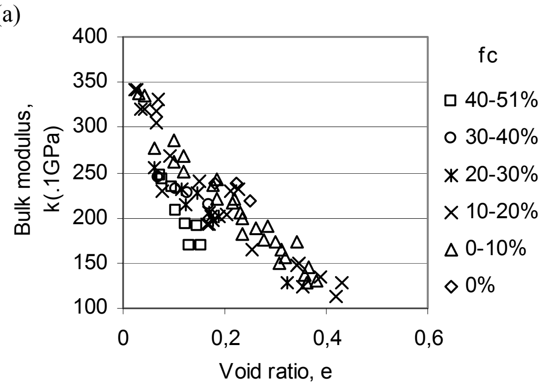


Figure 6 Shear wave velocity versus void ratio (a) and corrected void ratio (b) for different fines content fc, confining pressure 30 MPa (data from Han et al. 1986)

(a)



(b)

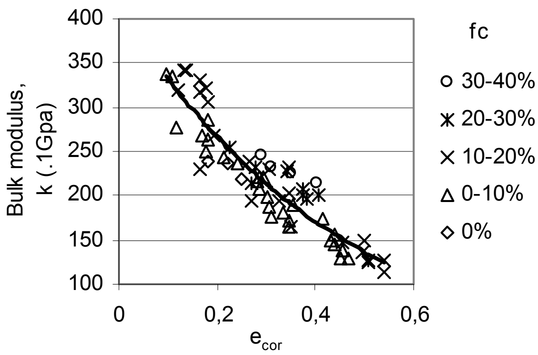
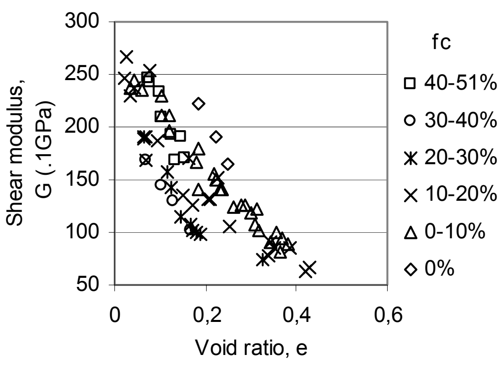


Figure 7 Bulk modulus versus void ratio (a) and corrected void ratio (b) for different fines content fc, confining pressure 30 MPa (data from Han et al. 1986)

(a)



(b)

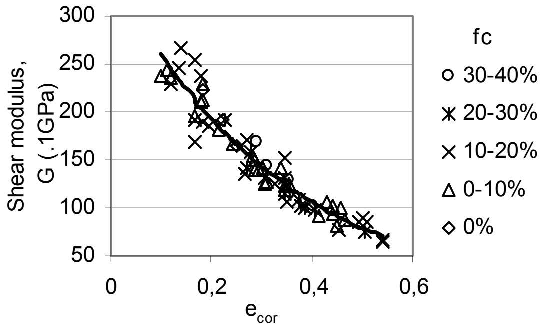
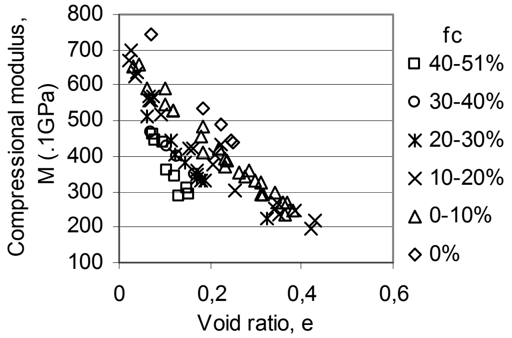


Figure 8 Shear modulus versus void ratio (a) and corrected void ratio (b) for different fines content fc,confining pressure 30 MPa (data from Han et al. 1986)

(a)



(b)

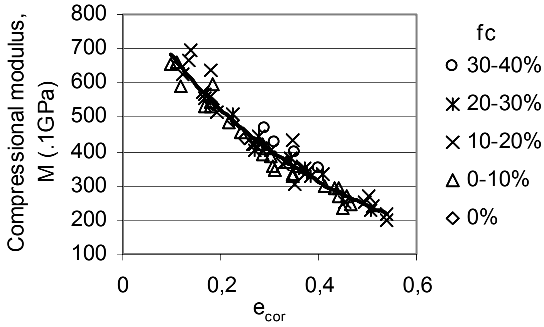
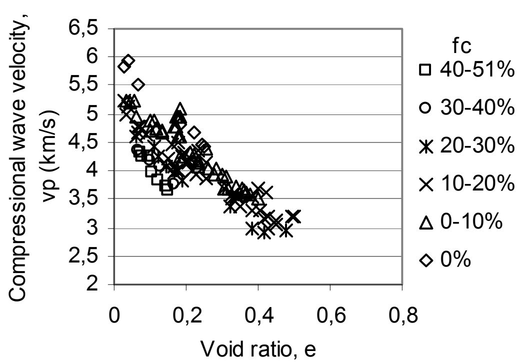


Figure 9 Compressional modulus versus void ratio (a) and corrected void ratio (b) for different fines content fc, confining pressure 30 MPa (data from Han et al. 1986)

(a)



(b)

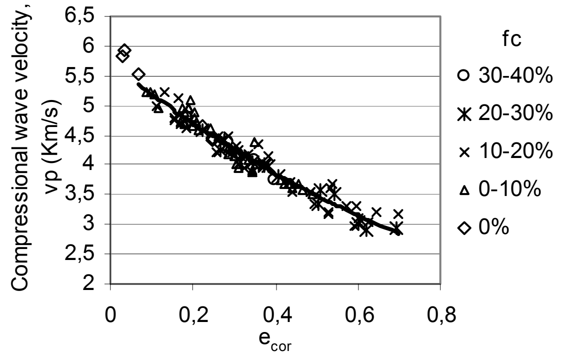


Figure 10 Compressional wave velocity versus void ratio (a) and corrected void ratio (b) for different fines content fc, confining pressure 40 MPa (data from Han et al. 1986 and Klimentos and McCann 1990)

Table 4 Regression equations for compression, shear wave velocities, bulk, shear and compressional moduli

Regression equation	Correlation coefficient	Number of samples	rmse
Linear regression			
$Vp=5.54-4.51\ e_{cor}$	0.953	65	0.120
$Vp^*=5.48-3.97e_{cor}$	0.935	110	0.156
$Vs=3.44-3.16e_{cor}$	0.903	65	0.124
$K=362.4-472.5e_{cor}$	0.843	65	24.3
$G=279.4-427.0e_{cor}$	0.895	65	17.5
$M=735.1-1043\ e_{cor}$	0.925	65	35.4
Exponential regression			
$Vp=5.77\exp(-1.09\ e_{cor})$	0.955	65	0.117
$Vp^*=5.74\exp(-1.00\ e_{cor})$	0.947	110	0.140
$Vs=3.64\exp(-1.29\ e_{cor})$	0.909	65	0.120
$K=416.3\exp(-2.23\ e_{cor})$	0.855	65	23.4
$G=349.2\exp(-2.98\ e_{cor})$	0.921	65	15.2
$M=878.5\exp(-2.59\ e_{cor})$	0.946	65	30.1

Where

Vp	Compressional wave velocity
Vs	Shear wave velocity
G	Shear modulus, $G=\rho Vs^2$
M	Compressional modulus, $M==\rho Vp^2$, ρ, density
K	Bulk modulus, $K=\rho\ (Vp^2-4Vs^2/3)$, ρ, density
Vp*	Vp based on Figure 10,
rmse	Root mean square error
e_{cor}	Corrected void ratio, referring to the corrected intergranular void ratio

6 DISCUSSION AND CONCLUSIONS

Petrophysical and geotechnical models to predict the porosity or void ratio of sand-clay or sand-silt mixtures were presented in this paper. They are based on an assumption of ideal binary packings. The geotechnical model was used successfully to predict the geophysical properties of sandstones with different fines content. The analysis indicated that for a given corrected void ratio, sandstones with different clay content show the same compressional and shear wave velocity as clean sandstone. This study suggests a good potential to use this model for interpreting geophysical properties of sediments.

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